INTERMEDIATE PHYSICS

VOLUME I

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PREFACE

"Outlines of Intermediate Physics" was originally written by Prof S. C. Ray Chowdhury and the book was so well received by the readers that it had six editions to its credit in a few years. The same book has now been very thoroughly and carefully revised by Prof. D. B. Sinha and is being presented to the readers with the name "Intermediate Physics" under the joint-authorship of both. It has been split up into two volumes for the convenience of readers, but each volume is complete in itself, Vol. I embodying General Properties of Matter, Heat and Sound, while Vol. II embodying Light, Magnetism, Electricity and Aeronautics. In former editions of the book, occasional instructions in connection with important experiments followed by actual data were given for the guidance of the students in the practical class. In a theoretical text-book the scope for such instructions is bound to be ultimately very limited whereas the necessity for such guidance has assumed a much wider importance now-a-days with the introduction of the system of university examinations in Practical Physics. Many text-books on Practical Physics are already available in the market to meet these requirements and so these instructions have been mostly omitted in this edition,

It is customary that in the preface the authors should draw the attention of readers to the special features of the book, which according to their opinion distinguish it from contemporary publications. Firstly, in modern text-books on light the new convention of signs viz. *all real distances are positive and all virtual distances negative*, is used in connection with optical calculations. This system of signs is less artificial than the older system and so in Art. 67(b), Vol. II, we have given an account of these new conventions illustrating the relative advantages in their application. The students can make use of them, though in the book the old system has still been retained. Secondly, in Electro-statics and Current electricity, all

nations of electric phenemena have been sought to be given on basis of the modern electronic theory and this will surely help produce in the minds of the students continuity of ideas all throughout. Thirdly, we have thought it fit to add a chapter on nentary Aeronautics (Appendix A), because it has proved to be a very useful branch of Physics and is already included in the I.Sc. syllabus of the Patna University. Fourthly as the study of Physics requires some knowledge of Trigonometry from the very beginning, some elementary relations of Trigonometry have been treated in Appendix B. Fifthly, a large number of sums have been selected from a very wide field of applications and they have been worked out so that they may serve as guides to the students.

In preparing the book in its present form, very considerable changes have been made throughout the old text but we will record here the more important additions to topics only, leaving out the rest. The new topics added, in Vol. I, are : the Watts Governor, S. H. M. of pendulum, Shear strain, Filter pump (in Part 1), Equation for progressive waves, Hydrophone, Mathematical theory of beats, Demonstration of accoustical interference, Tempered scale and the Theory of transverse waves in a string (in Part III), while in Vol. II, are : Annular eclipse of the sun, Hartle's Optical disc, Refractive index of a solid by total reflection, Mathematical deduction of the condition of minimum deviation, Huygens' principle (in Part IV). Coulomb's torsion balance, Equipotential surface, Parallel plate condenser with compound di-electric, the Voss machine, the Van de Graaff generator (in Part VI), modern Electronic theory of the voltaic cell including Kelvin's theory of electric double layer together with the Theory of electrolytic dissociation, Edison cell, Cosmic rays, etc. (in Part VII). Out of 615 old blocks 183 have been replaced by newly designed ones and 66 additional blocks have been used in the two volumes for properly illustrating the subjects dealt with in the book.

It is quite possibe that some errors or misprints should occur in the book hurried as it has been through the press, notices for

which will be thankfully received by the authors and attended to in the next edition The author will deem their labours amply repaid if the book in its present form proves to be of greater service to the readers.

Calcutta
July, 1948.

S. C Ray Chowdhury,
D. B. Sinha.

Preface to the Second Edition

We could little anticipate that the reorientated first edition of the book would be exhausted in the course of a few months only. This has been possible due to our patrons and collegues in the various colleges of India, Pakistan, Burma and Nepal. We are glad to say that we have fully utilised the opportunity of this second edition to revise the book as thorough as possible. In this connection we must thank those esteemed colleagues who gave us valuable suggestions during this revision work.

The topics on 'Lambert's Cosine Law, value of velocity of light, the earth as a magnet, support of molecular theory of magnetism, maps of isogonic and isoclinic lines, tubes of force, intensity in terms of lines or tubes of force, rotation of magnet round a current, mercury arc lamp, electric lamps and their progress, Coolidge tube' are important new additions. Considerable changes have been made in the writing of the topics on 'intensity of illumination and illuminating power, band spectra, dispersive power, Fizeau's method of determination of velocity of light, para and diamagnetic bodies, action of a telephone circuit and wireless telephone.

Ninety four old blocks have been replaced and *twenty two* new blocks have been added Index has been provided at the end of the text. These changes have resulted in increasing the volume of the book but the price of the book has been retained as before considering the hardship of the students.

Calcutta,
July, 1949.

S. C. Ray Chowdhury,
D. B. Sinha.

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ABBREVIATIONS

The following abbreviations have been used for examination questions.

C. U.—Calcutta University
C. P.—Central Province University
Pat.—Patna University
Dac.—Dacca University
All.—Allahabad University
P. U.—Punjab University
Mysore—Mysore University
U. P. B.—United Province Board
L. U.—London University
L. M.—London Matric
S. C.—School Certificate (Oxford and Cambridge)
C. L.—Cambridge Local
O. L.—Oxford Local
C. M. B.—Cambridge Medical Board
M. U.—Madras University
Utkal—Utkal University

PART IV

LIGHT

CHAPTER I

Nature, Propagation, Photometry

1. Light.—Our knowledge of the various objects in this world and the diverse phenomena that occur in nature is derived from the impressions they produce upon our different senses. By the sense of touch we can know the shape of an object, by the sense of hearing we percieve the sound of a body and similarly, by the sense of sight we see objects around us, though there is no visible link between the eye and the objects seen.

The objects we see send something to our eyes which creates the sensation of vision. This something which enables us to see objects is called *Light.*

Light is, therefore, the cause of the sensation of sight. It is a form of **radiant energy** to which our eyes are sensitive. The sun is the source of all natural light which it emits because it is hot. Almost all the sources of artificial light are also at high temperature.

The branch of Physics which deals with the phenomena of light is known as *Optics.* The subject of Optics is divided into two parts.—

(a) **Geometrical Optics.**—Here the formation of images is dealt with in accordance with certain observed laws purely by geometrical methods without entering upon any theory regarding the nature of light.

(b) **Physical Optics.**—It deals with the theories as to the nature and propagation of light and explains the experimental laws with the help of those theories.

2 (i). Light is energy.—*Light is a form of energy.* Like all other forms of energy it is also indestructable and can undergo transformations into different forms of energy, such as mechanical. chemical, electrical etc. Like any other form of energy, it also can be derived from these forms.

(*a*) *Light energy and Mechanical energy.*—The production of fire by the rubbing of two pieces of dry wood, or by pressing a piece of iron

against a rotating stone wheel, is an example of how mechanical energy can be first converted into heat energy which is then transformed into light energy. The reverse process, where light is transformed into mechanical energy, can be found in the fact that when light falls on a surface it exerts pressure on the latter. In 1900 Lebedew actually made a thin vane rotate by this mechanical pressure.

(*b*) *Light energy and Chemical energy.*—The burning of a candle, of a gas in case of an incandescent gas lamp etc., are examples where the substances combine chemically with the oxygen of the air, and the energy of the chemical combination is transformed into heat and light.

The reverse phenomenon, when light energy is transformed into chemical energy, occurs in ordinary photographic plates where chemical decomposition takes place of certain silver salts, when exposed to light. The changes so produced are then *developed* by chemical treatment to reveal the picture.

(*c*) *Light energy and Electrical energy.*—Modern electrical glow lamps are examples which illustrate how electric energy is transformed first into heat and then into light energy.

Light energy can also be converted into electric energy. It has been found that electrons are readily emitted by certain metals, e. g. potassium, cæsium, etc., when light falls on them. This property of the above metals has been utilised in the construction of photo-cells, which are extensively used in modern 'talkies' and 'television' (see Ch. X, Part VII, for the production of electric current).

2 (ii). **Light is invisible.**—Light is a form of energy and is itself invisible. It makes the things visible to us. Light as such is not seen, but the objects around are seen with the help of light. The track of a beam of sun-light entering through any slit into a dust-free room can not be visualised but when dust particles are scattered in the path of the beam, the track becomes noticeable. The light falls on the particles, which are then seen by the light scattered by them.

3. Definitions.—

Ray.—A *ray* of light is the path along which the light travels. Rays are invisible. They travel straight in the same medium and are represented by straight lines with arrow-heads which indicate the direction of travel.

Beam : Pencil.—A collection of rays along a definite direction is called a *beam.* A narrow beam is called a *pencil* of light. A ray is single and a beam is a bundle of rays.

A beam of light may be (a) parallel, (b) convergent, or (c) divergent, (Fig. 1).

Parallel beam.—A beam is said to be parallel when the rays making the beam are parallel to each other, *i.e.* when the rays do not meet each other if produced backwards or forwards. When light comes from a point source placed at a very large distance, the rays within a limited space are sensibly parallel. For this reason, the sun's rays are taken to be parallel.

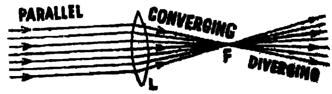


Fig. 1

Convergent beam.—When the rays making a beam are directed towards a point, *i.e.* when the mutual distance between the rays decreases gradually and progressively till they meet, the beam is said to be convergent.

Divergent beam—When the rays making a beam originate from a point and diverge out, *i. e.* when the mutual distance between the rays increases gradually and progressively as the light travels forward, the beam is divergent. The rays from any point of a source of light spread out in all directions in the form of a divergent pencil.

A substance, or any portion of space through which light can pass, is called an **optical medium.**

A medium is called **isotropic or homogeneous** when it has the same property, i. e. composition, density, etc., throughout ; and a medium having different properties at different points is called **heterogeneous.**

Luminous and Non-luminous Bodies—*Luminous bodies* are those which emit light of themselves, such as the sun, the burning lamp ; and *non-luminous* bodies, like wood, stone, etc., are those which do not themselves emit light, but which become illuminated by the light of other luminous bodies and serve as objects

Non-luminous bodies may be transparent, translucent, or opaque.

Transparent bodies are those through which light can pass with only negligible absorption and, as such, an object can be seen distinctly through them, *e. g.* glass, water, air, etc.

Translucent bodies are those which allow a part of the light incident on them to pass through them and scatter the rest. Through them vision is possible but it is not so distinct as in the case of transparent bodies. Greased paper, ground glass, paraffin wax, etc. are translucent.

Opaque bodies are those which do not allow any light to pass

through them, and through which no object can be seen, *e. g.* stone, wood, etc.

The difference between transparent and opaque besides is often a question of thickness. Thus a thin leaf of silver can transmit some light but a thick slab of silver is opaque. Though water is transparent, an object at a great depth of water cannot be seen.

4. Light travels in straight lines.—In a homogeneous transparent medium light travels in straight lines. In other words, propagation of light is rectilinear.

Expt.—Take two card-board screens A and B (Fig. 2) and make a small pin-hole in each. Place a candle flame C and the two screens in such a way that the two holes A and B and the middle of the flame are in the same straight line Now, placing the eye E behind the screen B in the same straight line, the flame will be visible. If, however, any of the screens is displaced, the light is immediately cut off.

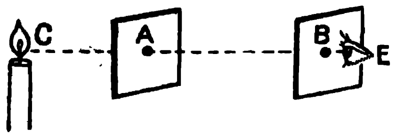


Fig. 2

The most common illustrations of the rectilinear propagation of light are, (1) the formation of shadows, and (2) the formation of inverted images by a pin-pole.

4(a). Shadow.—

When an opaque body is placed in front of a source of light, the rays in that direction are intercepted and consequently the space behind it, where light cannot penetrate, is in darkness. The boundary of this darkness formed on a screen held behind the body is a geometrical form and is called the *shadow of the body.* Thus a shadow is a consequence of the recitilinear propagation of light. Formation of shadows is possible because light can not bend round the corners of an opaque body, as *sound* can.

Different Cases of Shadows :—

(i) **Point Source and Extended Obstacle :**—Let L be a point source in front of which a spherical opaque object AB is a placed and S, a screen held behind AB. Join L to A and B, the extremities of a diameter of the body and produce the straight lines LA and LB up to the screen at A' and B'. If light travels in straight lines in an isotropic medium (*e.g*, air) the cone of light ALB will be intercepted by the

body AB. The projection of this cone on the screen S will be a circular plate of darkness $A'B'$ which, therefore, is the shadow of AB. The shadow, in this case, is a diverging one.

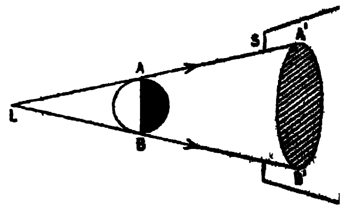


Fig. 3

(ii) **Extended Source Smaller than the Obstacle :**—Let S be a source smaller than the opaque spherical object G (obstacle) behind which the screen DA is placed [Fig. 4 (a)]. The extended source S may be regarded as a collection or assembly of point sources only. Two extreme points (at the ends of a diameter) are taken on the source S, corresponding to each one of which a cone of light is stopped by the obstacle G. Thus the area limited by C and A on the screen does not receive any light corresponding to one of the points referred to above and again the area limited by D and B does not get any light from the other point.

The area limited between D and A, generally known as the shadow, is not uniform in character everywhere. The nature of the shadow will be a circular plate of complete darkness surrounded by a less dark ring of darkness. The area limited between C and B does not receive any light from either of the two extreme points of the source, *i.e.* it does not receive any light from any point of the source at all and is completely dark This region is called the **umbra** of the shadow. No part of the source canbe seen from any point within this cone. The area limited between D and C, or between B and A, is under a different condition. Such areas receive light from some parts of the source but do not get light from the other parts.

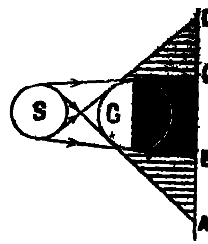


Fig. 4(a)

These areas are, therefore, partially dark or partially lighted. From such regions, some parts of the source can be seen while the other parts are invisible. Such regions are called **penumbra**. The umbra and the penumbra in this case both increase or decrease as the distance of the screen from the obstacle increases or decreases, *i. e.* they are diverging in character.

Umbra and Penumbra :—Umbra is that portion of the shadow of an object which is completely dark and from which no part of the source of light is visible. Penumbra is that portion of the shadow which is only partially dark and from which some parts of the source are visible.

(iii) Extended Source Larger than the Obstacle :—Let S be the source, G the obstacle and DA the screen. By considering two extreme points on the source, it will be found that the two divergent cones of light from them, which are intercepted by the obstacle G, produce the umbra CB and the penumbra CD and BA. The penumbra is *divergent* but the umbra *convergent*. If the screen is shifted away from the obstacle, the umbra progressively reduces in size and finally when the screen is at the apex of the umbral cone, the umbra reduces to a point. After the crossing of the apex when the screen is at D_1 A_1, an area C_1B_1 is found which is not at all in shadow but is lighted. If an observer looks to the source from such a region, the middle portion of the source will not be seen due to the obstacle G but the peripherial portion (outer portion) of the source will be visible.

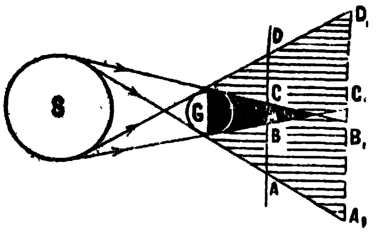


Fig. 4(b)

5. Eclipses of the Moon and the Sun.—

Both the lunar and the solar eclipses can be explained from the principle of the formation of shadows. These are, some *natural instances* which illustrate the truth of the recitilinear propagation of light. The sun is a very large luminous source round which the earth moves in its orbit with a definite periodic time while the moon moves round the earth, in an orbit inclined at an angle of about 5° degrees with respect to the orbit of the earth, in a definite period of its own.

Lunar Eclipse.—It occurs during *the full moon period, i.e.* when the earth lies between the sun and the moon (Fig. 5). As the sun is larger than the earth (which acts as an obstacle), umbra and penumbra will be produced as shown in Fig. 5, where the umbra is converging. The size of the earth is such that the apex of its umbral cone is always well beyond the moon's orbit. When the moon, in course of its rotation round the earth, passes completely into the umbra of the earth's shadow, the moon is totally out of view from the earth's surface and the phenomenon is called the total eclipse of the moon. If the moon is partly in

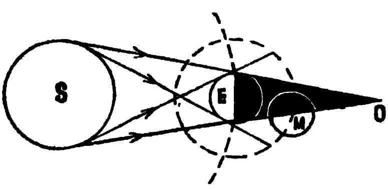


Fig. 5—Lunar Eclipse

the umbra and partly in the penumbra as is depicted in Fig. 5, the eclipse observed is partial.

The lunar eclipse does not occur at every full moon, because the shadow of the earth is not always formed on the moon, the orbit of the moon being inclined to that of the earth by about 5° degrees.

Solar Eclipse.—It occurs during the *new moon period*, *i.e.* when the moon lies between the sun and the earth [Fig. 5(a)]. The moon acts as an obstacle and as it is very small compared to the sun, the umbra produced is too much convergent and as a result, the umbral cone is cut by the surface of the earth at only a little earlier than the apex. A small portion CB of the earth's surface, by which the umbral cone is intercepted by the earth, is in complete darkness though facing the sun. To the people in this area, the sun will be completely out of view and for them, it will be a case of total eclipse of the sun. To the people of the penumbral zones like CD and BA, round the zone (BC) of total eclipse, the sun will be only partially in view and for them it will be a case of partial eclipse of the sun. The nature of the partial eclipse will depend on the position of the observer.

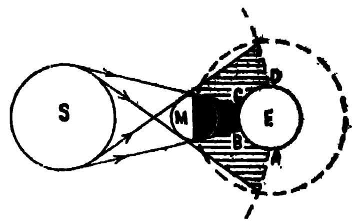


Fig. 5(*a*)—Solar Eclipse

The umbral and penumbral cones do not touch the earth's surface at every new moon, because (i) the plane of the moon's orbit is inclined to that of the earth by about 5°, and (ii) the distances of the earth from the sun and the moon vary and consequently the earth often goes too much beyond the umbral cone. So the solar eclipse does not occur at every *new moon*.

Annular Eclipse of the Sun.—If the earth is placed a little

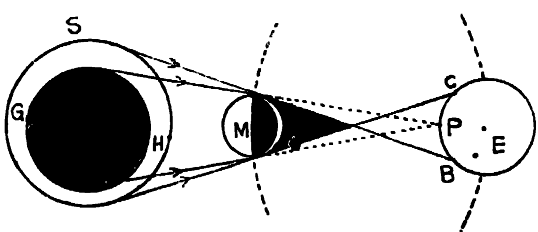


Fig. 6—Annular Eclipse of the Sun.

beyond the apex of the umbral cone of the moon during a *new moon period*, which of course is not a frequent event, an observer P (Fig. 6) lying

within the geometrical prolongation of the umbral cone limited by C and B, will see the outer ring of the sun's disc round the central part GH which will be in darkness due to intervention of the moon. Such a phenomenon is called the annular eclipse of the sun.

6. The Pin-Hole Camera.—This is another example of the rectilinear propagation of light. It consists of a rectangular box $EFGH$ (Fig. 7) having a hole O of the size of a pin-prick in the front wall and a ground glass screen FG at the back. The interior of the box is preferably painted black to avoid internal reflections. Consider a luminous object AB Light is emitted from every point of the object in all directions. Of this, a narraw pencil passes through the aperture O of the camera and produces on the ground glass screen placed at FG, a small patch of light B_1A_1, which is the image of the object. On the screen, A_1, is the image of a point A, and B_1 that of the point B, assuming light to travel in straight lines in the same medium. Thus an invertd image is obtained on the screen. The inversion of the image, which has been possible due to the rays proceeding straight and cutting each other at the pin-hole, establishes the truth of rectilinear propagation of light. The size of the image is directly proportional to the distance of the screen from the aperture ; for,

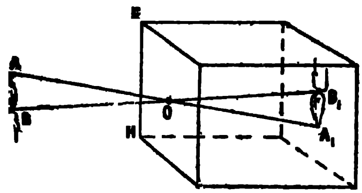


Fig. 7—Pin-Hole Camera.

$$\frac{AB}{A_1B_1} = \frac{\text{distance of object from the pin-hole}}{\text{distance of image from the pin-hole}}$$

Effect of Enlargement of the Hole.—By making the *aperture large*, a bright image is obtained, but it becomes blurred ; and if the aperture is made very large, then, instead of any image being formed on the screen, only a general illumination on a portion of the screen will be produced. For, a large aperture may be considered to be made of a number of small apertures due to each of which an image is formed on the screen ; by the overlaping of these images the boundary of the resultant image formed becomes blurred and no distinct image is visible.

A photograph may be taken with the help of a pin-hole camera, by substituting a photographic plate for the screen, but the size of the aperture being small, a much longer exposure is required than with the usual lens-camera. The similalarity of the photograph in every details of the picture with corresponding positions of the object is also a result of the rectilinear propagation of light.

7. **Radiation of Light**.—A body which is hotter than the medium surrounding it radiates energy into that medium. This energy is radiated in the form of heat only, if the temperature of the body is below that at which it becomes "red-hot". At the "red-hot" temperature the body emits *light* in addition to *heat*. At a higher temperature still, when the body becomes "white-hot", the light emitted gives the impression of *white-light*. When light energy is radiated by a *luminous* body, always some heat energy is also simultaneously emitted.

$$\text{The ratio } \frac{\text{energy radiated in the form of light}}{\text{total energy radiated by the body}}$$

is called the *radiant efficiency* of the body and it obviously depends upon the temperature of the body.

Definitions.—

Luminous flux (flux of light) is the *light energy radiated per second* from a luminous body. It is, therefore, a form of power.

The *illuminating power* (*intensity of emission*) in any given direction is the luminous flux radiated per unit solid angle in the direction in which the intensity is required.

If $Q=$ total quantity of light energy emitted by a body per second, supposed uniformly in all directions, the *luminous flux radiated per unit solid angle is* $\frac{Q}{4\pi}$ since the whole solid angle about any body is 4π. Again, if a hollow sphere of unit radius is drawn with the luminous body as centre, the amount of light that will fall per second on each unit area on the internal surface of the sphere will also be $\frac{Q}{4\pi.1^2}$, *i.e.* $\frac{Q}{4\pi}$ So the illuminating power may also be defined as follows.

The Illuminating Power of a source of light is given by the amount of light which falls per second on unit area of a surface placed at unit distance from the source, the rays falling perpendicularly on all points of the surface.

Unit of Illuminating Power.—

Candle-Power.—It is the unit of measurement of illuminating power. The illuminating power of a *standard candle* is said to be one candle-power (abbreviated as c. p.) The candle-power of a source, then, expresses how many times the illuminating power of a source is greater than that of a standard candle. When it is said that

an electric lamp is of 25 c. p., it means that it can produce the same illumination at a point as 25 standard candles placed in its place together can. *The Mean Horizontal Candle-Power* (M. H. C. P.) of a source of light is the average value of the candle-powers in all directions in a horizontal plane passing through the source of light.

The Mean Spherical Candle-Power (*M. S. C. P.*) of a source is the mean of the candle-powers in all directions from the source of light, and is the same as the candle-power of a source of light which radiates the same total flux uniformly in all directions.

Standards of Candle-power.—

The Standard Candle.—It is a candle made of *sperm-acetic* wax, $\frac{7}{8}$ inch in diameter, weighing one-sixth of a pound and burning at the rate of 120 grains per hour.

This was the old British standard for candle-power. The term "candle-power" is derived from this standard. This standard, however, has been found not to fulfil the requirements of a *true* standard, for its illuminating power varies, though slightly, with the factors such as pressure, temperature, humidity, CO_2-content of the air, the shape of the wick, etc.

Vernon Harcourt Pentane Lamp.—This lamp has superceded the old British standard and is now-a-days used as the British standard of candle-power. In it a flame is produced by burning the vapour of pentane oil, which is a light volatile oil derived from paraffin. There is no wick in this lamp. It has also been adopted as the *International Unit.* When constructed under the standard specifications and the flame properly adjusted, its illuminating power at the specified conditions of pressure, temperature and humidity is equal to 10 international units of candle-power. Therefore, one candle-power is *one-tenth of the illuminating power of a standard Vernon Harcourt Pentane Lamp.*

Hefner Standard Lamp:—This lamp in used as the German standard of candle-power. A specially constructed wick of untwisted cotton is used in this lamp and the fuel used is a pure grade of *amyl acetate.* When properly adjusted, the illuminating power of the flame reduced to standard conditions of pressure, humidity and CO_2-content, is one *Hefner Candle-power.* One Hefner candle-power is equal to 0·9 British candle-power.

The modern British standard lamp (Vernon Harcourt Pentane Lamp) has the advantages, as compared with the Hefner, of higher candle-power, better colour and greater steadiness of flame. It is also less affected, as regards candle-power, by the height of the flame. Its

disadvantages, are its lack of portability, complicated structure and bulk.

Lumen :—For measurement or comparison purposes, some standard of flux must be fixed up. This *standard unit of flux* is the Lumen, which is the "*luminous flux*" emitted in unit solid angle from a source of unit candle-power (situated at the apex of the solid angle), the radiation being uniform in all directions. That is, the total quantity of light emitted per second by a standard candle is 4π lumens. Thus it is also the same as the amount of light falling per second on unit area placed normal to the rays of light from a standard candle held at unit distance.

The Brightness of a source is the *luminous flux* emited per unit area of the surface of the source in a direction perpendicular to the surface.

8. Degree of Illumination.—

When a surface is illuminated by any source of light, the amount of light received by the surface depends on (*i*) the area of the surface ; (*ii*) the distance of the surface from the source ; the surface will be brighter if the source be nearer ; (*iii*) the inclination of the surface with respect to the rays of light ; (*iv*) the *illuminating power* of the source ; the surface will be brighter when held in front of an electric lamp instead of a candle, and (*v*) the nature of the medium.

The **Intensity of Illumination** (degree of illumination) at a point on a surface is the amount of light falling *per second* on unit area surrounding the point. This depends on the illuminating power of the source, the distance of the surface from the point, the inclination of the surface with respect to the rays and the nature of the medium

Evidently, the illuminating power of a source of light is numerically equal to the intensity of normal illumination produced at unit distance.

If I be the intensity of illumination, and Q, the amount of light falling per second upon any area A, then, $I = Q/A$.

Units of Intensity of illumination :—

In **British Units**, the illuminating power (P) is measured in candle-powers, distance (d) in feet and the intensity of illumination is expressed in *foot-candles*.

The **Foot-candle** is the measure of unit intensity of illumination. It is the intensity of illumination on the inside white surface of a hollow sphere of one foot radius having a standard candle burning at its centre.

Evidently, one ft.-candle is equal to one lumen per sq. ft. That is,-lumens = ft.-candles × area in sq. ft.

It may be noted that the intensity of the light due to the *Full-Moon* is approximately equal to one foot-candle, and that due to the *Sun* is 600,000 foot-candles.

In **C. G S. Units**, the illuminating power (P) is measured in candle-powers, and the distance *(d)* in cms., or metres; the intensity of illumination is then expressed in *cm.-candles* or *metre-candles*. 1 c.p. at a distance of 1 foot produces an intensity of illumination of 1 ft.-candle. So in the British system, the intensity at a distance of 1 foot from the standard candle is unity; at a distance of 10 feet from a lamp of 25 c.p., the intensity of illumination $= \frac{25}{10^2} = \frac{1}{4}$ foot-candle only. The best intensity of illumination for reading and writing purposes is about 3 to 6 ft.-candles.

8 (a). Laws of illumination :—

(i). The Law of Inverse Squares.—When illumination on a surface is due to normal incidence of rays, the variation of intensity takes place according to this law. Let S be a point source of light placed at the centre of a hollow sphere A of radius r_1 (Fig. 8). Each unit area of the sphere will be uniformly illuminated, as light is emitted equally in all directions. If there be another hollow sphere B of radius r_2 and with the same centre, then, in the absence of A, the same total quantity of light, which falls on the inside surface of the sphere A of area $4\pi r_1^2$, will fall on that of B of area $4\pi r_2^2$. Consider also a concentric sphere C (not shewn in the figure) of unit radius within the sphere A. If Q be the total quantity of light emitted by the source per second, then the quantity of light falling on unit area of C per second, *i. e* the intensity of illumination, I, at the inside surface of the sphere C, (which is equal to the illuminating-power P of the source S, since the inside surface of C is at unit distance from S) will be given by,

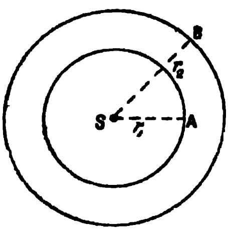


Fig. 8

$$I = \frac{Q}{4\pi \times 1^2} = \frac{Q}{4\pi} = P \text{.................}(1)$$

If I_1 and I_2 be the intensities at the inside surfaces of the spheres A and B respectively, we have,

$$\therefore I_1 = \frac{Q}{4\pi r_1^2}\text{ ; and } I_2 = \frac{Q}{4\pi r_2^2}\text{ ; whence } I_1 = \frac{P}{r_1^2} \text{ and } I_2 = \frac{P}{r_2^2}\text{, from (1).}$$

That is, *the intensity of illumination at a point*

$$= \frac{\textit{Illuminating power of the source}}{\textit{Square of the distance}}.$$

Or, the intensity of illumination at a point varies inversely as the square of the distance from the source. This is the law of inverse squares.

(ii) Lambert's Cosine Law for Oblique Illumination :—

This law states that, if a surface is inclined with respect to the direction of the *luminous flux*, so as to make the angle of incidence θ, then the degree of illumination of the surface is reduced from that given by the Inverse Square Law stated above in the ratio Cos θ : 1.

Let O [Fig 8(a)] be a point source of light and AB a small plane surface of area s placed such that the rays incident at the central part of the surface make an angle θ with the normal to the surface. The area being small, the angle of incidence θ may be taken to be the same for the whole of the surface. The total light incident on the surface AB will be limited by the cone AOB, whose normal section at B is, suppose, given by BA′. If q is the quantity of light passing through any section of this cone per second, the intensity I' on AB will be given by,

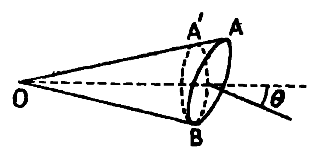


Fig. 8(a)

$I' = q/s$..............(1).

The intensity on the normal section will be given by,

$I = q/\text{area of section} = q/s \cos\theta$.............(2), $= I'/\cos\theta$.

$\therefore \quad I' = I \cos\theta$.

9. **Photometry** :—The measurement of illuminating power of any source of light by comparison with that of a standard source is called photometry (Gk. *photos*, of light and *metron*, a measure). Photometers are instruments by which the comparison is made.

Two methods are given below for comparing the illuminating powers of various sources. The *Lummer-Brodhun photometer* is used where greater accuracy is required for the measurement of illuminating power.

Optical Bench :—

It is a long and narrow bench, metallic or wooden, placed on stands having levelling screws by means of which the bed of the bench may be made truly horizontal. A scale is attached to the

bench or etched on it along its entire length. On the bench there are some sliding carriages each provided with a stand or upright which can hold an optical element such as a lens, mirror, slit or screen. Each carriage has on its front face a vertical mark, called **the Index mark,** which gives the position of the optical element the carriage holds and the position of the optical element on the bench is determined by the reading of this mark against the scale. Any error made in the fixing of this mark introduces an error, called the Index error, in the deermination of the position of the optical element concerned.

Index error, for the distance between two optical elements mounted on the stands, is determined as follows. A thin rod, called the *index rod,* whose length is accurately measured by means of a scale, is held between the two objects parallel to the bench such that the ends just touch the two objects. Thus the real distance, say b_1, between the objects at those positions is equal to the length of the rod. Read the positions of the two index marks on the carriages concerned against the scale of the bench and hence find out the apparent distance, say b_2, between the objects. So $(b_2 - b_1)$ is the index error and the correction to be applied will be, $(b_1 - b_2)$, to be added algebraically to the apparent distance. This error and so the correction will remain constant even if the distance between the objects is changed, provided the objects always remain normal to the bench.

(a) **Rumford's Photometer.**—In this photometer an opaque rod C is vertically fixed in front of a ground glass screen S (Fig. 9). Two lights A and B whose illuminating powers are to be compared are placed in front of the rod, and their distances from the screen are so

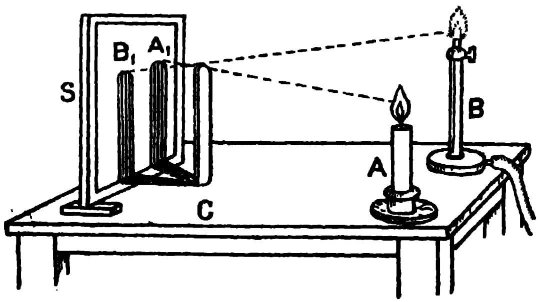


Fig. 9—Rumford's Photometer

adjusted that two shadows A_1, B_1, are formed side by side on the screen corresponding to them and are equally dark. Both the shadows being equally intense, the intensity of illumination on the screen (*i.e.* in the position of the shadows) due to both the sources, is the same.

The shadow cast by a rod which cuts off rays from one source, is illuminated by the other source, that is, the shadow cast by the candle A is illuminated by the gas lamp B, while the candle illuminates the shadow cast by the gas lamp.

If P_1 be the illuminating power of the candle and d_1 be its distance from the shadow cast by the gas lamp, and P_2 be the illuminating power of the gas lamp and d_2 be its distance from the shadow cast by the candle, then the intensity of illumination on the screen due to the candle $=P_1/d_1^2$, and that due to the gas lamp $=P_2/d_2^2$.

Since the intensities are equal, we have, $\frac{P_1}{d_1^2}=\frac{P_2}{d_2^2}$; or $\frac{P_1}{P_2}=\frac{d_1^2}{d_2^2}$.

That is, the illuminating powers of the two sources of light are directly proportional to the squares of their distances from the screen at the position of balance.

(b) Bunsen's Grease-spot Photometer.—In this photometer (Fig. 10), on an optical bench (EF) the two sourcess of light, A and B, are placed on opposite sides of a white paper screen C. The screen C, called the photometer head consists of a piece of white paper with a spot of grease or oil in the middle. The height of the two sources of light and the greased spot are so adjusted that their centres are on the same horizontal line. The grease-spot being more translucent than the rest of the paper allows more light to pass through it, and so, if the screen is illuminated by one of the sources only and observed from the same side, *the spot appears darker* than the rest of the paper, as the amount of the light incident on the spot transmits more and

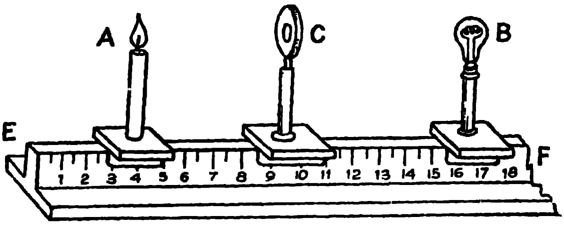


Fig. 10.—Bunsen's Photometer.

sends back less to the eye by reflection than in case of the rest of the paper. When looked at from the other side, *i.e.* by transmitted light, *the spot appears brighter* than the rest. The same thing happens in case the other source in used. If now the screen is illuminated by both the sources, one from each side, and the distances of the sources from the grease-spot are so adjusted that the spot is equally bright

on either side, *i.e.*. both sides look alike, the intensities of illumination on the screen due to both the sources are equal. If d_1 and d_2 be the distances of the two sources (having illuminating powers P_1 and P_2) from the screen,

we have, $$\frac{P_1}{P_2}=\frac{d_1^2}{d_2^2}.$$

[When the intensities of illumination on the two surfaces of the spot are equal, each surface will lose by transmission to the other side and gain by transmission from the other surface such quantities of light as will be identical in case of both the surfaces. So the condition of equal intensity will be the condition of equal brightness of the two surfaces. At this condition of equal brightness of the two surfaces of the spot each surface will, however, be less bright than the rest of the white paper of the screen. For, considering any one face, the light incident on the rest of the paper is almost wholly reflected; whereas, of the light incident on the spotted portion, the loss due to transmission to the other side will not be wholly compensated for by the light gained by transmission from the other side, because an appreciable portion of this transmitted light is absorbed by the grease or oil. So, for each face of the screen, the quantity of light reaching the eye of the observer from the greese-spot will be less than that from the rest of the white paper. That is why, the greese-spot will be less bright than the rest of the paper.

In otherwords, the position of the photometer head for equal illumination on either side will really be the case of *equal contrast in illumination* between the opaque and transparent portions of the screen].

Improved Photometer Head.—

In Bunsen's photometer the screen with the greased spot is often provided with two plane mirrors inclined at equal angles so that an observer may see both the faces of the greased spot at the same time by reflection from the two mirrors as in Fig. 11, each face of the screen being looked at by one eye. This arrangement helps the observer in making his judgement of equal illumination by observing both the surfaces simultaneously without movement of the eye.

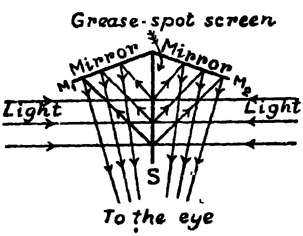


Fig. 11

To Verify the Law of Inverse Squares with the help of the Bunsen's photometer, take four candles (arranged close together) on one side and one candle on the other. Proceeding as suggested in the

above experiment, it will be found that the row of four candles is twice as far from the grease-spot as the single candle is.

Then, we have, $\frac{P_1}{P_2} = 4 = \frac{d_1^2}{d_2^2}$. $\frac{d_1}{d_2} =$

This shows that four candles must be placed at twice the distance of a single candle to give the same illumination ; and thus the law is verified.

Mathematical Treatment of the Bunsen's Photometer.—Let q_1, q_2 be the quantities of light falling per second on unit area of the screen C (Fig. 10) from the sources A and B respectively. Suppose a is the fraction of unit quantity of incident light, diffusively reflected from the greased portions of the screen. Then $(1-a)$ will represent the fraction that should be transmitted. But, owing to the partial opacity of the spot, exactly this fraction will not be transmitted. Let $K(1-a)$ be transmitted, where K is a constant, less than unity, depending on the power of absorption of light by greese.

Hence, the total quantity of light reaching the eye, on the side of the source A, from unit area of the greased surface $= q_1 \times a$ by reflection $+ q_2 K(1-a)$, by transmission. Similarly, the quantity of light reaching the eye on the other side $= q_2 \times a + q_1 K(1-a)$.

$\therefore$ At the position of balance, we must have,

$$q_1 a + q_2 K(1-a) = q_2 a + q_1 K(1-a).$$

$$\therefore \quad q_1\{a - K(1-a)\} = q_2\{a - K(1-a)\} ; \quad \therefore \quad q_1 = q_2.$$

Considering the ungreased portion of the screen, the same result may also be arrived at. The only difference in the two cases will be in the values of the constants, a, K, and the total quantity of light reaching the eye on either side of the screen in the latter case will be greater. Therefore, at the position of balance, q_1 being equal to q_2, we have,

$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2}$ where are the illuminating powers of the lamps and d_1, d_2 the respective distances of the lamps from the photometer head at the position of balance.

10 Notes on Photometers.—In *Rumford's photometer* only one side of the screen is used for comparison, and so scattered light, coming from sources of light other than those compared, affect both the shadows equally. Therefore, in this case, *a dark room is not so much necessary.*

In *Bunsen's Photometer*, opposite sides of the screen are optically matched and these two sides are not equally affected by scattered light from other sources. So in this case *a dark room is necessary.*

When comparing the illuminating powers of two lights by any of the photometers, it is assumed that light is radiated uniformly in all directions. This assumption, however, is not quite correct.

Most sources of light differ greatly in colour, in consequence of which it becomes difficult with all types of photometers to compare the shadows produced by them. For instance, a candle or gas flame gives yellowish light, and a metal filament is rich in bluish light. Due to this, the shadow cast by the candle will appear different from that cast by the filament lamp even when the intensities are matched. For this reason, in photometer experiments, the light to be compared should be of the same colour.

When the lamps to be compared give lights of the same, or similar, colour, the Bunsen photometer gives good results, but if the colours of the two lights differ appreciably, a "*Flicker photometer*" probably gives better results.

10 (a). Determination of Percentage of Light transmitted by a Glass Plate—A Bunsen's photometer is set up as usual between any two sources of light and the distances between the sources are adjusted until their illuminations are equal. The glass plate is then placed between the photometer and any of the two sources. Evidently the intensity of illumination (as seen on the photometer screen) due to the source of light near the plate is diminished, and so it will be necessary to move this source towards the photometer inorder to make the illuminations again equal.

If d_1 be the initial distance of this source from the photometer and d_2 the final distance when the glass plate is interposed, the intensity of illumination in the beginning is P/d_1^2, where P is the illuminating power of the source, while the final value is xP/d_1^2, where x is the fraction of light transmitted by the glass plate. The other source being at the same place as before, these illuminations are equal, that is,

$$\text{we have } \frac{P}{d_1^2} = \frac{xP}{d_2^2}\,; \quad \text{or} \quad x = \frac{d_2^2}{d_1^2}.$$

Hence the percentage of light transmitted by the plate is $100x = 100d_2^2/d_1^2$.

Examples.—1. *Two electric lamps of 64 and 16 candle-power respectively are placed 200 cms. apart. Where would you place a screen on the line joining them in order that it may be equally illuminated by each of them ?* (*Pat. 1929*).

Let x cm. be the distance of the screen from the 16 c. p. lamp. Then its distance from the 64 c. p. lamp is $(200-x)$ cms. Now, for equal illumination on the screen, we have, $\frac{16}{x^2}=\frac{64}{(200-x)^2}$; whence $3x^2+400x-40000=0$;

i.e. $x=66{\cdot}6$ cms., *or* -200 cms.

This means that there are two possible positions for the screen ; (1) 66·6 cms. from the 16 c.p. lamp., and $(200-66{\cdot}6)$ cms. or 133·4 cms. from the 64 c.p. lamp ; (2) 200 cms. to the left of the 16 c. p. lamp on the line joining the two, and 400 cms. from the 64 c. p. lamp. The former corresponds to the Bunsen's photometer while the latter to Rumford's photometer.

2. In a grease-spot (Bunsen) photometer, light from a lamp with a dirty chimney is exactly balanced by that of a candle distant 10 cms. from the spot. When the chimney is cleaned the candle has to be shifted by 2 cms. to obtain a balance. Calculate the percentage of light absorbed by the dirty chimney.

(Pat. 1931 ; cf. All. 1924).

Let P_1 be the illuminating power of the lamp with the dirty chimney, P_2 that with the clean chimney, and d the distance of the lamp from the grease-spot. Then, we have, $P_1/d_1{}^2=1/10^2$ ... ... (2)

Next, the distance of the candle becomes $(10-2)$ cms. when the chimney is cleaned. Hence, we have, $P_2/d_1{}^2=1/8^2$ ... ... (2)

From (1) and (2), $\frac{P_1}{P_2}=\frac{8^2}{10^2}=\frac{64}{100}$; $\therefore P_1=\frac{64}{100}P_2$.

Hence the dirty chimney absorbs $(1-\frac{64}{100})$ or $\frac{36}{100}$ of the total light. So the percentage of the light absorbed $=\frac{36}{100}\times 100=36\%$.

3. A photographic print is found to be satisfactory when the exposure was for 15 seconds at a distance of 2 feet from a 16 candle-power lamp. At what distance must the same paper be held from a 32 candle-power lamp in order that an exposure of 20 seconds will give the same result ? *(Pat. 1928 ; Del. U. 1938).*

The amount of light falling perpendicularly upon unit area of the photographic plate in one second, *i.e.* the intensity of illumination at a distance of 2 feet from the 16 c. p. lamp $=16/2^2$; hence the amount of light falling in 15 seconds $=16/2^2\times 15$.

Again, if the paper be held at a distance d from the 32 c.p. lamp, the amount falling in 20 seconds $=32/d^2\times 20$.
In order that the result in both the cases be equal, $32/d^2\times 20=16/2^2\times 15=60$.

or $d^2=\frac{32\times 20}{60}=\frac{32}{3}$; whence $d=3{\cdot}23$ ft.

4. A book is to be read from a distance of 5 ft. from an electric lamp, for which the best intensity of illumination is taken to be 5 ft-candles. Assuming 20% of the light reaching the book does so by reflection at the ceiling and walls, calculate the candle-power of the lamp which should be used for the purpose.

Direct rays from the lamp reaching the book provide 80% of the light, whose intensity of illumination $=\frac{5\times 80}{100}$ ft.-candles. $=4$ ft.-candles.

If x be the candle-power of the lamp required, the intensity of illumination at a distance of 5 ft. $= \frac{x}{5^2}$ ft.-candles. $\therefore$ $\frac{x}{5^2} = 4$. That is, $x = 100$ c.p.

Hence, a 100 c. p. lamp will be required.

5. *A standard candle and a gas flame are placed 6 ft. apart, the gas flame being of 4 candle-power. Where must a screen be placed on the line joining the candle and gas flame so that it may be equally illuminated ?* (*C. U. 1931*)

Let d ft. be the distance of the screen from the standard candle and $(6-d)$ ft. the distance from the 4 candle-power source. For equal illumination on the screen due to both sources, we have $\frac{1}{d^2} = \frac{4}{(6-d)^2}$;

whence, $d^2 + 4d - 12 = 0$; *i. e.* $d = 2$ or -6.

This shows that there are two possible positions for the screen ; (1) 2 ft. from the standard candle and 4 ft. from the gas flame ; (2) 6 ft. to the left of the standard candle on the line joining the two and 12 ft. from the gas flame.

6. *In a Rumford's Photometer the screen is at distances of 100 cms. and 64 cms. respectively from a gas burner and a candle when the illumination due to the gas burner is matched to the extent of a quarter only by the illumination due to the candle. Find the candle-power of the gas burner.* (*Pat. 1930*)

The intensity of illumination on the screen due to the candle $= 1/64^2$, and that due to the gas burner $= G/100^2$, where the candle is of 1 c. p and the gas burner is of G. c. p.

According to the condition given in the example, $\frac{1}{4}\left(\frac{G}{100^2}\right) = \frac{1}{64^2}$;

or, $G = \frac{4 \times 100 \times 100}{64 \times 64} = 9{\cdot}7$ c.p. That is, the gas burner is of 9·7 candle power.

Questions.

Art. 4.

1. Distinguish between umbra and penumbra. Indicate the formation of umbra and penumbra due to a spherical obstacle, when the source of light is a luminous sphere, (*a*) when the latter is larger than the obstacle, (*b*) when it is smaller, (*c*) when the spheres are equal. (C. U. 1918, '29)

2. How would you demonstrate experimentally that light travels in straight lines. (Dac. 1923 ; C. U. 1948).

Art. 5.

3. Give a general explanation of the eclipses of the sun and the moon. (Pat. 1925, '27 ; C. U. 1937)

4. Define Umbra and Penumbra. The diameter of the sun being taken as 9×10^5 miles, and its distance from the earth 9×10^7 miles, and the diameter of the moon 2100 miles, find the distance of the earth from the moon at the time of a solar eclipse when the eclipse is total only at a single

point on the earth. Also, find the diameter of the area on the earth within which the eclipse is total when the distance of the moon from the earth is 209,000 miles. The earth is assumed flat for this purpose. (P. U.)

[*Ans :* 210,000 miles : 10 miles].

Art 6.

5. Describe a pin-hole camera and explain its action. (C. U. 1930)

What is the effect of (*a*) enlarging the hole of a pin-hole camera, (*b*) doubling the distance from the small hole to the sensitised plate or translucent screen. (Pat. 1931 ; C. U. 1930)

6. A solar eclipse can be watched if a beam of sun-light is allowed to enter a room through a very fine hole, but it is a failure when the hole is big. Explain this. (See Art. 6). (Pat. 1932)

Arts. 7, 8 & 9.

8. What do you mean by the intensity of illumination and illuminating power ? (Dac. 1932 ; *cf.* C. U. '47, '49)

State the relation between them. (C. U. 1949).

9. State what is meant by the candle-power of a lamp, and explain how it can be determined by a shadow photometer. (*Cf.* C. U. 1941)

10. What is meant by the intensity of illumination at a point ? How would you verify the laws governing this intensity ?
(Pat. 1926, '36 '37, ; *cf.* C. U. 1931)

11. Prove that the intensity varies inversely as the square of the distance from the source of light. (All. 1919 ; Dac. 1930 ; *cf.* C. U. '41, '47. '49)

12 Two lamps of 10 and 20 c. p. respectively are placed 100 cms. apart. Show that there are two positions on the line through the lamps in which a screen may be placed so as to receive equal illumination from each.

13. Explain the principle of a grease-spot photometer. What is meant by candle-power ? (C. U. '28 ; *cf.* C. U. '47)

A 10 c. p. lamp is placed one metre from a surface. At what distance must a gas flame of 18 c. p. be placed so as to produce an equal illumination of the surface ? (C. U. 1928)

[*Ans :* 1·26 metres].

14. How would you compare the illuminating powers of two sources of light ?

There are two electric lamps directly above a table ; one of 32 c.p. is at a height of 4 feet and the other of 50 c. p. at a height of 10 feet. Compare the illuminations of the table when both lamps are alternately burning. (All. 1931)

[*Ans :* $P_1 ; P_2 :: 4 : 1$].

15. Two sources of light whose candle-powers are in the proportion of 2 : 1 are 2 metres apart. At what position must a screen be placed in order that both sides may be equally illuminated ? (C. M. B.)

[*Ans :* 117·2 cms. from higher c. p.]

16. Describe a simple method of comparing the candle-powers of two lamps. Explain the theoretical principles underlying the method.
(Dac. 1933 ; *cf.* C. U. '47)

17. Two sources of light, each 2 candle-power, are placed on the same side of a Bunsen grease-spot photometer. One is at a distance of 1 foot from the spot and the other at 2 feet. Where must a third source of 5 candle-power be placed in order that the appearance on each side of the photometer may be the same ? (L. U.)

[*Ans :* At 1·414 ft. on the other side of screen].

Art. 10(a).

18. Two lamps balance on a shadow photometer at distances of 60 and 42 cms. from the screen. The stronger lamp is, then, covered with a glass shade which transmits 80% of the incident light. How far must the other lamp be displaced in order to restore balance ? (Pat. 1942)

[*Ans :* 4·95 cms.]

19. A lamp produces a certain intensity of illumination on a screen when situated at a distance of 85 cms. from it. On placing a sheet of glass between the lamp and screen the lamp must be moved 5 cms. nearer to the screen to produce the same illumination as before. What percentage of light is stopped by the glass). (L. U.)

[*Ans :* 11·42].

CHAPTER II

Reflection of Light

11. Definitions.—

Reflection.—When light travelling in *one medium* falls on a *second medium*, three effects are produced.—

(*a*) A portion of the incident light is turned back, *i.e. reflected* from the surface of the second medium into the first medium again.

(*b*) A portion is *absorbed* by the second medium.

(*c*) A third portion passes through and is bent, *i.e.* *refracted* into the second medium.

Two Kinds of Reflection.—There are two kinds of reflection: (*i*) *regular* reflection, and (*ii*) *irregular* or *diffused* reflection (scattering).

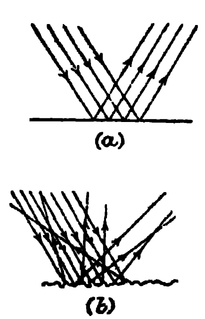


Fig. 12

(*a*) Regular Reflection.
(*b*) Irregular (or Diffused) Reflection.

(*i*) *Regular reflection* takes place when a beam of light falls on a smooth surface, as, for example, a mirror [Fig. 12(*a*)]. It is so called because the phenomenon is governed by *definite laws*, called the laws of reflection.

(*ii*) *Irregular reflection* takes place when the surface is not smooth, as for example, a wall or the ceiling of a room, or a piece of unglazed paper, unpolished wood, etc [Fig. 12(*b*)]. A beam of light, when incident on such a surface, does not proceed along a definite direction after reflection, but the **rays are reflected** in *diverse directions* depending on the nature of the **surface. All** opaque bodies are rendered visible by such diffused light.

So **the incident light = reflected light (regular and irregular) + absorbed light + transmitted light.** In the case of a polished metal reflector no part of the incident light is transmitted (unless the metal is very thin) and very little is absorbed. Such a reflector is in this respect superior to a plane mirror, which is only a glass silvered at the back.

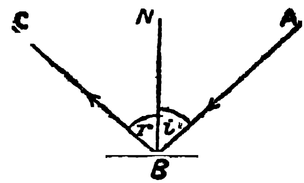


Fig. 18

Regular Reflection.—If a ray of light AB (Fig. 13) strikes a polished plane surface, say a plane mirror, at B, it will be reflected along BC. Draw a line BN *at the point B perpendicular to the reflecting surface.*

Here AB is called the **incident ray** and BC the **reflected ray. The** *incident ray* is the direction in which the light travels before striking the reflecting surface.

The **reflected ray** is the direction of the path of the ray after it is turned back into the first medium by the reflecting surface.

The line BN which is *at right angles to the surface at the point of incidence B* is called the **normal.**

The angle between the incident ray and the normal is called the *angle of incidence*. The angle between the reflected ray and the normal is called the *angle of reflection*.

Thus, the angle $ABN(\angle i)$ is the angle of incidence and the angle $NBC(\angle r)$ is the angle of reflection.

12. The Laws of Reflection.—Regular reflection takes place according to these two laws :—

(1) *The incident ray, the normal to the reflecting surface at the point of incidence, and the reflected ray lie in one plane.*

(2) *The angle of incidence is equal to the angle of reflection.*

Normal Incidence.—In the particular case when a ray is incident normally on the surface of a mirror, *i.e.* when the angle of incidence $(\angle i) = O$, the reflected ray returns along the same path, because, in this case, the angle of reflection $(\angle r) = O$ (see Fig. 13). So *a normal ray retraces its own path.*

13. Verification of the Laws.—

(*a*) **Pin method.**—Fix a sheet of white paper on a drawing board and draw a line AB on it (Fig. 14). Place a thin plane mirror upright on the paper such that its silvered surface is on the line drawn (see note below). Fix two pins S and C vertically on the paper, C touching the surface of the mirror in such a way that the line through S and C is oblique to the plane of the mirror, *i.e.* to AB. Move your eye and fix a third pin D such that the reflection of the pin S in the mirror and the pins C and D appear continuous, *i. e.* they appear to lie in the same straight line. Take off the pins after marking their positions on the paper and remove the mirror. Draw the line SC which represents the incident ray, and CD which represents the corresponding reflected ray. Draw CL normal to the mirror at C. Measure the angles SCL and LCD, which will be found to be equal *This proves the second law.*

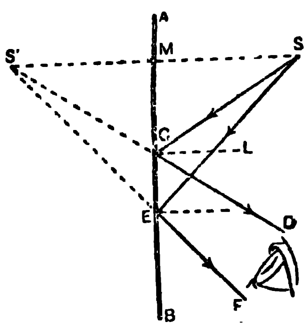


Fig. 14

The lines SC and CD representing the incident and the reflected rays respectively lie in the plane of the paper ; and the line CL, which represents the normal, is also in the plane of the paper. *This proves the first law.*

Note.—The rays are only partially reflected from the front surface of the mirror. Most of the reflection takes place from the back of the

mirror which is silvered ; however, due to refraction the mirror will appear to be only two-thirds of its real thickness [vide Art 50(b)].

So the reflecting surface acts as if it is at two-thirds of the thickness of the glass from the first surface of the mirror. Hence the reflecting surface should be taken at one-third of the thickness of the glass from the back of the mirror.

(*b*) **Hartle's Optical Disc Method** :—Hartle's disc [Fig. 14(a)] is essentially a plane circular disc (D) vertically mounted on a heavy stand and is capable of rotation in the vertical plane about a horizontal axis passing through the centre of the disc.

The disc is divided into four quadrants by a horizontal and a vertical diameter. Each quadrant is divided into 90 equal parts, each being a degree (or its submultiples), and the vertical diameter is marked $0°-0°$, while the horizontal $90°-90°$. Around the edge of the vertical disc is a curved metallic screen which is provided with an adjustable aperture The aperture can be closed by means of a slide which has one or more slits through which light from a source can be made to pass tracing its path along the disc.

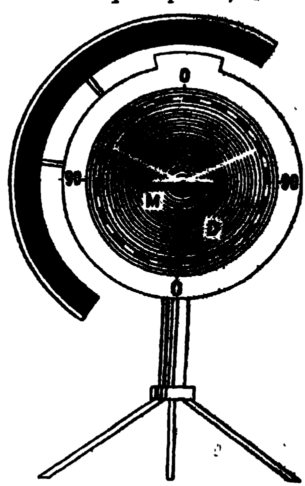


Fig. 14(a)

An optical element, say a piece of plane mirror *M*, may be held at the centre of the disc, with its plane normal to the disc, along the $90°-90°$ line by means of a suitable clamp attached to the disc.

An adjusted thin pencil of light is made to pass through the slit in the screen and trace its path along the disc and fall on the mirror at the centre of the disc so that the $0°-0°$ line marks the normal at the point of incidence. The trace of the reflected pencil is seen on the other side of the normal. Because the incident ray and the reflected ray lie in the same plane in which the normal lies, *i.e.* in the plane of the disc, the first law is verified.

The angle of incidence, as also the angle of reflection, can be read off from the circular scale on the disc and they are found to be equal. On rotating the disc (the screen containing the slit remaining fixed), the angle of incidence is changed and the corresponding angle of reflection found as above. It is seen that in every case the angle of

incidence equals the angle of reflection whatever is the angle of incidence. This verifies the second law.

14. Reversibility of Light.—In Fig. 13, any one of the rays AB and BC, or in Fig. 14, any one of the rays SC and CD may be taken as the incident ray when the other one will be the corresponding reflected ray. This fact may be expressed by saying that **the path of light is reversible.** This doctrine applies to the case of refraction (chapter IV) also. Thus, if a ray starting from a point P reaches another point Q along any path either by reflection or by refraction or by both, then the ray will travel back to P along the same path if the source be placed at Q, all other conditions remaining the same.

15. Optical Image—If rays diverging from a point source suffer changes of direction due to reflection or refraction such that the reflected (or refracted) pencil either actually converges to, or appears to diverge from, a second point, the second point is called the *optical image* of the first point. The image is either real or virtual.

Real and Virtual Images.—When a pencil of rays diverging from a point *appears*, after reflection or refraction, to diverge from a second point, the second point is called the *virtual image* of the first point. When the pencil of rays diverging from a point, after reflection or refraction, actually converges to a second point, the second point is called the *real image* of the first point. So the real image is formed by the *actual intersection* of the rays and is always inverted and can always be received on a screen. But the virtual image is formed by the *imaginary intersection* of the rays. It is always erect and cannot be received on a screen, as it has no real existence. The image S' of S in Fig. 14 is a virtual image which has got no actual existence, for it is behind the opaque mirror.

Note.—(1) The image of S seen at S' (Fig. 14) by an eye placed between DF is *virtual* because it is not formed by the actual intersection of the rays and has got no real existence. But if a convergent pencil of rays like DC, FE strikes the mirror, the rays will be reflected and the image will be formed at S by the actual intersection of the rays. So an eye placed near S will see a *real* image. It should be remembered that this is the only case in which a plane mirror gives a real image, otherwise **a plane mirror always gives a virtual image**

In chapters III and V, formations of real and virtual images by spherical mirrors and lenses have been respectively dealt with.

N. B. The image formed by a pin-hole camera is not an optical image at all for the following reasons :—(*i*) the course of the rays of light passing through the hole is not changed in any way by reflection, refraction, etc. ; (*ii*) the rays do not intersect on the screen in forming

the image ; (*iii*) the image is formed at any distance from the hole ; in the case of an optical image, the image is formed at some particular distance only.

16. Image formed by a Plane Mirror.—The image of a point source formed due to reflection by a plane mirror is (*a*) *as far behind the mirror as the object is in front* ; (*b*) *on the perpendicular drawn from the object to the mirror* ; (*c*) *always virtual.*

Verification.—

(1) **Pin Method** (*Sighting Method*).—Proceed as in the experiment in Art. 13 and determine the directions of the reflected rays corresponding to the incident rays SC, SE (see Fig. 14). Remove the mirror and produce the reflected rays backwards. They will converge to a single point S' which is called the image (virtual) of the point S. So an eye situated so as to receive some of these reflected rays will see the pin at S'.

We have, $\angle SCL = \angle LCD$; $\therefore$ $\angle SCA = \angle DCB = \angle ACS'$.

$\therefore$ $\angle SCE = \angle S'CE$; similarly $\angle SEC = \angle S'EC$; and CE is common to the triangles SCE and $S'CE$.

$\therefore$ These triangles are equal in all respects. $\therefore$ $SC = S'C$.

Again, in the triangles, SCM and $S'CM$, $SC = S'C$; MC is common to both ; and $\angle SCA = \angle S'CA$. $\therefore$ The triangles are equal in all respects. $\therefore$ $SM = S'M$; and $\angle SMC = \angle S'MC =$ one rt. angle, and so SS' is normal to AB.

Hence, *the image of a point is as far behind the reflecting surface as the object is in front along the normal to the surface drawn from the point. The image is virtual.*

(2) *Parallax Method.*—Consider for a moment that you are looking at a clock just at 11-30. If you look at it from the right, it will possibly be 11-31, while from the left it will appear to be 11-29. It is because there is *parallax* between the minute hand and the printed figures on the dial. So *parallax is the relative displacement of objects, if not in actual contact, when seen from different directions.* For example, hold two parallel rods in the upright position in front of the eyes and move the head to the right or to the left. It will be noticed that the more distant rod appears to follow the motion of the head while the nearer one moves in the opposite direction. If, however, the rods are made to approach each other, the apparent relative displacement of the rods would gradually diminish and finally disappear when ther ods are closest together. This relative change in position of the rods with the change in position of the eye is known as *parallax*, and the motion of the rods relative to each other with the movement of the eye is

known as *parallax motion*. Thus, when any two objects are coincident and there is no relative motion between them, parallax is said to be eliminated. So by this method coincidence of two objects can be tested.

Expt.—Fix a paper on a drawing board and place a mirror M perpendicularly to the plane of the paper (Fig. 15). Stick a long pin P in front of the mirror and place another long pin P' on the other side of the mirror opposite to P, so that the image of the lower part of the pin P appears to be continuous with the upper part of the pin P', seen above the top edge of the mirror, and, moving your eye from side to side, locate the exact position of the pin P' such that there is no relative displacement between the two, that is, there is no parallax, as it is called. Remove the mirror, and join the positions of P, P' on the paper. It will be observed that the position of P' is on the normal through P produced behind the mirror, and that the distance of P from the mirror is epual to that of P' from the mirror.

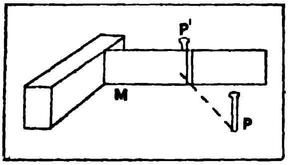


Fig. 15

The above method of locating the position of the *virtual image* is called the *Method of Parallax*.

17. Tracing the Rays by which an Eye can see the Image.—

(a) In order that an eye E (Fig. 16) can see the image I of a point O formed by a plane mirror M, it must receive rays of light reflected from some portion of the surface of the mirror. Hence, to see the image, (*i*) the source O is placed anywhere in front of the mirror M ; (*ii*) the eye must be placed within the space enclosed by straight lines drawn from the image through the boundary of the mirror.

Here the eye E will be able to see the image I of the object O formed by the mirror M as long as E is placed within the space enclosed by IA and IB.

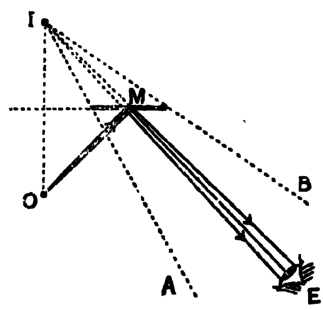


Fig. 16

In the case of an extended object, each of the two extreme points of it should be treated as the point O and the same principle should be adopted.

(b) The image of an Object (i.e. an extended Body) formed by a Plane Mirror.—The image of a point source has been considered, but when an actual object is put in front of a mirror, every point of it

may be regarded as a source of light. Thus the image of the object is formed by the point images of all its parts.

18. Lateral Inversion.—When we look at the image of our body in a fairly big mirror, we find that the whole of the right side of the image appears to be the left side of the body; if we move our right hand, the image appears to move its left; that is, the image is inverted as regards side. This is called *lateral inversion.* This happens because every point image is at the same distance behind the mirror as the point object is in front; and the front of the image faces the front of the object with the result that the sides of the object appear to be changed in the image.

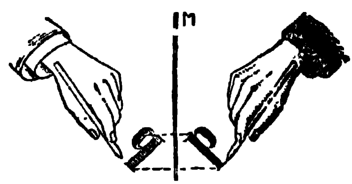


Fig. 17—Lateral Inversion

The size of the image is, however, equal to the size of the object. The image due to bodies having symmetrical sides is not affected by lateral inversion but image due to non-symmetrical bodies are seriously affected. Thus, in Fig. 17, the image of the letter '*p*' is laterally inverted and appears as the letter '*q*'. It should be noted that this is due to the image of every point of the letter '*p*' being formed at the same distance between the mirror as the point is in front.

Using two mirrors we can produce two lateral inversions and thus get the ordinary view of things.

19. Rotation of a Beam of Light by a Plane Mirror.—*When a plane mirror is rotated through an angle, the reflected ray is rotated through twice that angle.*

Let a ray MO be incident on the mirror AB at an angle i to the normal ON (Fig. 18). The angle of reflection NOP is also equal to $\angle i$.

Hence $\angle MOP = 2i$. When the mirror is rotated through an angle θ into the new position $A'B'$, the normal ON is also rotated through θ, to the position ON', so the angle of incidence for the ray MO, in this case, is $i + \theta$. The new angle of reflection $N'OP'$ is also equal to $i + \theta$.

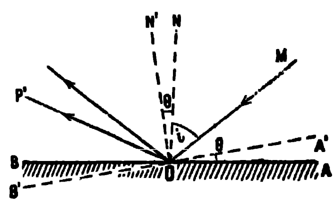


Fig. 18

$\therefore$ The $\angle MOP' = 2(i + \theta)$. So $\angle POP' = \angle MOP' - \angle MOP = 2(i + \theta) - 2i = 2\theta =$ the angle through which the reflected ray is rotated.

This principle is practically utilized in measuring the very small deflections of mirrors in case of reflecting galvanometers (see Art. 25, Part VII), the sextant, etc.

20. The Sextant.—This instrument, the working of which is based on the principle of double rotation of the reflected ray (Art. 19) when a mirror is rotated, is used for measuring the angular separation of two distant objects. Its important use is in measuring the altitude of the sun and other heavenly bodies while at sea, as the instrument is not affected by the motion of the ship. It consists of a rigid frame to which is attached at right angles a mirror M_2 of which the upper half is transparent and the lower half is silvered. Another mirror, M_1, also fixed at right angles to the frame, is attached to a radial index arm QV, fitted with a vernier V, which moves over a circular scale AB, the arc being 60°, *i.e.* one-sixth part (hence the name *Lat.*, sextant) of the complete circle. A telescope T is also attached to the frame directed towards M_2 as shown in the figure. If the index arm QV is moved to the zero position B of the scale, the mirror M_1 becomes parallel to the mirror M_2.

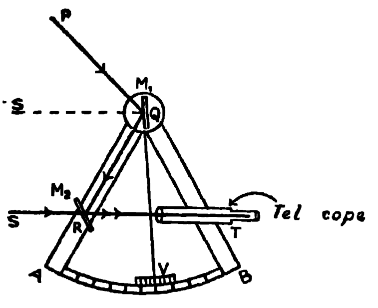


Fig. 19—Sextant

To find the altitude of the sun, or of a star, the instrument is placed in a vertical plane and the telescope is focussed so that a distant object in the horizon is viewed through the transparent portion. The mirror M_1 is now turned by means of the arm QV until light from the horizon, after reflection at Q and R, also enters the telescope, and the position of V on the scale is noted when these two images coincide. The arm QV is now rotated into the position shown in Fig. 19, so that light from the lower edge of the sun, or from a star, after reflection at Q and R along the path $PQRT$ forms an image coinciding with the horizon. The new position of V is noted and the angle BQV through which the mirror M_1 is rotated from the zero position is also noted. The altitude of the sun, *i.e.* the angle PQS, is twice the angle through which the mirror M_1 is rotated. The movement of the vernier V over the graduated scale is therefore half the altitude of the sun's lower edge. As the instrument is used to read the altitude directly, each half degree on the scale is labelled as a whole degree, the arc which is usually 60° being therefore numbered upto 120°.

It is evident that the instrument may be used to measure the

angle subtended by any two distant points if the instrument is placed in the proper plane.

21(a). Deviation produced by one Reflection in a Plane Mirror.—In Fig. 18, it will be seen that the ray MO which, in the absence of mirror AB, would have travelled straight in the direction MO, has taken the path OP due to reflection at the point O. The direction of the ray MO has, therefore, been changed by the angle $(\pi - MOP) = (\pi - 2i)$. This will be the *deviation* produced in any ray after being reflected once at a plane surface.

(b). Deviation due to two Successive Reflections at Two inclined Mirrors.—Instead of one reflection if the ray be reflected twice, *i.e.* by the two mirrors (Fig. 20), *inclined* at the angle θ, the change of direction due to the first reflection at P is $(\pi - 2i)$, and that at Q is $(\pi - 2i')$. Hence the total deviation produced by two reflections would be $(\pi - 2i + \pi - 2i') = 2\pi - 2(i + i')$, where i' is the angle of incidence at the second mirror. It is assumed, however, that the ray always lies in one plane.* On drawing the diagram it will easily be seen that $\theta = i + i'$. Thus, after two reflections, deviation $= (2\pi - 2\theta)$.

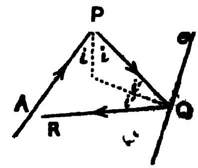


Fig. 20

That is, *the deviation of a ray undergoing two reflections depends only upon the angle between the two reflecting surfaces.*

Example .—

Rays of light strike a horizontal plane mirror at an angle of 45°. Show how you would arrange a second mirror in order that the reflected ray may finally be reflected from the second mirror horizontally. (*Pat. 1932*)

Here, in order that the final ray may be horizontal after two successive reflections, the deviation suffered by the ray will be $= 180° + 45° = 225°$. Hence, if the second mirror be inclined at $\theta°$ to the first, we have,

$$2\pi - 2\theta = 225° \; ; \quad or \quad 2\theta = 360° - 225° = 135° \; ; \quad \therefore \quad \theta = 67{\cdot}5°.$$

22(i). Two Mirrors Inclined at any Angle.—Let OA and OB represent two plane mirrors facing each other and inclined at an angle AOB (Fig. 21). Let P be a source of light. The image of P formed by reflection at the mirror OA is at P_1 on the normal PNP_1 to the mirror. This virtual image is formed as a result of reflection of the rays diverging from the point P and incident on the mirror OA. It can be shown with the help of geometry that $OP = OP_1$. Therefore, P_1 lies on the circle drawn with O as centre and radius OP. It will be seen

that other images formed will also lie on this circle. Now, P_1 serves as an object in front of the mirror OB. Draw a normal P_1P_2 on OB

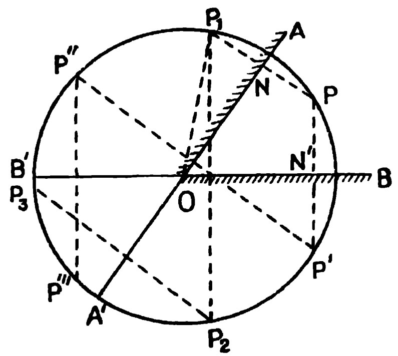


Fig. 21

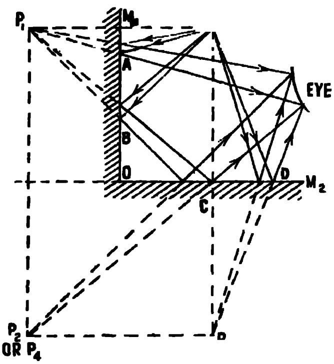


Fig. 22 -Two mirrors at rt. angles

so that P_1P_2 is bisected by OB. Then P is the image of P_1 formed by the mirror OB, and OP_2 will be equal to OP_1. Similarly, P_2 serves as an object in front of the mirror OA, and an image P_3 is formed on the other side of OA so that OP_2 is equal to OP_3. P_3 falls behind both the mirrors and no further image is formed.

In the same way, another series of images P', P'', P''', etc., will be formed, the first image beginning with the mirror OB. The process of formation of images by successive reflections at the surfaces of the mirrors goes on until an image has been formed behind both the mirrors, *i. e.* within the angle $A'OB'$.

The number of images formed depends on the inclination of the mirrors, and can be proved to be equal to $\left(\dfrac{2\pi}{\theta} - 1\right)$, where θ is the angle between the mirrors. Thus, when the angle is 90°, the number of images is 3 (Fig. 22) ; when it is 60°, the number is 5, and so on.

If, however, θ is not a submultiple of 2π, *i. e.* if $2\pi/\theta$ is not a whole number, the total number of images is the integer next greater than $\left(\dfrac{2\pi}{\theta} - 1\right)$.

(ii) **Two Mirrors at Right Angles.**—In Fig. 22 two mirrors M_1 and M_2 are placed at rt. angles to each other and facing the object P. Rays of light diverging from P have been drawn showing how by successive reflections images produced by M_1 and M_2 are seen by an

observer whose eye is suitably placed. P_1 and P_3 are the first images produced by M_1 and M_2 respectively. Each of P_1 and P_3 acts as a virtual object for the other mirror and produces images P_2 and P_4 respectively. In the case of two mirrors at rt. angles to each other, as shown in the figure, P_2 and P_4 merge into one image. So there are three images altogether seen by the eye.

Brewster's Kaleidoscope.—This is a toy constructed on the principle stated above. In this, two mirrors are kept inclined at an angle of 60° in a tube. Between the mirrors are placed several bits of coloured glass, each of which produces 5 images, and, as the tube is turned round, symmetrical hexagonal patterns are observed on looking into this tube.

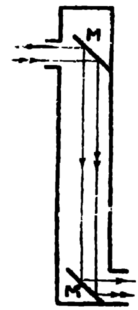


Fig. 23—Simple Periscope

23. Simple Periscope—This is an application of the principle of reflection. Fig. 23 explains the principle of a Simple Periscope which is formed by fixing two plane mirrors at angles of 45° to the axis of the tube and parallel to each other. Rays of light from distant objects are first reflected from the upper mirror along the axis of the tube and then being reflected by the lower mirror come out in the horizontal direction reaching the eye of the observer, thereby giving him a view of the distant objects. The instrument is used for looking over the heads of crowds by raising the upper mirror of the instrument above the obstacle, for observing enemy movements in trench warfare without any danger to the observer, for observing a performance in an enclosure from without, etc. Periscopes using totally reflecting prisms (Art. 52) are used in submarines.

24. Reflection in Two Parallel Mirrors.—Place two plane mirrors parallel to and facing each other. Let P be a luminous point between them. Through P draw a straight line perpendicular to both the mirrors and produce the line both ways, meeting the mirrors LK and MN at A and B respectively (Fig. 24) The image of P formed by the mirror LK will be at P_1 such that $AP_1 = AP$. Rays diverging from P will appear to come from P_1 after being reflected by the mirror LK. Some of those

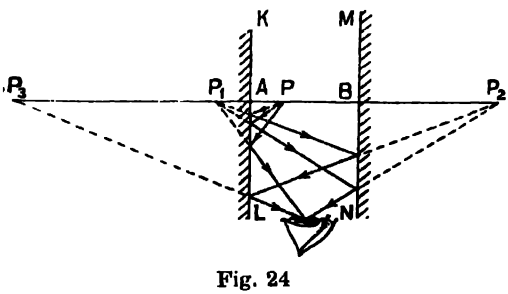


Fig. 24

reflected rays will fall on MN and will appear to diverge from P_2, being the image of P_1 which serves as an object in front of the mirror MN, so that $BP_2 = BP_1$. Again, in the same way, the image of P_2 is formed at P_3 behind LK Starting with the rays first reflected from the mirror MN, another series of images will be produced as in the above case.

Theoretically, the number of images thus formed in the case of parallel mirrors will be infinite, but as the images after each reflection

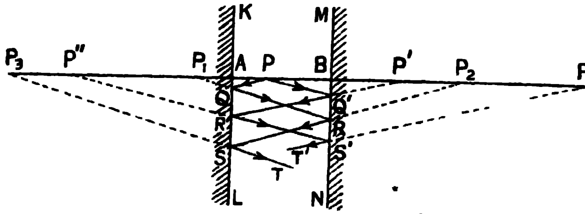


Fig. 24(a)

get fainter and fainter due to loss of light, the number of visible images is, however, limited, but quite large, no doubt. In Fig 24(a), the positions of the images formed by successive reflections from the two parallel mirrors have been shown. They extend, theoretically, upto infinite numbers on either side.

25. Multiple Images formed by a Thick Glass Mirror—When a candle flame at S is viewed *obliquely* in a mirror of thick glass silvered at the back, a train of images S_1, S_2, S_3, etc, is seen within the mirror (Fig. 25). Of these images the first is faint, the second S_2 is usually the brightest, and others gradually diminish in brightness. When an incident ray SA from a source, suppose, a candle flame, falls on the mirror at A, only a small portion of light is reflected at the surface in the direction AB forming a faint

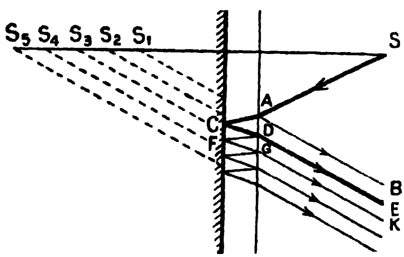


Fig. 25

image S_1, while a larger portion enters into the mirror and is reflected back from the silvered surface C; and, of this, again a large part emerges out of the glass in the direction DE forming the second and the brightest image S_2. The rest is reflected to the back surface again, and the process is repeated giving other faint images. All the images lie on the normal from the object to the mirror.

When incidence is nearly normal, by far the greater part of the rays emerge along DE making the second image the brightest. When we look to our image in a thick mirror, there is no confusion, because the images formed by the two surfaces overlap on each other almost exactly. With the obliquity of incidence increasing, more and more light is reflected at the first surface, and *with very oblique rays, the first image becomes the brightest.*

26 Irregular (or Diffused) Reflection : How Objects become Visible.—If in the dark room a beam of light is reflected regularly from a polished surface, say a mirror, it produces an image of the source of light, and an observer in the path of the reflected beam sees the bright image of the source. If the observer is outside the beam, the mirror will not be visible to him and he will not see the image either. But, if the beam falls on a rough surface, the rays are scattered in all directions , there is no regularly reflected beam and, therefore, no image. The surface is then visible from all directions. So, when the mirror is replaced by a sheet of white paper, the whole surface of the paper becomes more or less illuminated and becomes visible from different parts of the room ; because in this case the surface being rough, the rays of light are reflected from various parts of the paper in all directions, and what is seen is not an image of the source of light but different parts of the paper.

Such irregular or diffused reflections render visible the objects around us, such as unglazed paper, blotting paper, unpolished wood, etc., from which the reflection comes, and also other parts of the room which are in shadow.

The amount of the diffused light mentioned above depends upon the nature and colour of the surface on which it is incident and also on the colour of the incident light.

White walls and ceilings of a room diffuse larger amount of incident light while dark coloured walls will reflect less and absorb more. So, for proper illumination in a room, its walls and ceilings should be periodically white-washed.

The rays of the sun are visible on account of the dust particles floating in the air which diffuse the light. If the dust particles were removed the rays would be invisible. So, we see things because they send light to our eyes, but *light itself is invisible.* Light is a form of energy and is not visible, but it is the surrounding objects illuminated by light that are visible.

Twilight.—This is caused by diffused light. After the sun has set to an observer at a particular place, its rays falling upon dust and other floating particles in the air are scattered, some of which reach the place and continue the illumination for some time after sunset. This is

the twilight period. Twilight ends and darkness ensues when such diffused rays fail to reach the place.

27. Some Cases of Reflection —Some interesting examples of reflection are given below :—

(1) Movement of Object and Mirror.—*Prove that if an object in front of a plane mirror moves through a distance 'd' away from the mirror, the image moves through the same distance : whereas, if the mirror moves parallel to itself through a distance 'd' (the object remaining fixed) the image will move through a distance '2d.'* (*C. U. 1923, '46 ; Dac. '28*)

(a) *When the object moves.*—If the initial distance of the object be x in front of the mirror, then, when the object moves through a distance d away from the mirror, the distance from mirror becomes equal to $d+x$. Therefore the distance of the image will now be equal to $d+x$, *i.e.* the image also moves through the same distance d.

(b) *When the mirror moves.*—If the mirror moves through a distance d away from the object, the distance between the mirror and the object becomes $=x+d$, and the distance between mirror and first position of image $=x-d$. Therefore the distance between the two images $=x+d-(x-d)=2d$, *i e.* the image has moved through a distance $2d$ parallel to itself.

(2) Minimum Size of Mirror required.—(*a*) *Show by means of a diagram that a man can see the whole of his person in a mirror, the length of which is half his own height.* (*C. U. 1915, '25, '29 ; Pat. 1919.*)

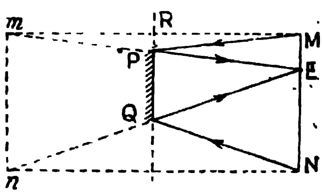


Fig. 26

Let MN represent the man, and E be the position of his eye (Fig. 26). Let mn be the image of the man formed by the mirror PQ. Both Mm and Nn are bisected by the surface of the mirror, or of the mirror produced.

The man sees the image m by means of rays coming from M, which, after reflection at P, appears to diverge from m. Similarly, the image n is seen on account of rays coming from N, which, after reflection, appears to diverge from n.

So, PQ is the minimum length of the mirror required to see the complete image mn equal to MN.

From the similar triangles EPQ, Emn, $\dfrac{PQ}{mn}=\dfrac{PE}{mE}$ ....(1)

Again, from the similar triangles mPR, mEM, $\dfrac{mP}{mE}=\dfrac{mR}{mM}$;

$$or,\quad \frac{mE-MP}{mE}=\frac{mM-mR}{mM};\quad or,\quad \frac{PE}{mE}=\frac{RM}{mM}=\frac{1}{2}.$$

From (1), $\frac{PQ}{mn} = \frac{1}{2}$, *or* $PQ = \frac{1}{2}mn = \frac{1}{2}MN (\because mn = MN)$, *i.e.* the minimum size of the mirror required is half the height of the observer.

(*b*) *Calculate the minimum size of a plane mirror fixed on the wall of a room in which an observer at the centre of the room can see the full image of the wall behind him.* (*C. U. 1929*).

Let PQ be the plane of the mirror which is fixed on the wall, and let ED be the observer whose eye is at E, MN being the wall behind the observer (Fig. 27).

From M, the top point of the wall, draw MA perpendicular on AB and produce it to M' making $AM' = AM$. Join $M'E$ meeting the wall AB at P. Join MP. So the ray MP, from the top point M, will be reflected from the point P of the mirror to reach the eye. So M' is the image of M. Similarly, N' is the image of the lower-most point N of the wall. So the minimum size of the mirror required is PQ.

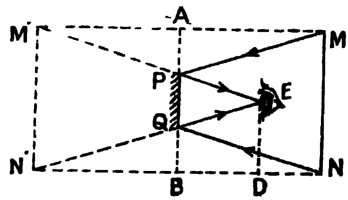


Fig. 27

Because the man ED stands in the centre of the room, $BD = \frac{1}{2} BN = \frac{1}{2}BN' = \frac{1}{3}DN'$.

Now in the Δ DEN', $BD = \frac{1}{3}DN'$, and DE is parallel to BQ; so by geometry $EQ = \frac{1}{3}EN'$. Similarly, $EP = \frac{1}{3}EM'$.

Hence in Δ $EM'N'$, $PQ = \frac{1}{3}M'N' = \frac{1}{3} MN$ *i.e.* the minimum size of the mirror $= \frac{1}{3}$ of the size of the wall.

(3) Time taken by Light to pass from one point to another by way of Reflection is a Minimum (Fermat's Principle) :—

PQ is an incident ray, and QR is the reflected ray at a plane surface. If Q' is any point on the reflecting plane, show that $PQ + QR$ is less than $PQ' + Q'R$. (*C. U. 1913*).

From P draw PN normal to the mirror and produce it to meet RQ produced in P'. Join $P'Q'$. Now P' being the image of P, $PN = P'N$; hence in the Δs PQN, and $P'QN$, $PQ = P'Q$.

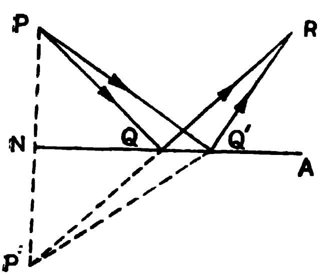


Fig. 27(*a*)

Similarly, by taking Δs PNQ' and $P'NQ'$, we have $PQ' = P'Q'$.

Now $P'Q' + Q'R > P'R$, *i.e.* $> P'Q + QR$;

$\therefore$ $PQ' + Q'R > PQ + QR$.

It, therefore, follows that the path taken by a ray in natural reflection is always the least. **This is the principle of least path or the principle of least time.**

Questions

Art. 15.

1. Distinguish between a real and a virtual image. How would you experimentally find the position of a virtual image? (Pat. 1919, '40; Cf. '33)

2. Define the 'image' of an 'object.' When is it called *real* and when *virtual*? (Pat. 1932)

Arts. 19 & 20.

3. Given a graduated scale, a stand, a source of light and a narrow slit, how would you measure small deflections of a plane mirror? (Pat. 1927)

4. Prove that when a plane mirror is rotated through any angle the reflected ray is rotated through twice the angle. (C. U. 1927, '46; Dac. '28, '34)

5. What is the relation between the turning of a mirror and that of a ray of light reflected from it? Apply this property to the construction of the sextant. (C. U. 1941)

Arts. 22 & 24.

6. Two plane mirrors are placed at right angles to each other, and an object stands between them facing their reflecting surfaces.

Draw a diagram to show how the images are produced by the mirrors. Draw a second diagram to show how the light travels to an eye looking at the last image through one of the mirrors. (C. U. 1949)

6(*a*). Show by a diagram (carefully drawn as large as your paper will allow) how many reflections of a pin you can obtain in two plane mirrors placed at an angle of 60°.

7. Explain, with the help of diagrams, the formation of multiple images by two mirrors, (*a*) when they are parallel, (*b*) when they are inclined to each other at 90°. (C. U. '19, '39, '47)

8. State the chief facts relating to images formed by plane mirrors.

A man sits in a room between two mirrors, one on the wall in front of him and the other on the wall behind him. An electric lamp is glowing over his head. Describe what he sees in the front mirror. How can he tell whether the mirrors are parallel? (Pat. 1926)

[**Hints.**—He will see a number of images of the lamp, one behind the other. If the mirrors are parallel then the line in which the images will appear is perpendicular to the wall, otherwise it will be inclined].

9. Explain the formation of the series of images of an object placed between two parallel mirrors. Draw a diagram showing the pencil of rays by which an eye sees an image formed by one reflection at each of the mirrors. Why is there a limit to the number of images visible? (Pat. 1918)

Art. 25.

10. Explain how a number of images is visible when a bright object is held in front of a thick mirror silvered at the back. Illustrate your answer by a neat diagram for a given position of the eye. (Pat. 1932)

Art. 27.

11. A plane mirror 2 ft. high is fixed on one wall of a room, the lower edge being 4 ft. 6 in. from the floor. If the opposite wall of the room is 14 ft. distant and 10 ft. high, draw a diagram to show from what point a man must look in order to see reflected in the mirror the whole height of the opposite wall, from floor to ceiling.

[**Hints.**—Draw the diagram to scale as in Art. 27, (2)b. E will be the position of the observer, the distance $AP = 3'6''$: $QP = 2'$: $QB = 4'6''$.]

12. A man running towards a plane mirror at the rate of 5 ft. per sec. approaches his image at the rate of 10 ft. per second. Explain. (C. U. 1948)

13. A large plane mirror stands vertically at a certain distance from a man who views his reflection in it. Compare the rate of motion of the image with the rate of motion (a) of the man, when the man moves towards the mirror, (b) of the mirror when the mirror is moved towards the stationary man. (C. U. 1946)

CHAPTER III

Spherical Mirrors

28. Definitions.—

Spherical Mirror.—A spherical mirror is a reflecting surface so curved that it forms part of a hollow sphere. It may be *concave* or *convex*.

Concave Mirror.—A spherical mirror is concave when the reflection takes place at the hollow side of the sphere of which the mirror is a part. So the middle point of such a mirror is the farthest position of the mirror to an observer.

Convex Mirror.—A spherical mirror is convex when the reflection takes place at the bulging or the raised side. Obviously when such a mirror is placed before the eye, its middle point will be nearest to the eye.

The **Pole** of a mirror is the middle point of its reflecting surface.

The **Centre of Curvature** (C) of a mirror (Fig. 29) is the centre of the sphere of which the mirror forms a part, and the radius of the sphere is called the **radius of curvature** (PC) of the mirror. Rays travelling along the continuation of a radius meet the mirror normally, and so after reflection they travel back along their own paths.

The **Principal Axis** is a direction obtained by the line joining the centre of curvature and the pole of the mirror ($C\ P$, in Fig. 29).

Any radial line passing through the centre of curvature other than the principal axis is called a **Secondary Axis.**

The **Principal Section** of a mirror is a section taken normal to the mirror and passing through the principal axis. Figures 28, 29, etc., are all *principal sections* of spherical mirrors.

The **Aperture** of a mirror is given by the angle subtended at the centre of curvature by the two extreme radii on the principal section of the mirror.

(a) Concave Mirror Fig. 28 (b) Convex Mirror

Principal Focus and Focal Length.—A pencil of rays parallel to the principal axis of a spherical mirror will after reflection either converge to [*as in the case of a concave mirror*, Fig. 28(a)] or appear to diverge from [*as in the case of a convex mirror*, Fig. 28(b)] a fixed point F on the axis. This fixed point F is called the **principal focus** of the mirror, and the distance of the focus F measured from the pole of the mirror is called the **focal length** ($A\ F$ in Fig 28).

29. Relation between Focal Length and Radius of Curvature.—The focal length of a spherical mirror, which is usually denoted by the letter f, is approximately half of the radius of curvature r of the mirror. A ray AB parallel to the principal axis PC (Fig. 29) of a spherical mirror will, after reflection, either actually pass through the principal focus F as in the case of the concave mirror [Fig. 29(*a*)], or appear to emerge

(a) Concave Mirror Fig. 29 (b) Convex Mirror

from F, the principal focus as in the case of the convex mirror [Fig. 29(*b*)]. In both cases, BC, the radius of curvature of the spherical surface, is normal to the mirror at the point of incidence.

(*i*) **Concave Mirror.**—

AB and PC are parallel so, $\angle ABC = \angle BCF$.

But $\angle ABC = FBC$, (2nd law of reflection) ;

$\therefore$ $\angle BCF = \angle FBC$; $\therefore$ $FC = FB$.

(*ii*) **Convex Mirror.**—

AB and PC are parallel ; $\therefore$ $\angle ABC = BCF$.

But $\angle ABE = \angle DBE$, (2nd law of reflection) ;

$\therefore$ $\angle BCF = \angle DBE = \angle FBC$, (vertically opposite angle).

$\therefore$ $FC = FB$.

If B is very near the pole, $FB = FP$, approximately in both the above cases ;

$\therefore$ $FC = FP$; *i.e.* $FP = \frac{1}{2}\ CP$; or $f = \dfrac{r}{2}$.

That is, **focal length is half the radius of curvature.**

30 (a). Rules for Signs —

(*i*) *All measurements must be made from the pole of the mirror.*

(*ii*) *Take measurements against the direction in which the incident light travels as* **positive**, *and those in the same direction as the incident light as* **negative**. *In other words, measurements* **towards** *the source of light are* **positive** : *and measurements* **away** *from the source of light are* **negative**.

(*iii*) *When solving any problem do not substitute the signs in the formula unless the actual value of the quantity is given.*

30(b). For New Convention of Signs, see Art. 67 (b).—According to this *all real distances are positive* and *all virtual distances are negative.*

31. General Formula for Spherical Mirrors.—

(1) Concave Mirror.—Let O represent the position of a luminous point on the principal axis of a concave mirror (Fig. 30). The ray OP coming along the axis will be reflected back along PO as it is normal to the surface. The ray OA incident at A will be reflected along AI making the angle of reflection CAI equal to the angle of incidence CAO, because AC is normal at A, C being the centre of curvature of the mirror. The intersection of these two rays determines the position I of the image of the point O.

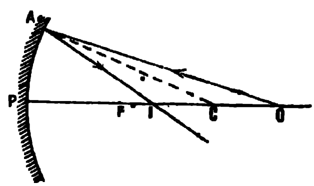


Fig. 30—Concave Mirror

Since AC bisects the angle IAO, we have $\frac{IA}{AO}=\frac{IC}{OC}$. Considering AP to be small, we may write, $IA=IP$, and $OA=OP$ (approx.).

$$\therefore \quad \frac{IP}{OP}=\frac{IC}{OC} \quad \dots \quad (1)$$

Let $OP=u$; $IP=v$, and $CP=r$. Then, we have, from (1),

$$\frac{v}{u}=\frac{r-v}{u-r}; \; or \; v(u-r)=u(r-v) \; ; \quad or \quad ur+vr=2uv.$$

Dividing by uvr throughout, we have, $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$.

But, if F is the principal focus, $PF=FC$. Therefore, $r=2f$ (where $f=PF=$ the focal length).

Hence $$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f} \quad \dots \quad \dots \quad (2)$$

(2) **Convex Mirror.**—In this case, every reflected ray from any point O on the principal axis appears to diverge from a point on the principal axis, so the image is virtual. In Fig. 30(a), the ray OP reflects back along PO, since OC is the principal axis and the incidence is normal. Any other ray OA reflects along AB. The two reflected rays seem to intersect at I, which gives the position of the virtual image formed in this case. AN is the bisector of the exterior angle OAB of the triangle IAO ; so,

$$\frac{IA}{OA}=\frac{IC}{CO}.$$

Considering AP to be small, we may write $IA=IP$, and $OA=OP$ (approx). $\therefore \frac{IP}{OP}=\frac{IC}{OC}$. Substituting with proper signs,

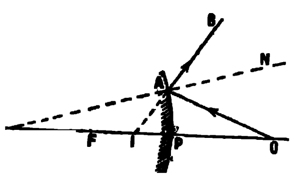


Fig. 30(a)—Convex Mirror

we have $$\frac{-v}{u}=\frac{-r-(-v)}{u+(-r)}=-\frac{-r+v}{u-r}$$

or $$\frac{v}{u}=\frac{r-v}{u-r}; \quad or \quad ur+vr=2uv.$$

Dividing by uvr throughout, we have,

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}.$$

Conjugate Foci—Two points on the axis of a spherical mirror are said to be **conjugate foci** when an object placed in one produces the

image at the other. In Fig. 30, a real object at O produces a real image at I and the positions are interchangeable. In Fig. 30(a), a real object at O produces a virtual image at I. A virtual object at I will produce a real image at O. Such points O and I are called **Conjugate Foci.**

32 Rules for Tracing the Image in the case of Spherical Mirrors.

(1) *Draw a ray from the top of the object parallel to the axis. This ray after reflection will pass through, or appear to pass from, the focus.*

(2) *Draw a ray fram the top passing, or appearing to pass, through the centre of curvature of the mirror. This ray being normal to the mirror is reflected back along the same path. If the bottom of the object be not on the axis, the position of its image may be determined exactly as above.*

(3) *The point to which any two of these reflected rays converge, or from which they appear to diverge, is the* **required image.**

(4) *A ray passing through the principal focus, and incident on the mirror, will be reflected in a direction parallel to the principal axis.*

33. Determination of the Position of the Image of an Extended Object by a Spherical Mirror.—An extended object may be supposed to be consisting of several points, and when the image corresponding to each point is found, the image of the whole object is obtained. *When the point source is on the principal axis, the image is also on it*, and when it is not on the axis (as P in Fig. 31), the image is obtained by the rules given above. Let AB be the principal section of a mirror (Fig 31 and Fig. 32) of which A is the *pole* ; C the *centre of curvature* ; AC the principal *axis*, and F the *principal focus*.

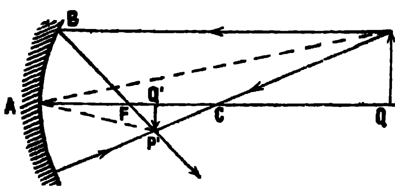


Fig. 31—Image by a Concave Mirror

Let PQ be the object perpendicular to the axis placed at Q beyond the centre of curvature. Draw a ray PB parallel to the axis which, after reflection, will pass through F, the principal focus. Then BF is the reflected ray in Fig. 31, (the reflected ray in Fig. 32, being produced backwards, passes through F). Draw another ray PC through C, the centre of curvature, which will be reflected back along the same path. The

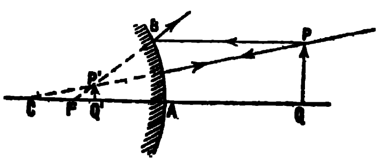


Fig. 32—Image by a Convex Mirror

point P' of intersection of the rays is the required image of P. So $P'Q'$ is the image of PQ, where $P'Q'$ is drawn perpendicular to AC. The size and position of $P'Q'$ will depend on the position of PQ with respect to the mirror. Following the above method of construction different cases may be examined.

Note.—(*i*) In case when the object PQ is very long, and the mirror (or the lens, Art. 68) is very small, the two rays PB and PC (or any one of them) may not meet the mirror. In such a case the image should be obtained graphically by producing the trace of the mirror. It should be remembered that the image is actually formed by a large number of rays proceeding from each point of the object and so it does not matter whether the two particular rays fall on the mirror or not.

(*ii*) The focal length of a *concave mirror is positive.* The focal length of a *convex mirror is negative. The focus in the case of a convex mirror* is a *virtual focus.*

(*iii*) *Real images are always inverted, whilst virtual images are always erect.*

34. Magnification.—The magnification (m) produced by a mirror is defined as follows.—

$$m = \frac{\text{Size of image}}{\text{Size of object}}.$$

Here size refers to linear dimension (e.g. length or breadth or height) and *not to area or volume.* So, m gives the **linear magnification** only.

(*a*) Magnification in the case of a Concave Mirror producing Real Image :—

In Fig. 31, join PA and $P'A$. Then $P'A$ is the reflected ray corresponding to PA, since P' is the image of P. AC is the normal at A. The two triangles PAQ and $P'AQ'$ are equiangular and, therefore, similar. So,

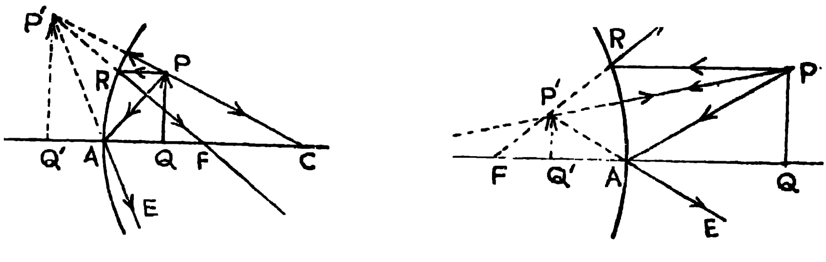
$$\frac{P'Q'}{PQ} = \frac{Q'A}{QA} = \frac{v}{u}.$$

Since, $P'Q'$ is inverted with respect to PQ, it may be mathematically represented by giving a negative sign before $P'Q'$.

$$m = \frac{\text{Size of image}}{\text{Size of object}} = -\frac{P'Q'}{PQ} = -\frac{v}{u}.$$

Thus, for a concave mirror producing real image, $m = -\frac{v}{u}$

(b) Magnification in the case of a Concave Mirror, or a Convex Mirror, producing Virtual Image.—



(i) (ii)

Fig. 32(a)

In Fig, 32(a), a concave mirror (left) and a convex mirror (right) have been shown to produce virtual image. Since P' is the image of P, the ray PA reflected at A as AE will pass, when the produced backwards, through P'. From the two similar triangles, PAQ and $P'AQ'$,

$$\frac{P'Q'}{PQ} = -\frac{v}{u}.$$

$$\therefore \quad m = \frac{\text{Size of image}}{\text{Size of object}} = \frac{P'Q'}{PQ} = -\frac{v}{u}.$$

Thus, for the virtual images produced by a concave or a convex mirror also, $m = -$

[N B Remember that in solving a problem, m is to be treated as positive when the image is virtual and negative when image is real, since virtual images are always erect and real images always inverted.]

Expression of Magnification in Various Ways.—

We may also find expessions for the magnification m in terms of u, v, f, and r as follows :—(*a*) we have, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

Multiplying each term by v, we have, $1 + \frac{v}{u} = \frac{v}{f}$.

$$m = -\frac{v}{u} \quad -\left(\frac{v}{f} - 1\right) = -\left(\frac{v-f}{f}\right).$$

$$m = \frac{v}{u} \quad \frac{v-f}{f} \qquad \ldots \qquad \ldots \quad (1)$$

(b) Also, by multiplying each term of the general equation by u, we get

$$\frac{u}{v}+1=\frac{u}{f}, \quad \text{or} \quad \frac{u}{v}=\frac{u-f}{f}, \quad \dots \qquad (2)$$

$$\therefore \quad m=-\frac{v}{u}=-\left(\frac{f}{u-f}\right) \quad \dots \qquad (3)$$

$$\therefore \quad \mathbf{m}=-\frac{v}{u}=\frac{-f}{u-f} \quad \dots \qquad (4)$$

(c) Again, taking the equation, $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$

we have, $\frac{1}{v}-\frac{1}{r}=\frac{1}{r}-\frac{1}{u}$, or $\frac{r-v}{vr}=\frac{u-r}{ur}$

$$\therefore \quad \frac{v}{u}=\frac{vr}{ur}=\frac{r-v}{u-r}, \quad \text{or } m=-\frac{v}{u}=-\left(\frac{r-v}{u-r}\right) \quad \dots \qquad (5)$$

$$\therefore \quad \mathbf{m}=-\frac{v}{u}=-\frac{r-v}{u-r} \quad \dots \qquad (6)$$

Plane Mirror.—The same equation for magnification has been obtained for both concave and convex mirrors. It should also hold for a plane mirror, which is only a spherical mirror of infinite radius (r). So the equation in the case of a plane mirror becomes, $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{2}{\infty}=0$, and so in this case $v=-u$;

$$\therefore \quad m=-\frac{v}{u}=1.$$

That is, the size of the image in a plane mirror is equal to the size of the object, and that the image is virtual, *i.e.* formed on the other side of the mirror.

35. **Complete Description of an Image.**—For the complete description of an image the following points are necessary—

(a) Its distance from the mirror : (b) whether erect or inverted : (c) whether real or virtual : (d) its magnification.

(I) Remember the signs of u, v, and f—

u	...	...	*+always*
v	...	...	*+for real image*
	...	...	*−for virtual image*
f	...	...	*+for concave mirror*
	...	...	*−for convex mirror.*

[See also Art. 67(b)]

36. Spherical Aberration (Caustic Curve)—All the rays parallel to the principal axis of a concave spherical mirror will meet at a point, called the principal focus, after reflection, when only a small portion of the surface is used as a mirror, *i.e.* only when the aperture of the mirror is small; otherwise the rays will fail to converge to a point. This failure of the mirror (or a lens, as the case may be, Art. 72) to bring the light rays to a point-focus is called **spherical aberration**.

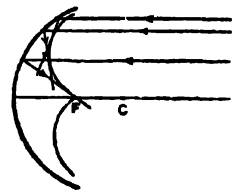


Fig. 33—Caustic Curve

It will be seen that when a large portion of a mirror is used, *i.e.* for a mirror of wide aperture, the reflected rays do not pass through F, the focus, unless they are incident near about the pole. As the aperture increases, the reflected rays cut the principal axis at points which are decidedly inside of F (see Fig. 33). The curve drawn tangentially to the reflected rays are called a **caustic curve** (Fig. 33).

The caustic curve is well seen when sunlight is allowed to fall on the side of a cup nearly filled with milk.

The spherical aberration may be remedied by diminishing the aperture by covering the outside margin with black paper so that the reflection takes place only near about the pole. Such a covering is called a *stop* or *diaphragm*.

The spherical aberration may, however, be avoided by using a **parabolic mirror**, where all the rays after reflection pass through the focus (see Art 40).

37. Nature, Position, and Size of the Image in Spherical Mirrors.—The nature, position, and size of the image formed by a mirror depend on the position of the object with respect to the mirror.

(1) Typical cases are explained by the following diagrams drawn according to the rules stated in Art. 32.

(a) Concave Mirror.—

(i) Object at Infinity—Rays from any point of the object at an infinite distance are parallel. These parallel rays, after reflection, converge to a point in the focal plane of the mirror (Fig. 34). The image is *real, inverted* and *diminished*.

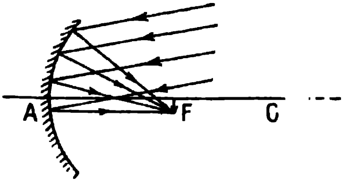


Fig. 34—Object at Infinity

(ii) Object between Infinity and the Centre of Curvature C.—(See Fig. 31). PQ is the object placed beyond C. $P'Q'$ is the image formed between C and the principal focus F, and is *real*, *inverted*, and *dimished*.

(*iii*) *Object at the Centre of curvature*—PQ is the object at C [(Fig. 34 (a)]. A ray from P parallel to the axis, after reflection, passes through F, and another ray $P'B'$ passing through F, after reflection, passes parallel to the axis. These two rays intersect at P', which is the image of P. As all rays from Q pass through Q after reflection, so the image of Q is formed at Q. $P'Q$ is thus the image of PQ, which is *real* and *inverted* : and it can be easily proved that $PQ = P'Q$. So the image is of the *same size* as the object.

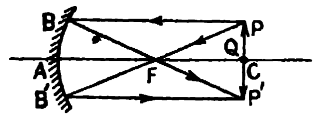


Fig. 34(a)—Object at C

(*iv*) *Object between the Centre C and the Focus F.* The ray PB is taken parallel to the axis and the other CP through C [Fig. 34(b)]. These two after reflection intersect at P' which is the image of P, and Q' is the image of Q. So the image is formed at $P'Q'$ which is *real*, *inverted*, and *enlarged*.

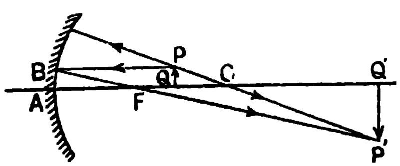


Fig. 34(b)—Object between C and F

(*v*) *Object at Focus.*—(Fig. 34(c)]. This case is the converse of case (*i*). Two rays from P, one parallel to the axis and the other coming in a direction through C, after reflection, become parallel and therefore meet at infinity. The image is *real*, *inverted*, and *infinitely enlarged*.

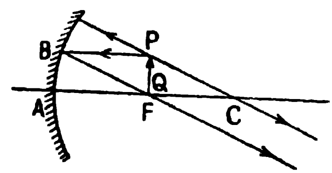


Fig. 34(c)—Object at F

(*vi*) *Object between Focus and Mirror, (i.e. distance less than the focal length).*—The ray PB [Fig. 34(d)], parallel to the axis, passes through F after reflection, and the other ray incident on the mirror and coming in a direction through C retraces its path after reflection and passes through C. The rays form a divergent pencil and will meet at P' behind the mirror, when produced backwards, which is the virtual image of P. The image $P'Q'$ in this case is *virtual*, *erect*, and *magnified*.

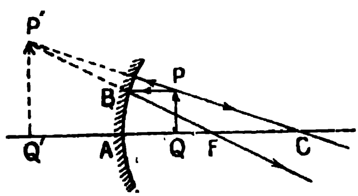


Fig. 34(d)—Object between F & A

(b) Convex Mirror.—This case is given in Fig. 32, where it has been found that the image $P'Q'$ is virtual, erect, and diminished, and it will be found also that for all positions of the object, the image is

virtual, *erect*, and *diminished*. As the object PQ is brought from infinity towards the pole A, the image $P'Q'$ increases from a point size to a size almost equal to PQ.

Remember that in the case of the concave mirror, when the object is at infinity, the image is formed at the focus F, and as the object moves from infinity to the centre of curvature C, the image moves from the focus to the centre of curvature C, being always real, inverted, and smaller than the object, gradually increasing in size. When the object is at C, the image is at C, being real, inverted and of the same size. As the object moves from C towards F, the image sets out from C to infinity, being always real, inverted, and enlarged ; and when the object is at F, the image is at infinity. As the object moves from F towards the pole of the mirror, the image moves from infinity, appears behind the mirror and approaches the pole, being always virtual, erect, and enlarged, and gradually diminishing in size. When the object is at the pole, the image is also at the pole, being virtual, erect and of the same size.

(2) Summary of Results.-

Figures for Mirrors	Position of Object	Position of Image	Nature of Image	Size
Concave Fig. 34	∞	F	Real, inverted	Diminished
Fig. 31	Between ∞ and C	Between F and C	,, ,,	,,
Fig. 34(a)	C	C	,, ,,	Same size
Fig. 34(b)	Between C and F	Between C and ∞	,, ,,	Enlarged
Fig. 34(c)	At F	∞	,, ,,	,,
Fig. 34(d)	Between F and pole	Behind the mirror	Virtual, erect	,,
	Pole	Pole	,, ,,	Same size
Convex	∞	F	Virtual, erect	Diminished
Fig. 32	Between pole and ∞	Between F and pole	,, ,,	,,

(3) Verification from the Formula—**(a)** The positions of the image formed by a **concave mirror** may be obtained from the *formula* as follows :—

(*i*) *Object at infinite distance* ; here $u = \infty$; $\therefore \frac{1}{u} = 0$.

$\because \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \qquad \frac{1}{v} = \frac{1}{f}$.

$\therefore \quad v = f$, *i.e.* image at focus.

(*ii*) *Object between* ∞ *and centre C. Here* $u > 2f$ *and* $< \infty$.

$\therefore$ From the general formula (Art. 31), $\frac{1}{v} > \frac{1}{2f}$ and $< \frac{1}{f}$.

$\therefore \quad v < 2f$ and $> f$, *i.e.* image between F and C.

(*iii*) *Object at C.* Here $u = r = 2f$.

or $\frac{1}{v} = \frac{1}{f} - \frac{1}{2f} = \frac{1}{2f} = \frac{1}{r}$; $\therefore v = r$, *i.e.* image at C.

(*iv*) *Object between C and F.* Here $u > f$ and $< 2f$.

$\therefore \quad \frac{1}{u} < \frac{1}{f}$ and $> \frac{1}{2f}$. $\because \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$,

$\frac{1}{v} > \frac{1}{f} - \frac{1}{f}$ and $< \frac{1}{f} - \frac{1}{2f}$, *i.e.* $\frac{1}{v} > 0$ and $< \frac{1}{2f}$;

$\therefore \quad v < \infty$ and $> 2f$, *i.e.* image between C and ∞.

(*v*) *Object at F.* Here $u = f$. $\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{f} = 0$,

i.e. $v = \infty$, *i.e.* image at ∞.

(*vi*) *Object between F and pole, i.e. mirror.* Here $u < f$.

$\because \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$, *i.e.* $\frac{1}{v}$ is negative. $\therefore$ v is negative. Image is behind the mirror.

Note.—It should be noted carefully that in order to obtain a sharp image with any spherical mirror, only a small part of the sphere of which the mirror forms a part (*i.e.* mirrors with small apertures) must be used. For, by using more of the mirror surface, all rays parallel to its principal axis, after reflection, will not pass through a single point focus and no sharp image is possible (see Art. 36).

(b) Case of Plane Mirror.—If the radius of curvature of a sphe-

rical surface becomes infinity, then the surface may be taken to be a plane surface. So, by putting $r = \infty$ in eq. (2), Art. 31, we get,

$$1 \quad \frac{1}{u} - \frac{2}{\ } = 0 \quad or \quad \frac{1}{v} = -\frac{1}{u} \quad or \quad v = -u$$

i.e. the image is formed on the side of the mirror opposite to the object at a distance equal to the object distance u.

(c) **Case of Convex Mirror.**—

Here $\frac{1}{v} + \frac{1}{u} = -\frac{1}{f}$, since f for a convex mirror is negative;

i.e. $\frac{1}{v} = -\frac{1}{f} - \frac{1}{u}$. Therefore, for all values of u, v is negative, *i.e.* the image is always virtual.

(*i*) When $u = \infty$, $v = -f$. That is, the image is at the focus.

(*ii*) When $u > 0 < \infty$, *i.e.* the object lies anywhere between infinity and pole, $v < -f$.

Thus, the image formed by a convex mirror lies between focus and pole. It is always virtual, erect, and diminished.

38. Conjugate Foci with the Principal focus as the Origin (Newton's Formula).—The equation (2), Art. 31, can be written in the following way.—

$$\frac{f}{v} + \frac{f}{u} = 1 \quad or \quad uv = uf + vf;$$

$$or \quad uv - uf - vf + f^2 = f^2 \quad or \quad u(v-f) - f(v-f) = f^2;$$

$$or \quad (u-f)(v-f) = f^2 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

So, if the distance of the object and its image due to a spherical mirror are measured from the principal focus, and are represented by x and y respectively, then $x = (u-f)$, and $y = (v-f)$; so the equation (1) becomes, $\mathbf{xy = f^2}$ ... ... (3)

Here f^2 is always positive, whether f is positive or negative, so x and y must be of the same sign, *i.e.* they are on the same side of the principal focus.

39. Experimental Method of finding the Radius of Curvature (or Focal Length) of Concave and Convex Mirrors.—

(1) Concave Mirror.

(a) U-V Method.—Mount the mirror M on a stand S placed on a table (Fig. 35) and place a lighted candle A in front of

it so that the middle of the flame is on the axis of the mirror. Inverted image C of the flame is received on the screen B on the same side as the object, when the object is placed at a distance greater than the focal length. Move the screen until the image is quite distinct. Measure the distances of the flame and its image from the mirror. Use the equation

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r} = \frac{1}{f}$$

to find r or f. The expt. may be better performed in a dark room on an *optical bench*.

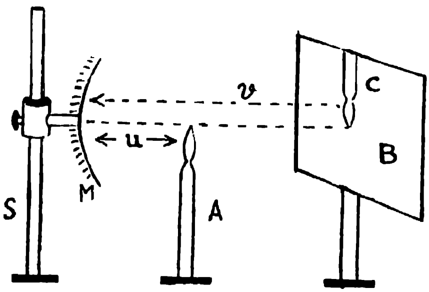


Fig. 35—U-V method

(A pin suitably mounted can also be used as the object and another pin may be placed so that it coincides with the image and there is no parallax between the two, *i.e.* the image and the second pin move together on moving the eyes side to side. In this case no dark room is necessary).

(b) Radius of Curvature and Focal length from Graph.—

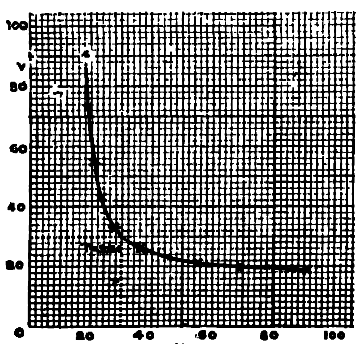


Fig. 36

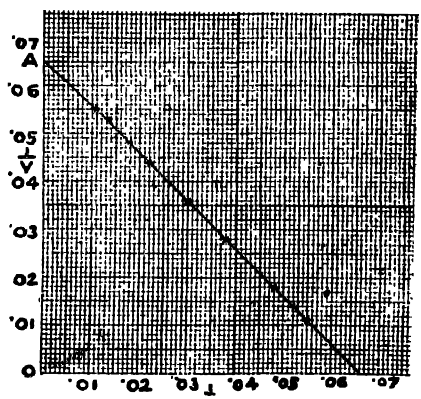


Fig. 37

Plot graphs of (*i*) u and v (Fig. 36), and (*ii*) $\frac{1}{u}$ and $\frac{1}{v}$ (Fig. 37). It is better to choose the same scale for u and v.

(*i*) From the graph of u and v, which is a rectangular hyperbola, we can find r ; for when $v=u$, each $=r=2f$,

$$\left(\because \frac{1}{u}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}\right).$$

On the curve the point for which $v=u$ is that where the line *bisecting* the angle between the axes cuts the curve (when u and v are given in the same scale).

(*ii*) Let $\frac{1}{u}=x$ and $\frac{1}{v}=y$, then the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ takes the form $y+x=\frac{1}{f}=$ a constant. This is the equation of a straight line. Now, from the graph of $1/u$ and $1/v$, which is a straight line (Fig. 37), we can find f because it cuts each axis at a distance equal to $\frac{1}{f}$, for when $u=\infty$, $\frac{1}{u}=0$, and $\frac{1}{v}=\frac{1}{f}$; or $v=f$. In Fig. 37, OA represents the distance for $1/v$ when $1/u=0$. $\therefore$ OA (or the intercept on the $1/v$ axis) $=1/f$. Similarly the intercept on the $1/u$ axis is also $1/f$.

(*iii*) Draw a graph with u along the y-axis and u/v (or $1/m$) along the x-axis. Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, on multiplying every term by u we have $\frac{u}{v}+1=\frac{u}{f}$; or $u=f+f(1/m)$. This is of the form $y=f+fx$; so the graph will be a straight line. When $x=0$, $y=f$; so f is obtained by taking the intercept on the y-axis, when $x=0$.

N. B. Art. 77, which deals with the determination of focal length of a convex lens, may also be read and compared in this context.

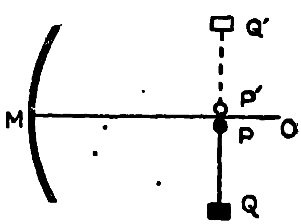


Fig. 88

(c) **Parallax Method.**—In experiment (1) above, using two pins, one as object and the other for location of image, after getting the inverted image move the object pin in such a way that the image and the object become coincident in position, no matter from whatever

position they are viewed,—that is, find the position for *no parallax* (Fig. 38). This is possible only when the object is placed at the centre of curvature. Hence, the distance of the object from the mirror is the radius of curvature of the mirror, and one half of this value will be the focal length.

(2) Convex Mirror.

(a) **Direct Method.**—Clamp the mirror on a stand and place it on a table. Take a piece of paper and draw three clear parallel straight lines (AB, OP, ED) about half an inch apart (Fig. 39). Fold the paper perpendicularly across the lines drawn and arrange the paper, mounted on the edge of a drawing board, so that its plane, when continued, would pass through the centre of curvature of the mirror. The crease of the paper should touch the surface of the mirror, say, at O.

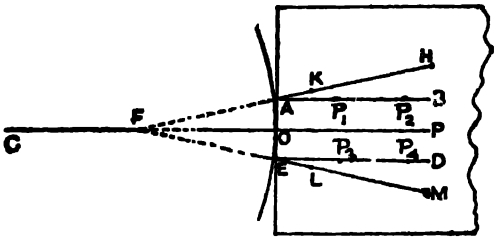


Fig. 39

Adjust the board so that the image of the central line OP and the line itself are in one line, *i.e.* the central line forms a part of the principal axis of the mirror. Find the images of the other two parallel lines AB, ED, which represent rays parallel to the principal axis, by the pin method, *i.e.* placing two pins, say P_1, P_2, on the line AB and locating the image by the other two pins K and H. Similarly, fix two pins P_3, P_4, on ED and locate the images by L and M. Now unfold the paper, join H, K, and M, L, and produce the two lines so obtained to meet at F on the central line. The focal length is the distance of F from the point O.

(b) **Convex Lens Method**—Take a convex lens L (chapter V) of focal length shorter than the radius of curvature of the convex mirror M and place it in front of the mirror (Fig.40). Then put a pin PQ in front of the lens at a distance greater than the focal length of the lens, when a real image of the pin will be formed on the same side of the lens after being reflected back by the mirror. Now adjust the position of the object pin till the image coincides with the object and there is *no parallax* between them. For this condition the rays of light from P, after refraction through L,

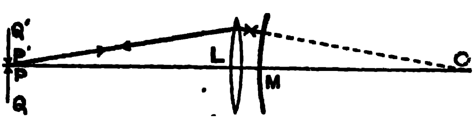


Fig. 40

must fall normally on the mirror, when they will be reflected back along the same path, and this would happen when the distance MO is the radius of curvature of the mirror.

Thus, after proper adjustment is made, measure the distance LM, keeping everything undisturbed. and then remove the mirror. Now take another pin and locate, by the method of parallax, the position O of the real and inverted image of PQ formed by the lens alone without disturbing the position of the object PQ and the lens L. Therefore, the radius of curvature of M

$$=(LO-LM)=MO=2\times \text{ focal length of } M.$$

Take several readings as above and take the mean value as the correct focal length. (This method should be read after Chapter V).

40. Distinction between Mirrors.—A plane, a concave, or a convex mirror, can be distinguished from each other by the *images* formed by each.

(1) Plane Mirror.—A plane mirror gives an *erect* image of the *same size* as the object situated as far behind the mirror as the object is in front. Our *looking glasses* are plane mirrors. A good looking-glass should be of *uniform thickness* and have a *plane surface*. The *silvering* at the back should also be good. Costly mirrors are, sometimes, made by silvering the front face. Ordinary cheap looking-glasses are neither plane nor of uniform thickness.

If the surface is not plane, concavity or convexity of different amounts, though small, will be present at different parts. As a result, different magnification will be caused at different parts of the image which thus will appear distorted.

If the thickness is non-uniform, the angle between any two reflected rays will not be the same as between the relevent incident rays. As a result, the image and the object will not look alike.

If the silvering at the back is good, the image formed by rays reflected at the back will be much more bright than the faint image formed by reflection at the front face and there will be no confusion resulting from multiple images. Due to defects of silvering and use of low quality glass, ordinary cheap mirrors give unsatisfactory images.

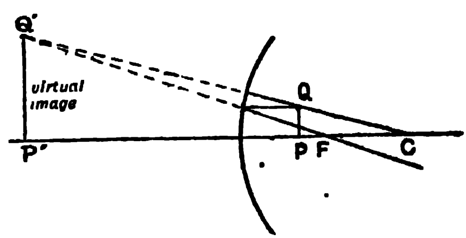


Fig. 41

(2) Concave Mirror.—A concave mirror gives a *magnified*, *errect* image (*virtual*), if it is brought very near to the eye (Fig. 41),

otherwise an *inverted* image is formed, if the observer *(PQ)* stands beyond the focus F of the mirror.

A **Shaving-glass** is a useful form of a concave mirror. A virtual magnified image of the face is produced when the mirror is brought near it, the distance being less than the focal length.

Sometimes surgeons, while examining the internal parts of the ear, nose, or throat, use a small *concave mirror* to throw a narrow but sharp beam of light into the affected parts.

Bright *concave mirrors* are very often used behind a source of light (placed at, or a little behind, the focus of the mirror) as **reflectors** in order to reflect the rays in a certain direction which would otherwise proceed towards the back of the source and would be useless. Thus it increases the intensity of light in a certain direction. Reflectors of table lamp belong to this class.

Parabolic Mirror.—In a concave *parabolic mirror* (Fig. 42) all parallel rays after reflection from any part of the surface pass through the focus, and reversibly the rays from a point source at the focus emerge parallel after reflection. **Parabolic Reflectors** are used for the head-lights of motor-cars, tram-cars, bicycles, etc., where the source of of light is placed just behind the focus in order to produce a slighty divergent beam, which will reach a great distance in front. If the source of light is too far forward, the reflected rays cross each other after only a short distance in front of the reflector.

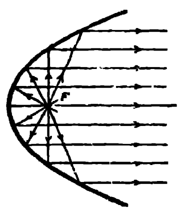


Fig. 42—Parabolic Reflector

Search Lights in steamers, ships, etc., are fitted with concave parabolic reflectors, and generally an *arc lamp* is placed almost at its focus and thus the reflected beam is thrown over a very great distance.

(3) Convex Mirror.—*A convex mirror always gives a virtual, diminished, erect image situated within the focal length* (Fig. 32).

In the case of a *convex mirror*, more objects around the mirror become visible (or, in other words, a larger field of view is obtained) than in the case of plane and concave mirrors. For this reason, **a convex mirror is used (as a reflector) in motor-cars** in order that the drivers can view the approach of vehicles from the rear.

A *plane mirror* to give satisfactory results for the same purpose would have to be far too large to be convenient, but the advantage with the plane mirror is that objects seen by means of it appear in their correct sizes and distances.

Fig. 43 makes these points clear. For the convex mirror BD, the field of view is between the extreme rays XB and YD. For a plane mirror BD (shown by the dotted line) of the same size as the convex mirror, the field of view is only between AB and CD.

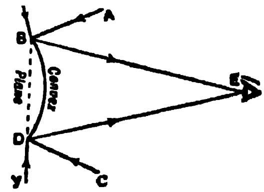


Fig. 43

It should be noted that **street lights** are provided with *convex reflectors* in order that light can be diffused over an extended area, while table lamps are provided with *concave reflectors* so that the rays may converge over a limited area.

(4) **Cylindrical Mirror.**—This kind of mirror forms part of a cylinder and will act partly like a plane mirror and partly like a spherical mirror. Thus distorted images of a body, say tall and thin, or short and fat, are obtained on standing in front of such a mirror. They are very often used for shop-window-advertisements.

40(a). Identification of Mirrors.—

Nature of Image	Mirror
Erect, same size as object ...	Plane
Erect, magnified	Concave
Inverted, magnified or diminished	Concave
Erect, diminished	Convex

41. Solution of Examples.—In solving numerical examples follow the rules given below.

(*a*) *First put down the data of the example with their proper signs.* (*b*) *In the general formula of mirrors, substitute the numerical values of u, v, etc., with their proper signs, and do not change the signs of any of the distances whose numerical values are not given. Then solve the equation and draw the conclusion from the sign of the distance or magnification, as the case may be.*

Conclusion.—(*i*) *The positive sign of the focal length (or radius of curvature) will mean that the mirror is concave, while the negative sign will mean that the mirror is convex.*

(*ii*) *If the value of* **m** *is negative, the image is real and inverted and is formed in front of the mirror, and if* **m** *is positive, the image is erect and virtual and is formed behind the mirror.*

Examples.—*1. A pin 3 cms. long is placed with its middle point at a distance of 1·5 metres from a concave spherical mirror whose radius of curvature is 50 cms. Find the position and the size of the image formed.* (*C. U. 1925*)

1·5 metres = 150 cms. For a concave mirror, we have $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$.

$\therefore \quad \frac{1}{v}+\frac{1}{150}=\frac{2}{50}=\frac{1}{25}$; whence $v=30$ cms.

$\therefore$ The image is a distance of 30 cms. in front of the mirror.

Again, $\frac{\text{size of image}}{\text{size of object}}=\frac{v}{u}=\frac{30}{150}=\frac{1}{5}$. $\therefore$ Size of image $=3\times\frac{1}{5}$ cm. $=0{\cdot}6$cm.

2. The sun subtends an angle of half a degree at the pole of a concave mirror which has a radius of curvature of 15 metres. Find the size of the image of the sun formed by the concave mirror. (*Pat. 1943*)

Here, u = distance of sun, v = image distance ; $f=15/2=7{\cdot}5$ metres $=(7{\cdot}5\times 100)$ cms. $=v$. We have $\frac{1^\circ}{2}=\frac{1}{2}\times\frac{\pi}{180}$ radians.

Again, angle in radians $=\frac{\text{arc}}{\text{radius}}=\frac{\text{diameter of sun}}{\text{distance of sun}}=\frac{\pi}{360}$.

But $\quad \frac{\text{diameter of image}}{\text{diameter of sun}}=\frac{v}{u}=\frac{\text{distance of image}}{\text{distance of sun}}=\frac{7{\cdot}5\times 100}{\text{distance of sun}}$

$\therefore \quad \frac{\text{diameter of image}}{7{\cdot}5\times 100}=\frac{\text{diameter of sun}}{\text{distance of sun}}=\frac{\pi}{360}$

$\therefore$ Diameter of image $=\frac{7{\cdot}5\times 100\times 22}{360\times 7}=6{\cdot}46$ cms.

3. How far from a concave mirror of radius 2 ft. would you place the object to get an image magnified 3 times? Would the image be real or virtual? (*All. 1932*)

Here, magnification $=v/u=3$; *or* $v=3u$; $r=2$

(*i*) If the image is real and inverted,

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}. \therefore \text{ We have } \frac{1}{3u}+\frac{1}{u}=\frac{2}{2}=1 \text{ ; } or \quad \frac{4}{3u}=1.$$

or $u=\frac{4}{3}$ ft. and $v=3\times\frac{4}{3}=4$ ft. In this case the image is real and inverted.

(*ii*) If the image be virtual, then v is negative ; thus we have,

$$-\frac{1}{v}+\frac{1}{u}=\frac{2}{r}\text{ ; } or \ -\frac{1}{3u}+\frac{1}{u}=\frac{2}{2}=1\text{ ; whence } u=\frac{2}{3}\text{ ft.}$$

In this case the image is virtual and erect.

4. A convex and a concave mirror of radii 10 cms. each are placed facing each other and 15 cms. apart. An object is placed midway between them. Find the position of the final image if the reflection first takes place in the concave and then in the convex mirror. (*Pat. 1928, '30*)

As the object is placed midway between the mirrors, the object distance in the first case $u = \frac{15}{2}$ cms. ; $r = 10$ cms. ; $v = ?$ We have,

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}; \quad i.e. \quad \frac{1}{v} + \frac{1}{\frac{15}{2}} = \frac{2}{10} = \frac{1}{5} \quad \text{whence } v = +15 \text{ cm.}$$

That is, the image is formed on the pole of the convex mirror and hence the final image is also formed on the pole of the convex mirror. ∴ The final image will be virtual, inverted and of the same size as the image formed by the concave mirror.

5. *An object is placed 18 cms. away from a concave mirror whose focal length is 10 cms. ; find the position and the size of the image, if the object be 4 mms. broad by 12 mms. long.* *(Pat. 1928)*

Here, $u = 18$ cms. ; $f = 10$ cms. ; $v = ?$

We have, $\frac{1}{v} + \frac{1}{18} = \frac{1}{10}$ whence $v = 22.5$ cms.

Magnification $= \frac{45}{2} \div 18 = \frac{5}{4}$ Length of the object $= 1.2$ cms., and breadth $= 0.4$ cm. ∴ Length of the image $= 1.2 \times \frac{5}{4} = 1.5$ cms.

Breadth of the image $0.4 \div \frac{5}{4} = 0.5$ cm.

∴ Size of the image $= (1.5 \text{ cm.} \times 0.5 \text{ cm.}) = 0.75$ sq. cms.

6. *The image of a gas-flame standing at a distance of 6 ft. from the screen should be magnified three times. Where would you hold the mirror and what sort of mirror you would require ?*

Let x ft. be the distance of the mirror from the gas-flame, then the distance of the mirror from the screen is $(6+x)$ ft. Here $u = x$ ft. ; $v = (6+x)$ ft.

Because the image is three times the size of the object, we have

$\frac{6+x}{x} = 3$; whence $x = 3$. Then $u = 3$ ft. ; $x = 3 + 6 = 9$ ft.

Hence $\frac{1}{9} + \quad \frac{1}{f}$; whence $f = +2.25$

The positive sign shows that the mirror is a concave one of 2·25 ft. focal length and it should be held 9 ft. from the screen or 3 ft. from the gas-flame. (No real image is possible for a convex mirror).

7. ***A convex lens of focal length 24 cms. is placed 12 cms. in front of a convex mirror. It is found that when a pin is placed 36 cms. in front of the lens, it coincides with its own inverted image formed by the lens and the mirror. Find the focal length of the mirror.*** ***(Pat. 1944)***

(See Fig. 40 and read the experiment carefully). In the absence of the mirror the image of P by the lens would form at O. We have,

$$\frac{1}{v} - \frac{1}{36} = -\frac{1}{24}, \text{ whence } v = -72 ;$$

Or $LO = -72$. ∴ $MO = -(72-12) = -60 = 2f$. ∴ $f = -30$ cms.

Questions

Art. 31.

1. An object is placed 28 cms. from a concave mirror whose focal length is 10 cms.; find where the image is. Is it real or virtual ? (C. U. 1926)

2. An object 1 cm. high is placed at a distance of 20 cms. from a concave mirror, and the image is found to be 2 cms. in height and is real. Find the focal length of the mirror. Where must the object be placed in order to give a virtual image 2 cms. high ?

[*Ans* : $13\frac{1}{3}$ cms. : $6\frac{2}{3}$ cms. from pole].

3. A object 3 cms. in height is placed perpendicular to the axis of a concave mirror of 10 cms. focal length and 4 cms. from the mirror. Show where the image is formed. If an observer's eye is 25 cms. from the mirror, what is the least diameter of the mirror necessary for the whole image to be visible at once ? (L. M.)

[*Ans* : $-6\frac{2}{3}$ cms. : $3\frac{18}{19}$ cms.]

3(a). Explain "Conjugate Foci" as applied to a concave mirror. Describe a method of determining the focal length of a concave mirror by finding the distances of the conjugate foci. If these distances are 5″ and 10″, calculate the focal length. (C. U. 1948)

[*Ans* : 10/3 inches.]

Art. 37.

4. You are required to form an enlarged real image of a certain object. How will you obtain it, if the rays are not allowed to suffer refraction. (Pat. 1922)

5. Draw diagram to illustrate the formation of (i) real images, (ii) virtual immages by a concave mirror. (Dac. 1929)

6. Explain by means of diagrams, how the position and size of the image vary with the position of an object for a convex spherical mirror. (C. U. 1922)

7. Distinguish between real and virtual images. Explain and illustrate by sketches the formation of each kind of image in a concave mirror. Explain why only virtual images are formed in convex mirrors. (C. U. 1922)

8. A concave mirror of focal length of 8 cms. is made to approach a rod of length 4 cms. placed perpendicularly to the axis of the mirror. Show by means of typical diagrams on a squared paper the changes in the nature and size of the image. (C. U. 1913, '18)

9. An object is at a distance of 10 cms. from a mirror, and the image of the object is at a distance of 30 cms. from the mirror on the same side as the object. Is the mirror concave or convex ? What is its focal length ? (C. U. 1920)

[**Hints.**—Since the object and the image lie on the same side of the mirror, the mirror is a concave one]. [*Ans* : $f = 7·5$ cms.]

10. Describe the appearance and position of the image produced by a concave mirror as the object moves from infinity towards the mirror. (All. 1932 ; *Cf.* C. U. '33 ; Pat. '37)

11. Show from the formula the variation of the position and the nature of the image when a real object is moved from a great distance up to the mirror. (C. U. 1940)

12. Explain by giving diagrams how a concave mirror can give images of the same linear dimensions when the object is placed at two different distances from the mirror. (Pat. 1929)

Art. 38.

13. x and y are the distances of an object and its image from the focus of a spherical mirror. Show that $xy = f^2$, where f is the focal length of the mirror.

An image produced by a convex mirror is $1/n$th of the size of the object ; prove that the latter must be at a distance $(n-1)f$ from the mirror. (Pat. 1939)

[See Art. 38, and Art. 34, eq. (4), where $m = \frac{1}{n} = \frac{f}{u+f}$ ($\because$ f is $-ve$) ; or deduce from the general equation taking v and f both negative].

Art. 39. (b).

14. Plot on a graph the following values of the distances of object and image for a given concave mirror :—

u...250, 200, 150, 120, 100, 80, 70 cms.

v...60·9, 65·2, 73·2, 84, 96·5, 127·5, 166·5 cms.

State and explain how from this graph you will determine the focal length of the mirror. Find from the graph, or otherwise, the distance of the object from the mirror for which a real image, magnified 1·5 times, will be produced. (Pat. 1938)

(Read the case of convex lens from Art. 77).

Art. 40.

15. You are asked to decide whether a given mirror is convex or concave without touching it. What method would you adopt to ascertain this ? (Pat. 1924 ; All. 1918 ; Dac. '27 ; C. U. 1941.)

16. The driver of a motor car is supplied with a convex mirror in order that he may see the roadway behind him. Explain how he is able to do this. Will a plane mirror serve the purpose equally well ? (Pat. 1938)

17. Explain why the reflection of objects seen in a cheap looking-glass gives distorted images ? (Pat. 1932 ; cf. Dac. '27 ; C. U. '29)

CHAPTER IV

Refraction of Light

42. Refraction.—When a ray of light travels in the same homogeneous medium, it travels straight; but when it passes *from one medium to another* of different density, it will suffer change of direction at the surface of separation between the two media and is said to be *refracted*, except when the ray is normally incident. If the second medium be **denser** than the first, the ray is bent **towards** the normal, and if **rarer** than the first, the ray is bent **away** from the normal. *This phenomenon is known as* **refraction.** The path of the ray in the first medium is called the **incident ray,** and that in the second medium is called the **refracted ray.** In Fig. 44, AO is the incident ray, OB is the corresponding refracted ray, and, because the second medium (glass) is denser than the first medium (air), the refracted ray OB is bent towards the normal PP_1.

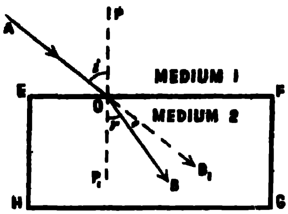


Fig. 44

Historical.—The discovery of the law of refraction is rather interesting. The famous astronomer Ptolemy had some idea of it, but he did not discover the law. The experiment of coin and water described in Art. 50 (Fig. 51) had been known to the ancients. Ptolemy measured the angle of refraction for different angles of incidence, the rays coming from water to air, from water to glass, and from glass to air, but he did not discover either the cause of the phenomenon or establish the law. In the ninth century, the Arab Alhazen made experiments on refraction and in the sixteenth century Vitello, a Polish philosopher, also made experiments, working from the results of which, the famous astronomer, Kepler, obtained a formula for calculating the angle of refraction. Then Snell, professor of mathematics at Leyden, discovered the law of refraction in 1621, but he died in 1626 at the age of thirty-five, without publishing his results. After eleven years the mathematician Descartes, who might have seen Snell's manuscript, published the law and claimed the discovery as his own.

Laws of Refraction :—

1. *The incident ray, the refracted ray, and the normal to the refracting surface at the point of incidence all lie in one plane.*

2. *The sine of the angle of incidence bears to the sine of the angle of refraction a constant ratio for the same two media for the same colour of light.*

This second law was formulated by Snell, and is known as **Snell's Law**, or simply **the Law of Sines.**

43 Refractive index.—When a ray of light passes from a medium (*a*) into another medium (*b*), the ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) is a constant whose value depends on the colour of light. This constant is called the refractive index of medium (*b*) relative to the medium (*a*) and is represented by

${}_a\mu_b$ or μ_b^a ('μ' is pronounced *mu*). That is, ${}_a\mu_b = \dfrac{\sin i}{\sin r}$

When the first medium (*a*) is vacuum, *i.e* the ray travels from vacuum into the medium (*b*), the value of the constant is **maximum**, and is called the **absolute index** of refraction of the medium (*b*).

44. Verification of the Laws of Refraction :—

(i) **Pin method.**—Place a rectangular glass slab $AB\ CD$ on a sheet of paper fixed on a drawing board and draw its outline by a pencil [Fig 44 (a)]. Fix a pin P into the paper in contact with face AB. Fix another pin P_1 at a short distance from the slab so that the direction P_1P is oblique to AB. On looking at these pins through the slab from the side of the face DC, two other pins are fixed, Q_1 in contact with the slab and Q at some distance, so that the four pins appear to be in the same straight line.

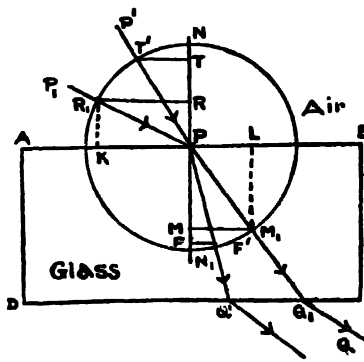


Fig. 44(a)

Remove the slab and join P_1P, QQ_1 and PQ_1. Then P_1P represents *the incident ray, and PQ_1 the refracted ray.* Draw NPN_1 normal at P to the face AB. The angles P_1PN and Q_1PN_1 are the angles of incidence and refraction respectively. With centre P and any convenient radius, lesser in length than the refracted ray PQ_1, draw a circle cutting PP_1 at R_1 and PQ_1 at M_1. From R_1 and M_1 draw perpendiculars, R_1R and M_1M on NPN_1. Now denoting the angle P_1PN by i and Q_1PM by r, we have,

$\sin i = \dfrac{R_1R}{PR_1}$, $\sin r = \dfrac{M_1M}{PM_1}$, and $PR_1 = PM_1$ (being radii of the same circle)

$$\therefore \frac{\sin i}{\sin r} = \frac{R_1R/PR_1}{M_1M/PM_1} = \frac{R_1R}{M_1M}.$$

Measuring R_1R and M_1M carefully with a scale, the ratio can be determined which will be about 1·5. Similarly, considering other incident rays (like $P'P$) and corresponding refracted rays (like PQ'), the above ratio can be determined.

This ratio, for the different incident rays, will be found to be the same. The constancy of this ratio (*refractive index*) proves *Snell's Law* (second law).

To prove the *first law*, fix the pins P_1, P so that they have the same height above the paper, and fix other two pins Q_1, Q such that the four pins are in the same straight line and also the heads of all the pins may appear coincident. Now, on measurement, the pins Q_1, Q will be found to be of the same height as those at P_1 and P. So it shows that a ray of light passing through the heads of the first two pins (P_1, P) passes after refraction through the heads of the other two pins (Q_1, Q). Hence the incident and the refracted rays lie in the same horizontal plane, and, the refracting face of the slab $ABCD$ being vertical, the normal at the point of incidence also lies in the same horizontal plane.

Thus the *First Law* is proved as the traces of the incident ray, the normal and the refracted ray all lie in the same plane.

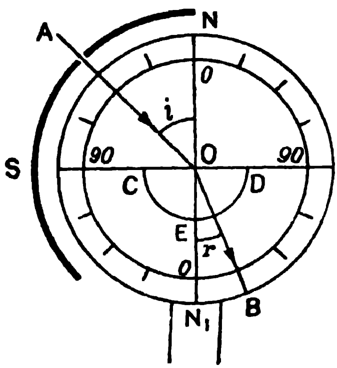


Fig. 44(b)

(ii) Hartle's Optical Disc Method.—A semi-circular glass plate CED is held at the centre of the disc with its plane face CD lying along the $90° - 90°$ diameter of the optical disc in such a way that the $0° - 0°$ diameter passes normally through the centre of the plate [Fig. 44(b)]. A narrow pencil of light AO is adjusted through the silt in the screen (S) to trace its path along the disc and become incident at the centre O of the plate obliquely. The refracted pencil being radial, meets the curved face CED normally and passes out undeviated as OB. The angle of incidence AON and the angle of refraction BON_1 are read off directly from the graduations of the circular scale on the disc. This angle of incidence is then changed by rotating the disc while keeping the screen S fixed and the corresponding angle of refraction is again found out. The operation is repeated for different angles of incidence. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is found to be the same in each case. This proves the second law.

The incident ray, the normal at the point of incidence ($0°-0°$ diameter), and the refracted ray all lie on the plane of the disc, *i.e.*, on the same plane in each case. This proves the first law.

45. Cause of Refraction—The bending of a ray of light entering a different medium where the velocity of light is different is known as refraction. Since a ray of light is refracted towards the normal when it passes from air to glass, *i.e.* from a rarer medium to a denser one, it follows, according to the wave theory of light (see Ch. VIII) that the velocity of light is less in a denser medium than in a rarer one, and it is shown that the refractive index of a medium is inversely proportional to the velocity of light in that medium. Thus,

$_{air}\mu_{glass} = \dfrac{\text{velocity of light in air}}{\text{velocity of light in glass}}$. It has been proved by experiment that the above result is true.

. So, the absolute refractive index of any medium, say glass, is given by,

$$_{vac.}\mu_{glass} = \frac{\text{velocity of light in vacuo}}{\text{velocity of light in glass}}.$$

45(a). Path of the Refracted Ray by Geometrical construction.—

Suppose the refractive index for any two media a and b is known. Given $_a\mu_b = p/q$. It is required to find the direction of the refracted ray in the second medium.

Let AB be the surface separating the two media, and P_1P be any ray incident at P on the surface AB [Fig. 44(a)].

From PA and PB cut off PK and PL equal to p and q units respectively. From K drop a perpendicular KR_1, cutting P_1P at R_1. With centre P and radius PR_1 draw a circle. From L drop a perpendicular LM_1 to cut the circle at M_1. Then PM_1 is the refracted ray. Drop R_1R and M_1M perpendiculars on the normal at P. We have,

$_a\mu_b = p/q = \dfrac{PK}{PL} \dfrac{R_1R}{M_1M} = \dfrac{R_1R}{PR_1} \div \dfrac{M_1M}{PM_1}$ ($\because PR_1 = PM_1$ being radii of the same circle) $= \dfrac{\sin P_1PN}{\sin M_1PM} = \dfrac{\sin i}{\sin r}$. Hence PM_1 is the refracted ray corresponding to the incident ray P_1P.

45(b). Path of light is Reversible.—

It can be shown by experiment that if a ray M_1P [Fig. 44(a)] is taken in the second medium, it will, after refraction, proceed along PP_1 in the first medium. So the path of a ray is reversible.

So, if μ denotes the index of refraction of the medium b with respect to the medium a, and μ' that of a with respect to b, we have,

$$\frac{\sin i}{\sin r}=\mu, \quad \text{and} \quad \frac{\sin r}{\sin i}=\mu' ; \quad \therefore \; \mu'=\frac{1}{\mu}.$$

Hence, if the medium a be air and b be glass, and if μ represents the refractive index of glass with respect to air, the refractive index of air with respect to glass is given by $1/\mu$. Thus the refractive index from air to glass is $\frac{3}{2}$ and that from glass to air is $\frac{2}{3}$.

46. Refraction through a Parallel Plate.—Let AB, DC be the parallel faces of a glass plate and let OP be a ray incident at the point P making an angle i with the normal, and PQ the corresponding refracted ray in glass (Fig. 45). It is bent towards the normal having r as the angle of refraction. The ray PQ again emerges out into air in the direction QR bending away from the normal.

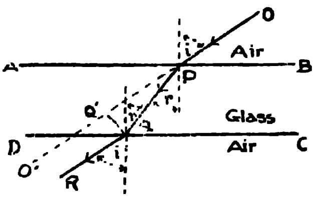


Fig. 45

Lateral Displacement—The emergent ray QR is parallel to the incident ray OP, and it is not deviated by the parallel-sided block of glass but **only displaced**, that is, moved sideways. This sideways shifting is known as *lateral displacement*, the magnitude of which (QQ' in Fig. 45) is measured by the perpendicular distance between the paths of incident and emergent rays. The amount of lateral displacement depends on (a) the thickness of the block, (b) the angle of incidence, and (c) the value of μ.

We have, $$\frac{\sin i}{\sin r}={}_a\mu_g \qquad \ldots \qquad \ldots \qquad (1)$$

Again, for refraction at the second face, $$\frac{\sin r'}{\sin i'}={}_g\mu_a \qquad \ldots \qquad (2)$$

But $$\frac{\sin r'}{\sin i'}=1\Big/\frac{\sin i'}{\sin r'}=1\Big/{}_a\mu_g \qquad \ldots \qquad \ldots \qquad (3)$$

$\therefore$ $\dfrac{\sin i'}{\sin r'}={}_a\mu_g$. Hence, from (1) and (3), $\dfrac{\sin i}{\sin r}=\dfrac{\sin i'}{\sin r'}$

But the two normals are parallel to each other as the sides AB and DC are parallel, so the angle r = the angle r'.

Hence $\sin i=\sin i'$; or $i=i'$; or QR is parallel to OP.

Thus, *a ray of light in passing through a parallel-faced plate emerges parallel to the original direction having suffered a lateral displacement only.*

Normal Incidence—If the ray OP meets the surface normally, then $i=0$, and since $\sin 0^\circ=0$, we have,

$$\frac{0}{\sin r}=\mu \text{ (from equation 1) ; } \therefore \sin r=0\ ; \quad or \quad r=0.$$

Hence the ray passes straight all throughout without deviation or any lateral displacement.

Lateral Displacement—It is given by QQ'. But $QQ'=PQ \sin QPQ'$ $=PQ \sin (i-r)$. Again, $\cos r=\frac{\text{thickness of slab}}{PQ}$;

$$\therefore \quad QQ'=PQ \sin (i-r)=\frac{\text{thickness of slab}}{\cos r}\times \sin (i-r).$$

If thickness, the value of refractive index $\left(\text{cf. } \mu=\frac{\sin i}{\sin r}\right)$, and the angle of incidence are given, the lateral displacement QQ' can be found.

47. Passage of a Ray through Several Media.—

If, instead of two media, air and glass, we have got air, water, glass (Fig. 46), and if the last medium be again air, then an incident ray OP, after passing through these media, will again emerge parallel in air as RS, the surfaces of these media being supposed to be parallel to one another.

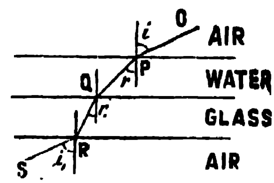


Fig. 46

$$\text{Then} \qquad {}_a\mu_w=\frac{\sin i}{\sin r}\ ;\ {}_w\mu_g=\frac{\sin r}{\sin r_1}\ ;\ {}_g\mu_a=\frac{\sin r_1}{\sin i_1}.$$

$$\therefore \quad {}_a\mu_w\times{}_w\mu_g\times{}_g\mu_a=\frac{\sin i}{\sin r}\times\frac{\sin r}{\sin r_1}\times\frac{\sin r_1}{\sin i_1}=1 \ (\text{since } i=i_1$$

from Art. 46).

The refractive index is generally expressed with reference to air.

Example.—*The index of refraction of water is $\frac{4}{3}$, and of glass $\frac{3}{2}$. Find out the index of refraction from water to glass.*

We have, as in Art. 47, ${}_w\mu_g\times{}_g\mu_a\times{}_a\mu_w=1$; *or* ${}_w\mu_g\times\frac{2}{3}\times\frac{4}{3}=1\ (\because {}_g\mu_a=\frac{2}{3})$

$\therefore$ ${}_w\mu_g=\frac{9}{8}$, *i.e.* The index of refraction from water to glass $=\frac{9}{8}$.

48. Deviation of a Refracted Ray.

—The angle of deviation of a refracted ray is the angle between the directions of the incident and refracted rays.

In Fig. 47 (*a*), the ray PO is passing from a rarer to a denser medium, and it will be seen that in this case the refracted *ray OQ is deviated towards the normal.* The angle of deviation D is $P'OQ$

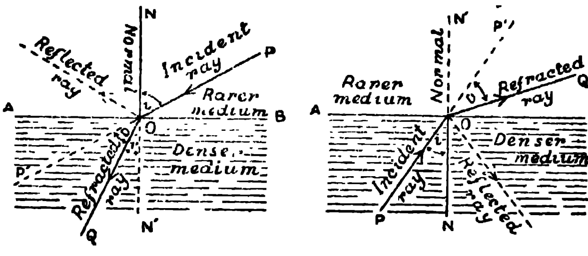


Fig. 47—Deviation of a Refracted Ray

which is equal to $(i-r)$. In Fig. 47 (*b*), where the ray PO is passing from a denser to a rarer medium, *the refracted ray OQ* is *deviated away from the normal.* The angle of deviation D is $P'OQ$, which, in this case, is equal to $(r-i)$.

49 Familiar Illustrations of Refraction.—The phenomenon of refraction explains many peculiar appearances of which *six familiar illustrations* are given below,—

(1) When an Object is in a Denser Medium and viewed from a Rarer Medium.—In this case (Fig. 48), the rays diverging from the object P, which is placed in the denser medium, are bent away from the normal (*i.e.* made more diverging) when emerging into the rarer medium, and so the image P' appears to an eye to be nearer than the object P. *This explains why in reflection experiments with plane mirrors, the silvering (i.e. the reflecting surface) was taken to be at two-thirds* [Art. 53(b)] *of the real thickness from the front side of the glass* (see Art. 13).

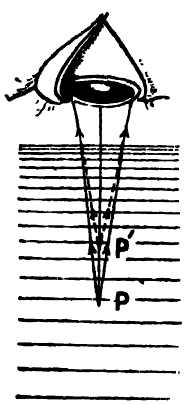


Fig. 48

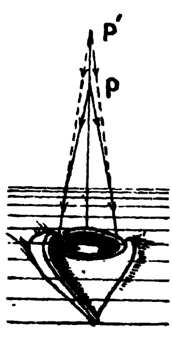


Fig. 49

(2) When the object is in a Rarer Medium and viewed from a Denser Medium—In this case (Fig. 49), the ray from the object P

after entering into the denser medium will be deviated towards the normal. So, to an eye, placed in the denser medium, the object P appears to be raised, *i.e.* the image P appears to be farther from the object.

(3) Put a coin P (Fig. 50) on the bottom of an empty basin, and look in such a way that the coin is no longer visible on the edge of the basin. In this case, the rays coming from the coin pass just above the eye. Keeping the position of the eye fixed, pour water in the basin, and the coin becomes visible. In this case, the rays from the coin P are bent away from the normal on coming out of water, and to the eye they appear to diverge from the position P', *i.e.* the coin appears to be raised and thus becomes visible.

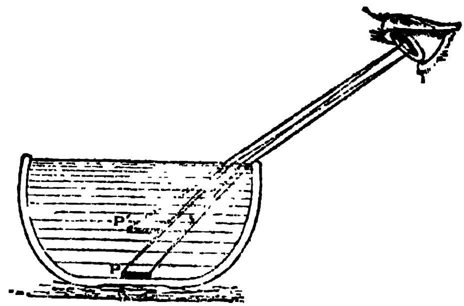


Fig. 50

(4) The same thing happens on immersing a straight rod ABC in water in an oblique position. The part of the rod BC under water appears shortened and the rod appears bent at the surface of separation B (Fig. 51). From the diagram it is clear that each point of the rod under water appears to be raised in proportion to its actual depth, and so the immersed proportion of the rod appears shorter and raised in position ; and thus it appears to be bent at the surface of water.

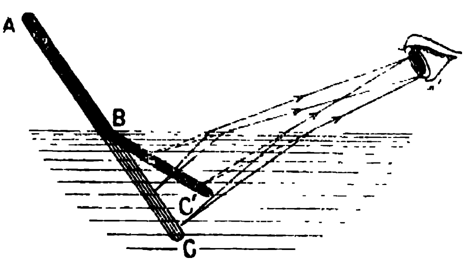


Fig. 51

If the rod is immersed vertically in the water, then the part of it under water will appear smaller by three-quarters of its actual length [vide Art. 53 (a)].

It will be evident from the above diagram that any object will appear to be raised more, when viewed more and more obliquely.

So, when standing at one side of a swimming bath, the water directly under you will appear to be shallower than it really is, and the water at the far side will appear to be even shallower, and the bottom more raised up.

(5) **Atmospheric Refraction.**—Atmosphere, we all know, is less and less dense as its height increases above the sea-level, and it is also noticed that the refractive index of a substance increases with its density. So the refractive index of air decreases upwards owing to the diminution in density. For this reason the rays of light proceeding from a heavenly body, such as a star, cannot travel in straight lines, but are refracted more and more towards the normal as they penetrate different layers of the atmosphere, and as an observer on the earth's surface sees the heavenly body in the direction of the rays reaching him, the altitudes of heavenly bodies always appear too great, *i.e.* they always appear higher up in the heavens than they are. Thus, an observer at A on the earth's surface sees the star S in the direction AS' (Fig. 52). Due to such atmospheric refraction, *the sun is visible for some time before it rises and after it sets, i.e.* when it goes below the horizon. Due to the same reason the *sun*, when near the horizon, *appears to be oval-shaped*, because rays from the lower edge, which have to pass through a greater thickness of air than rays from the upper edge, are refracted more, and so the vertical diameter of the sun appears to be diminished in size, whereas the horizontal diameter remains unaltered.

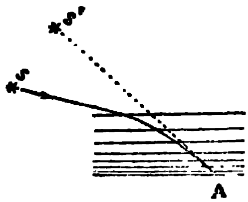


Fig 52

(6) **Twinkling of Stars.**—It is also caused by atmospheric refraction. Refractive index of air varies at different points of the atmosphere as the convection currents continually change temperatures of the air and so the paths of the rays of light from a star alter from instant to instant. Hence the rays travelling in a given direction are sometimes concentrated at one point and then at another, and so the amount of light reaching the eyes of an observer changes continually or appears to twinkle. Such variation in intensity is not appreciable in the case of planets which are nearer.

50. Mathematical Treatment of the Effect of Refraction on the Position of an Object—(a) *When an Object is placed in a Denser Medium and observed from a Rarer Medium.*—

Let EF [Fig. 53(a)], be the surface of separation of two media a and b, b being denser of the two. Consider a ray PO_1 coming from the point P in the medium b strikes the surface at O_1, very close to O, where the normal PO from P to the surface EF intersects the surface. The ray PO_1 emerges along O_1A_1 in the medium a. The emergent ray O_1A_1 produced backwards into the denser medium intersects the normal at P_1. Similarly, the ray PO_2 will refract along O_2A_2, which,

when produced backwards, will also meet the normal PO at P_1. Hence an eye receiving the rays between O_1A_1 and O_2A_2 would see the image of P at P_1, and OP_1 would be the *apparent depth* of the object when the real depth is OP.

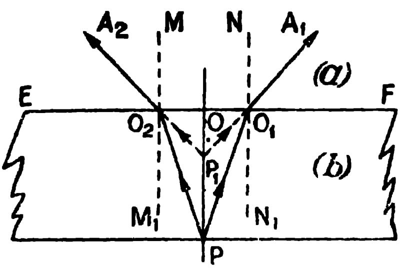


Fig. 53(a)

The emergent ray O_1A_1 makes with the normal at O_1 an angle $A_1O_1N = \angle P_1O_1N_1 = \angle OP_1O_1 = \angle r$, the angle of refraction, and $\angle PO_1N_1 = \angle OPO_1 = \angle i$, the angle of incidence.

$$\text{Hence,} \quad {}_b\mu_a = \frac{\sin i}{\sin r} = \frac{\sin OPO_1}{\sin OP_1O_1} = \frac{O_1P_1}{O_1P}.$$

In order that the pencil of rays considered may enter the eye, O_1 must be near O. So, $O_1P_1 = OP_1$ and $O_1P = OP$, approximately.

$$\text{Hence,} \quad {}_b\mu_a = \frac{OP_1}{OP} = \frac{\text{Apparent depth}}{\text{Real depth}}.$$

$$\therefore \quad {}_a\mu_b = \frac{OP}{OP_1} = \frac{\text{Real deapth}}{\text{Apparent depth}} = 1{\cdot}5 = \tfrac{3}{2}, \text{ in case of } glass.$$

$$= 1{\cdot}333 = \tfrac{4}{3}, \text{ in case of water.}$$

Note.—(*i*) It is evident that the refractive index of a medium can be determined by measuring the real depth and the apparent depth of any object in that medium when looked normally from air.

The above principle is utilised in determining the refractive index of a solid or a liquid by travelling microscrope (see Art. 53).

(*ii*) The result, obtained as above, is only true for the refraction of *nearly normal rays*. For oblique pencil, the apparent position is altered considerably.

50(b). Effect of Refraction in case of a Plane Mirror silvered at the Back.—In Fig. 53(*b*), a plane mirror silvered at the back surface PQ has been shown. A ray AB is incident at the first surface from which very little of it is actually reflected. It refracts into the glass and is reflected from the point of incidence O, which lies on the silvered surface PQ, and subsequ-

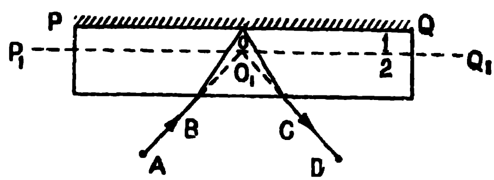


Fig. 53(b)

ently emerges out as CD after suffering refraction again at the first surface. AB and CD, when produced backwards, will meet at O_1, and not at O. That is, the point from which the reflection will appear to have taken place will be at O_1, while the actual point of reflection is O. From this consideration it is clear that the apparent position of the reflecting surface will be given by $P_1O_1Q_1$ as shown in the figure.

Since $\quad {}_{\text{air}}\mu_{\text{glass}} = \dfrac{\text{Real thickness of glass}}{\text{Apparent thickness of glass}}$

$$= \tfrac{3}{2}, \text{ from Art. 50 (a).}$$

∴ The reflecting surface will be at two-thirds of the thickness of glass.

50(c). When an Object is placed in a Rarer Medium and observed from a Denser Medium.—Suppose an object P placed in the rarer medium a is observed from the denser medium b [Fig. 53(c)]. The ray PO_1 falling normally on the surface of separation enters into the medium b undeviated. The slightly oblique ray PO, incident on the surface at O, refracts along OQ. Let NON_1 be the normal at O to the surface of water.

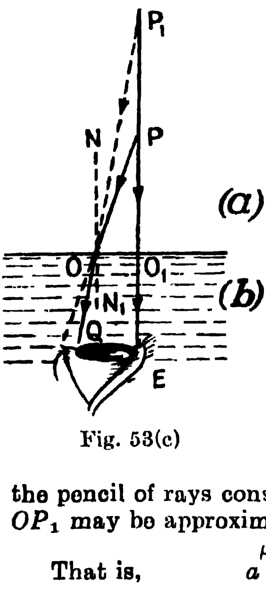


Fig. 53(c)

The two refracted rays produced backwards meet at P_1. Thus P_1 will be the image seen by the observing eye E. If ${}_a\mu_b$ = the refractive index of b relative to a,

$${}_a\mu_b = \frac{\sin PON}{\sin QON_1} = \frac{\sin OPO_1}{\sin P_1ON}$$

$$= \frac{\sin OPO_1}{\sin OP_1O_1} = \frac{OO_1/OP}{OO_1/OP_1} = \frac{OP_1}{OP}.$$

Now O must be very close to O_1 in order that the pencil of rays considered above may enter into the eye. So OP and OP_1 may be approximately be put equal to O_1P and O_1P_1.

That is, $\quad {}_a\mu_b = \dfrac{O_1P_1}{O_1P} = \dfrac{\text{Apparent height}}{\text{Real height}}.$

So, the apparent height of the object above the surface OO_1, as perceived by the observing eye E, will be given by,

$$O_1P_1 = O_1P \times {}_a\mu_b = \text{Real height} \times {}_a\mu_b.$$

50(d). Other Interesting Cases of Refraction.—(*i*) Glass is ordinarily transparent, but when it is powdered it appears white and opaque, because in that case light is reflected from the surfaces of in-

numerable tiny particles of glass and causes the powder appear white. If now water is poured over the powder, the rays are bent, or refracted, and little reflection takes place at the surfaces and so the glass appears transparent again.

Powdered coloured glass also appears white because the colour of the glass is due to the light which passes through it, but in this case the rays of light are reflected at the surfaces before they have penetrated far into the glass and so the powder appears white. The colour is restored by only moistening the powder, as in this case less reflection takes place.

Ordinary paper becomes transparent when oil is added to it, because, in this case, reflection at its rough surface is reduced and more light is allowed to pass through the paper.

(*ii*) It is known that the glare of an ordinary electric bulb, where the light enters the eye only from the direction of the filament, is reduced by frosting, *i.e.* by artificially roughening the inside surface of the bulb by which the light from the filament is refracted in all directions. If, however, the outside of the bulb is roughened, much of the light would meet the surfaces of glass at angles greater than the critical angle (see Art. 51), and thus the rays would be reflected back into the bulb.

(*iii*) If a colourless transparent object like glass is placed in a liquid having the same refractive index as that of the solid, neither reflection nor refraction takes place at the surface of the object, so the object remains completely invisible. An optical illusion may be prepared by keeping a ball of iron on a glass rod dipped in a bottle of glycerol (which has got the same refractive index as that of glass), then the ball will appear to be floating in the liquid.

51. Total Internal Reflection (Critical Angle).—When a ray of light BA passes from a denser medium like glass or water into a rarer medium like air, it is refracted away from the normal, like AF in Fig. 54, and *so the angle of refraction is always greater than the angle of incidence.* If the angle of incidence increases from $\angle BAN_1$ to $\angle CAN_1$, the angle of refraction also increases from $\angle NAF$ to $\angle NAG$; and, for a certain value of the

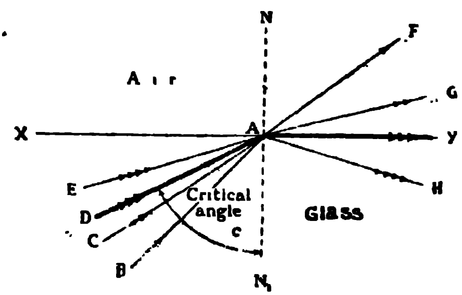


Fig. 54.

angle of incidence, say, $\angle DAN_1$, the angle of refraction becomes 90°,

that is, the refracted ray AY grazes the surface of separation XY. This particular angle of incidence is called the **Critical Angle**, the value of which depends upon the two media and the colour of light used.

If the angle of incidence be increased still further, say, to $\angle EAN_1$, there will not be any corresponding refracted ray. The ray AH, instead of emerging out into the rarer medium, air, is reflected back in the same medium, glass, obeying the law of reflection. The ray AH is said to be totally reflected and such reflection is called **Total Internal Reflection** to distinguish it from all other cases of reflection. This is so called because here no part of the incident light is refracted but the incident light is wholly reflected internally.

If C be the critical angle for the denser medium, and μ the refractive index of the denser medium with respect to the rarer one, we have (since here light passes from the denser medium to the rarer one),

$$\frac{1}{\mu} = \frac{\sin C}{\sin 90^\circ}; \qquad \therefore \quad \sin C = \frac{1}{\mu}; \quad \text{or} \quad \mu = \frac{1}{\sin C}.$$

In other words, the sine of the critical angle is equal to the reciprocal of the refractive index of the denser medium with respect to the rarer one.

Thus C being known, μ can be calculated and *vice versa*.

The value of μ for water is $\frac{4}{3}$. So the critical angle for water is the angle whose sine is $1/\frac{4}{3}$, *i.e.* $\frac{3}{4}$, which is $48^\circ 36'$.

Critical Angle.—*When a ray of light travels from a denser to a rarer medium in such a way that the angle of refraction is 90°, the corresponding angle of incidence is called the critical angle for those two media. It is so called because if the angle of incidence exceeds this angle, there is no refraction and the light undergoes total internal reflection.*

Total Internal Reflection.—*When a ray of light travelling in a denser medium is incident at the surface of a less dense medium such that the angle of incidence is greater than the critical angle for those two media, the ray is wholly reflected back into the denser medium, and is said to be totally reflected.*

Conditions of Total Internal Reflection.—It has been said (see Art. 45) that the velocity of light is less in a denser medium than that in a rarer one, so it follows that *total reflection is possible* only when a ray of light passes from a denser medium to a rarer one *where the velocity of light is greater*. The second condition is that the angle of incidence must exceed the critical angle for the two media.

52. Practical Applications of Total Reflection —

Expts.—(*i*) Insert a test tube into water contained in a glass trough. Gradually tilt the test tube and look at it from above. The tube presents a brightly polished metallic appearance. Rays after passing through water and striking the surface of the tube at an angle greater than the critical angle for water and air (48°36′) suffer total reflection, which gives the surface a shining appearance (Fig. 55).

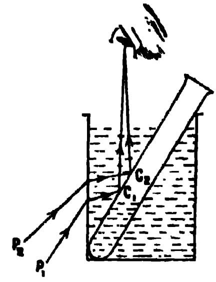


Fig. 55

Now pour water into the tube, and the shining appearance disappears as, in this case, the medium within the tube is changed into water, instead of air, and total reflection does no longer take place.

(*ii*) Similarly a smoked metal ball, introduced into a beaker of water, appears silvery white as water cannot come in intimate contact with the surface of the ball on account of a thin film of air intervening between the surface of the ball and water. Light coming through water is totally reflected at this air film.

When collecting gases in a gas jar, the shining appearance of the bubbles rising in the gas jar is due to light through the water reaching the surface of the bubbles at *angles greater than the critical angle* and so light is *totally reflected.*

(*iii*) **Totally Reflecting Prism** —If a parallel beam of light strikes normally on one of the faces forming a right angled isosceles glass prism, the rays pass undeviated through the first face, but, on meeting the face corresponding to the hypotenuse, they are totally reflected and pass straight through the other face (Fig. 56). The reason for the total reflection is that they are incident on the face corresponding to the hypotenuse at an angle of 45°, which is greater than the critical angle for glass and air, the value of which is about 42°.

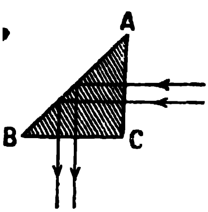


Fig. 56

Advantages of a Totally Reflecting Prism over a Plane Mirror.—

The **prism,** *placed as above,* **acts like a plane mirror** in turning the beam through 90°, and it has the following *advantages* over a metallic reflector or an ordinary mirror :—(*a*) In an ordinary plane mirror with silvering at the back two images may be seen, one due to reflection at the front glass surface and the other at the silvered surface, and the images are not so bright due to sharing of light between them, while in the prism the direction of the rays are changed with the *least loss of light* and thus only one single bright image is obtained. (*b*) In order to avoid the defect of having two images, a plane mirror may be sil-

vered on the front surface and it acts like a good metallic reflector, but both of these will easily tarnish while a prism *will not tarnish.* (An alloy consisting of 68·2 per cent. of copper and 31·8 per cent. of tin, however, has been prepared, the polished surface of which is not tarnished easily in contact with air). Such glass prisms are used in the **periscopes** of submarines by which persons inside the submarines can get a view of ships or other objects on the surface of water by the method described in (*v*) below.

(*iv*) The critical angles of many precious stones like diamond are small owing to their large refractive indices and the brilliancy of these stones is due to this fact. For diamond the critical angle is,

$$c = \sin^{-1}\left(\frac{1}{2{\cdot}47}\right) = 23°53'.$$

When light enters a piece of diamond or ruby at any of its cut faces, the light cannot come out at most of the other faces on account of the critical angle being low, and so it suffers total reflection again and again, for which the faces look brilliant.

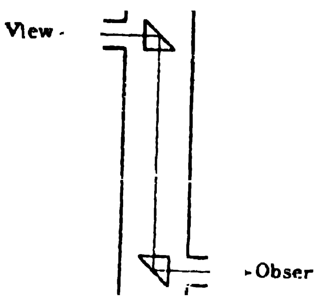


Fig. 57—Periscope

(*v*) **Periscope.**—A **prism periscope** essentially consists of *two right-angled glass prisms*, one fixed at the top and the other at the bottom (Fig. 57), and *a system of lenses* (not shown in the figure) placed between the prisms serving as two telescopes. Rays of light coming from a distant object, say a ship, enter the first prism and are reflected downwards by total internal reflection from the hypotenuse face. Then after passing through the telescopes (Ch. VI), they enter the second prism and are reflected at right-angles in a horizontal direction by total internal reflection again. Thus the observer sees a magnified image of the object from a depth (see also Art. 23).

(*vi*) **Erecting Prism.**—The image of an object formed by lenses as used in projection lanterns (see next chapter) is usually inverted, but in order that it may appear in the right way upon the screen, an **erecting prism** (Fig. 58) is placed in the path of the beam from the lantern. One angle of the prism is 90° and each of the

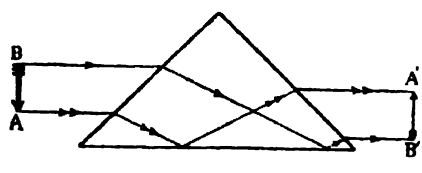


Fig. 58—Erecting Prism

other two angles is 45°. The image is made erect by total internal reflection from the hypotenuse face.

53. Determination of Refractive Index (μ).—

Expt —(Solid)—(1) Take a drawing board on which a paper is fixed. A straight line MN is drawn on the paper and an arrow mark is given at P (Fig 59). A glass cube is placed on the paper such that one of its edges is in contact with the arrow-head at P. A large pin S is fixed horizontally and it can be slided up and down. Now look into the block from the top and adjust the position of the pin so that there is no parallax between the image of the pin P seen through the glass and the end of the large pin seen in the air. Let the position of the pin be at P′. Then,

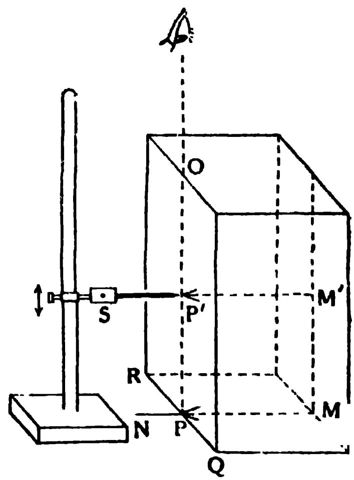


Fig. 59

$${}_a\mu_g = \frac{\textit{real thickness of glass}}{\textit{apparent thickness of glass}} = \frac{OP}{OP'}$$

(2) A travelling microscope EO, *i.e.* a microscope (E is the eye-piece and O the object glass) that can travel up and down a vertical scale S, is focussed on a fine pencil mark on a piece of paper, and the reading d_1 on the vertical scale S is noted [Fig. 59(a)]. A plate of glass is then placed on the mark, the microscope is focussed again on the mark seen through the glass and the position d_2 on the scale S is noted. A little lycopodium powder is then sprinkled on the top of the glass plate and the reading d_3 on the scale is again noted when the powder is best visible. Then,

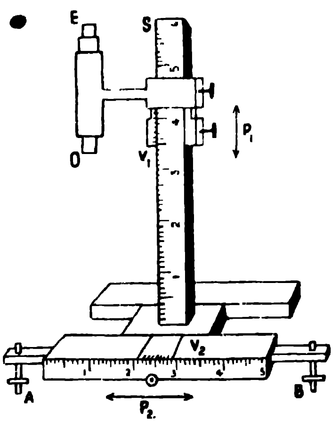


Fig. 59(a)—Travelling Microscope

$${}_a\mu_g = \frac{\textit{real thickness}}{\textit{apparant thickness}} = \frac{d_3 - d_1}{d_3 - d_2}.$$

Liquid.—The above method may be applied in the *case of a*

liquid by first focussing any mark on the bottom of a glass trough and then pouring the liquid and focussing again, and thirdly by focussing on the lycopodium powder sprinkled on the surface of the liquid.

(3) Refractive index of water can be roughly determined by looking down into a tall jar full of water, and marking on the outside of the jar the apparent position of its bottom. This method can be improved as follows.—

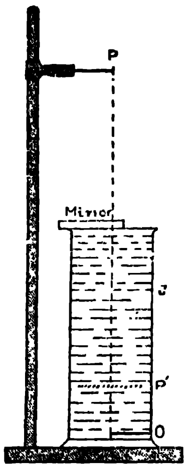


Fig. 60

Expt. Take a gas jar J and measure the depth d of the jar accurately (Fig. 60). Put a pin O in the jar and fill it with water up to the rim. Now place a plane mirror with its face upwards across the top. Fix a pin P in a clamp and adjust its distance so that there is no parallax between the image P' of this pin seen through the mirror and that of the immersed pin O as seen from above.

Then the distance of top pin from the back of the mirror gives the **apparent depth**, while the depth d of the jar gives the **real depth**. Take D to be the mean of several readings of the apparent depth.

$$\text{Thus } \mu = \frac{\text{real depth}}{\text{apparent depth}} = d/D.$$

(4) *Glass Prism Method.*—(See Art. 60).

Example.—The critical angle between glass and air is 42°. Prove that a ray of light incident on a face of a glass cube suffers total reflection at the adjacent face, whatever may be the angle of incidence. (Pat. 1944)

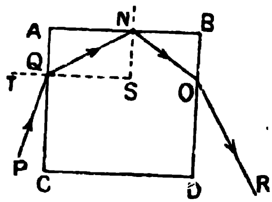


Fig. 61

Let $ABDC$ be a cube. PQ is a ray incident at Q and QN is the corresponding refracted ray which is reflected internally at N and meets the other side at O. OR is the emergent ray. NS and TS are normals. Total reflection will occur when the angle $QNS > 42°$. But $\angle QNS = 90° - \angle NQS$. $\therefore$ $\angle QNS$ is minimum when $\angle NQS$ is maximum. But the maximum possible value of $\angle NQS$ is 42° (when the angle of incidence at Q is 90°, *i.e.* for a ray of grazing incidence). $\therefore$ the minimum value of $\angle QNS = 90° - 42° = 48°$, which is greater than 42°. Hence total reflection will occur at N for any angle of incidence at Q.

(5) Liquid.—Refractive Index of a Liquid by Total Reflection Method.—When a large quantity of a liquid is available, the refractive

index of the liquid can be determined by making use of the relation $\sin C = 1/\mu$ (Art. 51), where C is the critical angle.

Expt.—A thin film of air is enclosed between two glass plates A, B cemented together, as shown in Fig. 62. The plates are so mounted

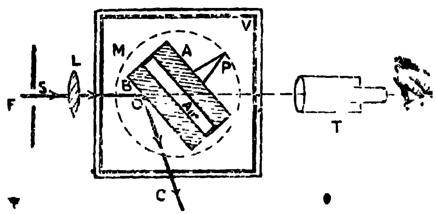


Fig. 62

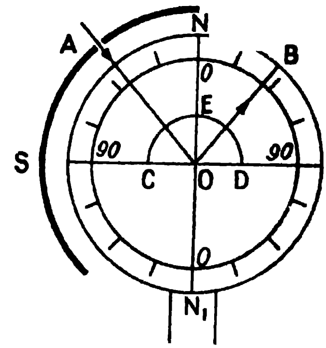


Fig. 63

that they can be turned about a vertical axis and the angle between any two positions can be measured by a pointer P, which moves over a a graduated circular scale M. The plates which are attached below the scale, are immersed in the liquid, say water, contained in a cubical trough V of glass. A beam of light from a sodium burner F proceeds through a silt in the screen S and falls normally on the plates through a convex lens L. The image of the silt is finally received by a telescope T focussed for parallel rays and placed on the other side of the trough and made distinct by adjusting the position of the lens L.

The observer can see the image of the silt when the plates are normal to the beam of light. But, if the plates are turned, a position will come when the angle of incidence from liquid to air is equal to the critical angle, and a little further turning of the plates gives rise to total reflection when the image just disappears. The position of the pointer here is noted. The plates are now rotated in the opposite direction until the image of the slit just disappears again.

The angle between the two positions of the plates, as given by the pointer, is twice the critical angle C for the liquid and air. The index of refraction μ of the given liquid is then given by $\mu = 1/\sin C$ (Art. 51).

(6) Solid—Refractive Index of a Solid by Total Reflection.—

A piece of semi-circular plate CED of the given solid (say, glass) is placed at the centre of Hartle's optical disc (Fig. 63) in such a way

that the plane face COD lies along the $90°-90°$ diameter and the $0°-0°$ diameter passes normally through the centre O, as shown in the figure. A thin pencil of light, coming through the slit and tracing its path along the disc, is made to fall on the curved edge CED. The pencil being radial, emerges out from the plane face COD through O (there being no refraction at the curved face). The disc is suitably rotated until the emergent ray just grazes the plane face COD. The angle of incidence AON at this stage gives the critical angle C. If μ is the refractive index of the solid,

$$\mu = \frac{1}{\sin C}.$$

Examples.—*1 A man is looking vertically downwards into a tank filled with water, the bottom of which appears to be at a depth of 4 ft What is the actual depth, the refractive index of water being 1·33 ?* *(All. 1925)*

We have, air $^{\mu}$ water $= \dfrac{\text{real depth}}{\text{apparent depth}}$; $\therefore\ 1{\cdot}33 = \dfrac{\text{real depth}}{4}$

$\therefore$ Real depth $= 4 \times 1{\cdot}33 = 5{\cdot}32$ ft.

2. A speck in the interior of a piece of plate-glass appears to an observer looking normally into the glass to be 2 mms. from the nearer surface. What is the real distance ? The index of refraction of glass may be taken as $\frac{3}{2}$. *(C. U. 1921)*

We have $_a\mu_g = \dfrac{\text{real distance}}{\text{apparent distance}}$; $\therefore\ \frac{3}{2} = \dfrac{\text{real distance}}{2}$.

Real distance of the speck from the surface $= \frac{3}{2} \times 2 = 3$ mms.

54. Mirage.—The mirage seen in deserts or over any flat heated surface is another example of total reflection. The layer of air nearest the earth is heated most ; it is thus the least dense ; and, in the absence of wind, the atmosphere may become a series of layers of density increasing upwards with the distance from the surface of the earth.

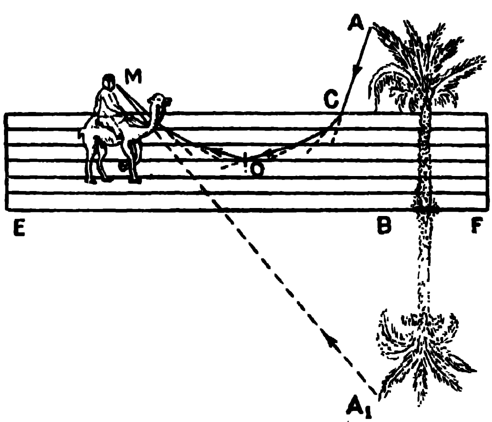


Fig. 64—Mirage in Deserts

In Fig. 64, a ray of light from a tree BA becomes refracted away from the normal in passing from one layer to another layer which is less dense. The refraction is thus increased as the ray passes to different layers and so it travels along a curved

path. The angle of incidence in this way may attain a value at which it just exceeds the critical angle for the two layers at a layer, suppose, at O, when the ray will be totally reflected (Fig. 64). The ray then passes upwards, and the upward ray enters the traveller's eye (at M) who sees the tree in the direction of the light last reaching his eye, *i.e.* he sees an inverted image BA_1 of the tree below the ground. This inversion of the image produces, by the law of association, in the mind of the traveller an impression as if the image is formed by a near pool of water. This optical illusion is called **mirage** Sometimes a thirsty traveller in a desert sees a palm tree and its inverted image by reflection in a pool, but on approaching it, the pool disappears and he thus becomes a cruel victim to this delusion. The quivering of objects seen over coke ovens or other very hot places is due to unequal refraction owing to constant change of density of different layers of air due to convection currents.

Mirage in Cold Regions.—In very cold regions, the lower layers of air are cooler and so the density of air gradually increases downwards. In this case, light from a distant object (the ship in Fig. 65) while proceeding upwards is refracted away from the normal, and the refraction increases more and more in different layers until it is totally reflected. The object ultimately appears to the observer to be hanging inverted in the air as shown in Fig. 65.

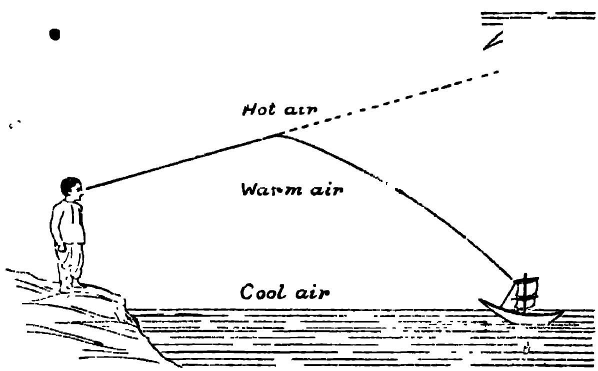


Fig. 65—Mirage in Cold Regions.

55. Upward Vision for an Eye placed under Water.—The greatest angle of refraction from air into water of the rays from objects above the surface of water is the critical angle C for water. That is, rays from objects making angles greater than C with the normals at the points of incidence will not be refracted at all and so the objects will not be visible. Thus an eye placed under water looking upwards will see external objects lying within a vertical cone of angle $2C$. Beyond this cone the eye will see, by total internal reflection, objects

lying below the water surface, and the water surface outside the cone will thus act as a mirror.

• **Example.**—*An eye is placed at a certain depth below the calm surface of water. Show that, to the eye, the surface appears like a reflecting plane mirror with a circular hole through which objects situated outside the water can be seen.*

Also prove that the radius of the hole is $h/\sqrt{\mu^2-1}$ *cm., where* μ *is the refractive index of water and h cm. the depth at which the eye is placed.* (*Pat. 1945*)

[See Art. 55]. If P be the position of eye in water and PO_1 is the ray which after emergence grazes the surface of separation, then the angle i ($\angle PO_1N_1$) in that limiting case (see Fig. 48) will be the critical angle. A ray incident at an angle greater than i will be totally reflected internally, and so the surface will appear like a circular hole of radius OO_1.

Here $OO_1 = h \tan i$ ($h = OP$ in Fig. 48). But $\sin i = 1/\mu$; or $\mu^2 = \text{cosec}^2 i$ $= \cot^2 i + 1$; or $\cot i = \sqrt{\mu^2 - 1}$. $\therefore$ Radius $OO_1 = h/\sqrt{\mu^2 - 1}$.

56. Refraction of Light through a Prism.—A **prism** is a portion of a transparent medium lying between two plane faces inclined at an angle.

The angle of inclination between the two refracting faces is called the refracting angle, or simply the **angle of the prism.** The **edge** of the prism is the straight line in which the refracting faces meet. Any section made by a plane perpendicular to the refracting faces is called the **principal section** of the prism.

Expt.—Place a prism on a sheet of paper fixed on a drawing board and draw its outline ABC (Fig. 66) by means of a pencil. Fix two pins vertically, one at P in contact with the prism and the other at Q at some distance, such that the line joining QP will meet the face of the prism in an oblique direction. Fix two other pins at R and S, on the opposite side of the prism, so that these pins and the refracted images of the pins at P and Q appear to be in the same straight line when looked through the prism. Now remove the prism. Join QP, PR and RS. Produce QP to T and SR to meet

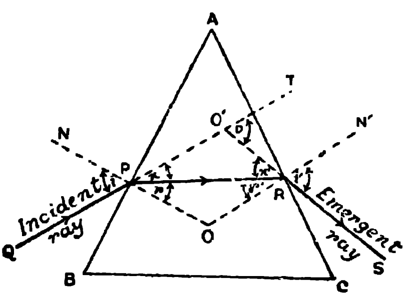


Fig. 66—Refraction through a Prism.

QT at the point O'. Then QP represents the **incident ray**, PR the **refracted ray** within the prism, and RS the **emergent ray**. The line $QPRS$ represents the complete course of the ray. Notice that at P the ray PR is bent towards the normal as it is passing from a rarer to a denser medium, and at R the emergent ray RS, in passing from a

denser to a rarer medium, is bent away from the normal, that is, deviated towards the base BC of the prism. *Thus a ray, in travelling through a prism, bends towards its thicker part.*

56 (a). Deviation of the Ray.—In the absence of any prism the incident ray QP would have proceeded in the direction QPT; so the ray has been turned through an angle $TO'R$ by the introduction of the prism in the path of the ray. This angle, *i.e.* the angle between the direction of incidence and the direction of emergence, is called the **angle of deviation**

Let i be the angle of incidence and r the corresponding angle of refraction at the first face (Fig 66); and let i', r' be the angles of emergence and incidence respectively at the second face. Suppose the normal NO at P meets the normal $N'O$ at R at the point O.

If D = angle of deviation, and A = angle of the prism, we have, $\angle APR + \angle ARP + \angle A = 2$ rt. angles, and $\angle APO + \angle ARO = 2$ rt. angles, for each of them is a right angle.

$$\therefore \quad \angle OPR + \angle ORP = \angle A; \quad or \quad r + r' = A;$$

$\therefore$ We have,

$$D = \angle TO'R = \angle O'PR + \angle O'RP$$
$$= i - r + i' - r' = i + i' - (r + r') = i + i' - A \qquad \ldots (1)$$

57. Measurement of Deviation of a Ray.—

Expt.—Take a sheet of paper fixed on a drawing board and draw a line XY almost in the middle of the paper (Fig. 67). Draw 7 or 8 lines such as NP, N_1P_1, etc., perpendicularly on the line XY (Fig. 67) a few inches apart from each other. Now draw lines such as PQ, P_1Q_1, etc., making angles i, i_1, etc., with each of the normals. Take the first angle i to be 30° and the other angles increasing by 5°, *i.e.* 35°, 40°, etc., up to 60° or 65°. Then place the prism ABC on XY so that one of its faces AB is just on the line and the normal NP is almost in the middle of AB. Draw the outline of the prism with a pencil. Insert a pin vertically at P along the face AB and another at Q, as far as possible, on the line PQ. Looking through the face AC fix two pins at R and S such that these two pins and the images of P and Q appear to be in the same straight line. Then PQ forms the incident and RS the corresponding emergent ray, and $QPRS$ the complete course of the ray. Remove the prism, draw the course of the rays. Produce QP to T and SR to meet QT at O. Then the angle

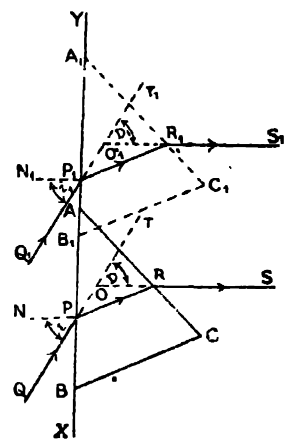


Fig. 67

TOS is the angle of deviation D corresponding to the incident angle i. Measure these angles by means of a protractor. Now place the prism in the position $A_1B_1C_1$ and proceed in the same way to find out the angle of deviation corresponding to the angle of incidence i_1. In this way measure the angles of deviation corresponding to the angles of incidence in each case and tabulate your readings.

It will be found that a ray, in passing through the prism, is on the whole deviated towards the base of prism.

57(a). Angle of Minimum Deviation.—If a graph (Fig. 68) is drawn plotting the angles of incidence as abscissæ against the corresponding angles of deviation as ordinates, called the $i-D$ curve, it will be seen that the deviation at first diminishes with the increase of the angle of incidence until it attains a *minimum value*, represented by the lowest position A' of the curve, after which the deviation increases again with further increase of the angle of incidence. The value of the *minimum deviation is represented* by the ordinate AA' of the lowest point of the curve.

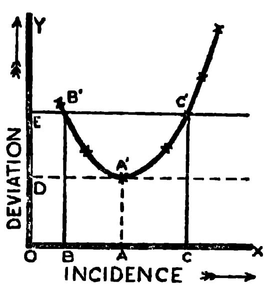


Fig. 68—$i-D$ Curve

Thus the deviation is minimum corresponding to the angle of incidence OA. For every prism *there is such a definite angle of incidence*, corresponding to the angle OA in Fig. 68, for which the deviation suffered by a ray of definite colour in passing through the prism, is minimum, *i.e.* the deviation would be greater both when the angle of incidence is greater or less than the above angle of incidence. It can be shown, both mathematically and experimentally, that when the deviation is minimum, the angle of emergence i' is equal to the angle of incidence i, *i.e.* the refracted ray PR (Fig 66) within the prism passes symmetrically through the prism. If the sides of the prism are of equal length, then in the case of minimum deviation when $i=i'$, the part (PR) of the ray within the prism is parallel to the base BC of the prism.

58. Determination of the Angle of Minimum Deviation.—

(a) i−D Curve Method.—Proceed, as in Art. 57, to measure the deviations D corresponding to angles of incidence i, beginning from a low value of the angle of incidence, say, 20°, which should be increased in steps of 5°, to say 60°. Plot on a graph paper the $i-D$ curve, as in Fig. 68, with the angles of incidence i as abscissæ and the corresponding values of deviation D as ordinates. From the lowest point, such as A', of the curve, draw a perpendicular $A'A$ on the

abscissa. The value of AA' measures the angle of minimum deviation (D_m).

(b) Direct Method.—The angle of minimum deviation may also be approximately determined by keeping the pin P fixed and rotating the prism about P. It will be noticed that the line SR (Fig. 66) and the images of P, Q will appear to move a certain distance in one direction and, after a certain value of D is reached on continuing to turn the prism in the same direction. the images appear first to stop and then to turn back in the opposite direction (opposite to the direction of rotation), *i.e.* they will then retrace the path. Stop the prism just when the change in direction occurs and obtain the directions of the incident and emergent rays in that position of the prism. The deviation measured in that position will be the minimum deviation.

(c) Symmetrical Method.—*It should be remembered that a ray of light suffering minimum deviation in passing through a prism must pass symmetrically through the prism, i.e. the points of incidence and emergence will be equidistant from the edge of the prism. and the angle of incidence will be equal to the angle of emergence : or, in other words,* $AP = AR$, *and* $i = i'$ (Fig. 66).

Proof.—Suppose for minimum deviation the angle i, which is the angle of incidence corresponding to the angle of minimum deviation, is not equal to i'. We know that the path of a ray of light is reversible, so for either of the values i or i', *i e.* whether light travels from Q or S (Fig. 66), the deviation has the same value. So, if minimum deviation occurs when the light travels from Q with the angle of incidence i, it will also occur when the light travels from S with the angle of incidence i'. Hence there will be two angles of incidence corresponding to the angle of minimum deviation which is contrary to experience there being only one such angle. Hence the only position for which the deviation is a minimum is that for which the ray passes symmetrically through the prism, *i e.* for which $i = i'$ and $r = r'$.

Expt—Place a prism on a sheet of paper fixed on a drawing board and draw its outline ABC by means of a pencil (Fig. 69). With A as centre, and any convenient radius, describe an arc of a circle cutting the sides AB and AC of the prism at P and R respectively. Now place the prism on its outline and fix a pin at P and another at R, both in

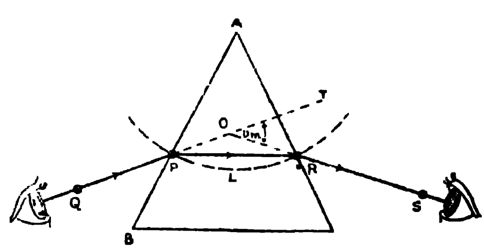


Fig. 69

contact with the prism. Then looking from the side AB fix a pin at Q, such that the pin Q and P and the refracted image of the pin at R appear to be in the same straight line Similarly, looking from the side AC, fix a pin at S, such that the pins S and R and the refracted images of the pins at P and Q appear to be in the same straight line. Now remove the prism and the pins, and draw $QPRS$ to represent the course of the ray. Produce QP and SR to meet at O. The angle TOR is the required angle (D_m) of minimum deviation.

Verification.—(i) This can be verified by noting from the curve (Fig. 68) the angle of incidence corresponding to the minimum deviation and fixing two pins on a line drawn through P making an angle with the normal equal to the same angle of incidence. On finding out the corresponding emergent ray RS, it will be the seen that $AP = AR$, and $i = i'$.

(ii) **By graph.**—Draw a $i - i'$ graph, *i.e.* a graph of the angles of emergence against the corresponding angles of incidence. This will be a branch of hyperbola (like Fig. 85). Now from the $i - D$ curve (Fig. 68) find the angle of incidence corresponding to the angle of minimum deviation, and from the $i - i'$ curve find the corresponding angle of emergence for this angle of incidence. It will be found that these two values are equal, *i.e.* in the position of minimum deviation the angles of incidence and emergence are equal. Draw these two curves ($i - D$ and $i - i'$) on the same piece of graph paper, as taken for Fig. 68, where from the lowest point A' of the $i - D$ curve (Fig. 68) a perpendicular should be dropped which will cut the $i - i'$ curve at a point P (see Fig. 85). This point P will be equidistant from both the axes, *i.e.* for P, $i = i'$. The line OP drawn from the origin O makes an angle of 45° with the axes (Fig. 85).

59. Deviation is minimum when the angle of incidence is equal to the angle of emergence.

(a) **Mathematical Proof.**—Applying Snell's law at the points of incidence and emergence (Fig. 66), we have,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i'}{\sin r'} = \frac{\sin i + \sin i'}{\sin r + \sin r'}$$

$$= 2 \sin \frac{i+i'}{2} \cos \frac{i-i'}{2} \Big/ 2 \sin \frac{r+r'}{2} \cos \frac{r-r'}{2}$$

$$= \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}} \times \frac{\cos \frac{i-i'}{2}}{\cos \frac{r-r'}{2}} = x \times y, \text{ (say)},$$

where x and y are both variable factors. With the change of deviation D, the value of x changes The value of y also changes with changes of i and i' and consequently of r and r'. But the product of x and y is equal to μ and is constant. So, when x is minimum, y must be maximum. But x is minimum when the deviation D is minimum. Thus, the condition of minimum deviation is identical with the condition of the maximum value of y.

$$\text{Now,}\quad y = \frac{\cos\frac{i-i'}{2}}{\cos\frac{r-r'}{2}} = \frac{\cos\theta}{\cos\phi},\quad \text{where } \theta = \frac{i-i'}{2} \text{ and } \phi = \frac{r-r'}{2}.$$

(a) Suppose $i > i'$, and consequently $r > r'$. Again $i - i' > r - r'$, because rotation of the incident ray in air is greater than the rotation of the refracted ray in the denser medium. Thus $\theta > \phi$. If $\theta > \phi$ and consequently $y < 1$.

(b) Suppose and consequently $r < r'$. Again $i - i > r - r$, since rotation of the incident ray in air is greater than the rotation of the refracted ray in the denser medium. Hence $\theta > \phi$.

$$\text{That is,}\quad y = \frac{\cos(-\theta)}{\cos(-\phi)} = \frac{\cos\theta}{\cos\phi}\ ;\ \text{but}\ \theta > \phi.\quad \text{So, } y < 1.$$

$$\text{(c) Suppose } i = i',\ i.e.\ r = r'.\ \text{Then } y = \frac{\cos 0}{\cos 0} = 1.$$

From the above considerations it is clear that y is maximum when $i = i'$, *i.e.* $r = r'$; it has smaller values both when i is greater and smaller than i'. So, the deviation D is also minimum when $i = i'$ (or $r = r'$).

(b) Graphical Proof.—From the relation $D = i + i' - A$ [Art. 56(*a*)], it is evident that the value of the angle of deviation is not changed by interchanging i and i', *i.e.* the angles of incidence and emergence. This fact is also evident from the graph in Fig. 68 ; for a line drawn anywhere parallel to the x-axis will cut the curve generally at two points, such as B' and C', on the two branches of the curve, where the corresponding values of angles of incidence, OB and OC, will possess the same value for deviation BB' or CC'. If one of these is taken as the angle of incidence for a ray, the other will be the angle of emergence, and *vice versa*. This fact is shown from the principle of reversibility of the path of light, for by reversing the direction of the ray $QPRS$ (Fig. 66) the angle of emergence becomes the angle of incidence and the angle of deviation remains unchanged.

Again, if the line $B'C'$ move parallel to itself towards the point A', the points B' and C' will gradually approach each other till they

coincide at A'. It follows then that, when the angle of incidence is equal to the angle of emergence, *i.e.* when $i = i'$ (for the value OA on the graph), the deviation (AA') is a minimum.

59 (c). Minimum Deviation Formula.—

We have, $$D = i + i' - A.$$

In the quadrilateral $APOR$ (Fig. 66),

$$\angle APO = \angle ARO = 90° ; \quad \therefore \quad \angle POR + A = 180° \quad \ldots \quad \ldots (1)$$

But the three angles of the Δ POR are equal to 180°, *i.e.* $\angle POR + (r + r') = 180°$. But when D is minimum, $i = i'$, and $r = r'$;

$$\therefore \quad \angle POR + 2r = 180° \quad \ldots \quad (2)$$

$\therefore$ From (1) and (2), $A = 2r$; or $r = A/2$.

Hence, if D_m denotes the angle of minimum deviation,

$$D_m = \angle TO'R = (i - r) + (i' - r') = i + i' - (r + r') = 2i - A \quad (\because \; i = i' \; ; \; r = r')$$

$$\therefore \quad D_m = 2i - A \; ; \quad or \quad i = \frac{A + D_m}{2}$$

So $$\mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

60. Determination of the Angle of Prism.—

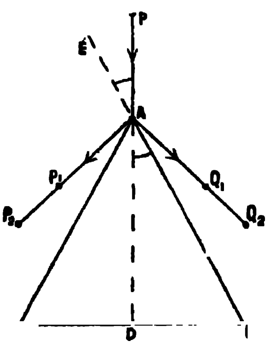


Fig. 70

Experiments.—(1) Place a prism on a piece of paper fixed on a drawing board, and draw its outline ABC (Fig. 70). From A draw a perpendicular AD on BC. Fix a pin at P on DA produced at some distance from A. A ray PA is reflected partly from the face AB and partly from the face AC. Fix two pins at P_1, P_2 such that these two, the edge of the prism, and the reflected image of the pin P appear to be in the same straight line. Similarly, fix two other pins Q_1, Q_2 on the other side. Remove the prism. Join AP_1P_2 and AQ_1Q_2. Measure the angle P_1AQ_1. Half of this value gives the angle of the prism.

Proof :—Produce CA to E. $\angle CAQ_1 = \angle EAP$ (AQ_1 is the reflected ray for the incident ray PA on the face EAC)

$= \angle DAC$ (vertically opposite angle).

$\therefore \angle DAQ_1 = 2\angle DAC$. Similarly $\angle DAP_1 = 2\angle DAB$.

$\therefore P_1AQ_1 = \angle DAP_1 + \angle DAQ_1 = 2\angle DAB + 2\angle DAC = 2\angle BAC = 2\angle A$.

(2) Another Method —The experiment can also be done by taking two parallel rays, one being reflected from the face AB and the other from AC (Fig. 71). Draw two long parallel straight lines DL and PM on a sheet of paper fixed on a drawing board and place the prism with the edge A almost midway between the two lines, and at a distance of about 3 or 4 inches from the upper ends of the lines DE and PQ. Draw the outline of the prism and fix two pins at D and E; and the other two pins at G and F, such that G, F, and the images of D, E appear to be in the same straight line due to reflection at AB. Similarly fix two pins P and Q and other two pins at R and S to get the reflected ray HRS at the face AC. Draw the two reflected rays TFG and HRS as in the last experiment. Now remove the prism and the pins F, G, R, S.

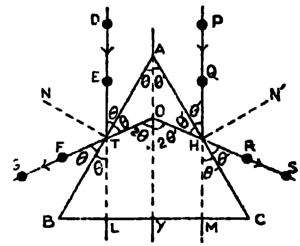


Fig. 71

It is clear from the diagram that the $\angle DTN = \angle NTG$; and the complements θ, θ are equal. $\therefore \angle GTL = 2\theta$; similarly $\angle SHM = 2\theta'$. But, $\theta + \theta'$ is the angle of the prism. If GFT and SRH are produced to meet at O, then since OY and TL are parallel, $\angle TOY = \angle GTL = 2\theta$. Similarly, $\angle HOY = \angle SHM = 2\theta'$; $\therefore \angle TOH = 2(\theta + \theta') = 2A$, *i. e.* twice the angle of the prism.

Measure $\angle TOH$ by a protractor, half of which gives the angle of the prism. Now take the prism a little below between the two parallel lines and repeat the experiment. In this way take several readings and take the mean value as the angle of the prism.

60(a). Determination of Refractive Index (μ) of a Prism :—

(*i*) **Solid.**—The refractive index of a solid in the form of a prism can be determined from the formula (3), Art. 59(c), by knowing the value of D_m, the minimum deviation and A, the angle of the prism.

(*ii*) **Liquid.**—In order to find μ of a liquid take a hollow prism formed by thin parallel plates of glass and provided with a stopper. Fill it up with the liquid. Now determine A and D_m as usual and calculate μ. Though the ray of light is refracted through the glass sides, both at the time of entering and leaving the liquid in the prism, the angle of deviation due to the liquid prism remains the same as the

angle between the directions of the incident and emergent rays (*e.g.* QP and RS in Fig. 66). because the paths in air corresponding to the path PR in the liquid (supposing the glass sides, of the prism to be absent) are parallel to the incident and emergent rays (QP and RS) respectively ; for a ray of light in passing through a parallel-sided plate is only leterally shifted and not deviated (see Art. 46).

61. Image Produced by a Prism.—The image of an object produced by a prism is sharp only when the prism is placed in the position of minimum deviation, where for a small change in the angle of incidence there is no appreciable change in the angle of deviation, so the rays diverging from any point, after refraction, will have almost the same angular separation as before incidence. Hence, when produced backwards, these rays will appear to meet at one point, which is the image of the first point. In any other position of the prism the rays from the point will be deviated unequally, and the angular separation of the rays in this case being different they will not appear to diverge from a single point after refraction. It will be noticed from Fig. 72 that **due to refraction through a prism a luminous object appears to be raised towards the edge** when viewed from the other face.

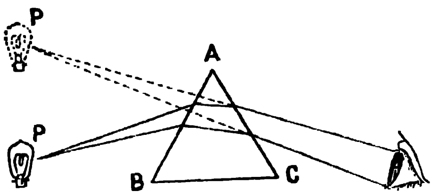


Fig. 72—Image by a Prism

62. Deviation produced by a Prism of Small Angle.—For a ray falling at a small angle on a prism having a small refracting angle (*i.e.* for a thin prism) the angle of refraction is also small.

We have, $D = i' + i - A$; and $\mu = \dfrac{\sin i}{\sin r}$.

But, for small angles, the angles can be substituted for their sines. $\therefore\ \mu = i/r$, *or* $i = \mu r$. Similarly $i' = \mu r'$.

$\therefore\ D = \mu(r + r') - A = \mu A - A = A\ (\mu - 1).$

Since $(\mu - 1)$ *is a constant,* we see that **the deviation produced by a very thin prism depends upon the angle of the prism** *and not on the angle of incidence.*

63. Limiting Angle of a Prism—It is evident from the equation $D = A(\mu - 1)$, that when $A = 0$, D is also zero, *i.e.* when the sides of the prism are parallel, or, in other words, the prism turns into a piece of parallel-sided glass there is no deviation, and it has also been shown that for a prism of small angle deviation does not depend on the angle

of incidence, but for a large prism, *i.e.* for a prism of large angle, deviation depends on the angle of incidence. Now when the value of A increases, a ray QP will be refracted in the direction PRS (Fig. 66). But there is a certain value of A for a prism of every substance, for greater values than which no ray can emerge out of the prism. This angle of the prism is *twice the critical angle* for the substance. If the prism has so large an angle that a ray incident on the first face at 90°, *i.e.* a ray of grazing incidence, strikes the second face of the prism at the critical angle, then a grazing emergent ray is possible.

For this limiting angle of the prism, both r and r' equal C, the critical angle ; and $A = r + r' = C + C = 2C$, and the prism is also in the position of minimum deviation. For a prism of angle greater than this, the ray will be totally reflected and no transmission is possible.

Hence r (or C) $= A/2$ and $\sin r$ (or C) $= 1/\mu$ (Art. 51). $\therefore$ $\sin A/2 = 1/\mu$.

For glass, $\mu = 1{\cdot}5$, critical angle $C = 41°\ 50'$, so the limiting value of A is $83°\ 40'$. So *for angles greater than this*, **no ray can pass through a prism of glass.**

[It can also be shown thus :—For a grazing emerment ray $i' = 90°$, for which the maximum value of $i = 90°$. We have,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i'}{\sin r'}\ ;\ (\text{but } i = i' = 90°)\ ;$$

$\therefore$ $\sin r = \sin r' = 1/\mu = \sin C$, the critical angle for the material of the prism.

$\therefore$ $r = r' = C$; Hence $A = r + r' = 2C$]

Examples—*1. The index of refraction of an equilateral prism is $\sqrt{2}$. If the angle of incidence of a ray of light on one of the faces of the prism is 45°, calculate the angle of emergence and the deviation of the ray.*

We have. $i = 45°$; and $\dfrac{\sin i}{\sin r} = \mu = \sqrt{2}$; *or* $\dfrac{\sin 45}{\sin r} = \sqrt{2}$;

or $\dfrac{1/\sqrt{2}}{\sin r} = \sqrt{2}$; $\therefore$ $\sin r = \frac{1}{2}$; *or*, $r = 30°$.

The angle of the prism $A = 60° = r + r'$ (see Fig. 66). $\therefore$ $r' = 60° - 30° = 30°$.

Again $\mu = \dfrac{\sin i'}{\sin r'} = \dfrac{\sin i'}{\sin 30°} = \sqrt{2}$; *or* $\sin i' = \frac{1}{2} \times \sqrt{2} = \dfrac{1}{\sqrt{2}}$; $\therefore$ $i' = 45°$.

Hence the angle of emergence is 45°.

Now, $D = i + i' - A = 45° + 45° - 60° = 30°$. Hence the angle of deviation $= 30°$.

2. The minimum deviation produced by a hollow prism filled with a certain liquid is 30° ; if the refracting angle of the prism is 60°, what is the index of refraction of the liquid ? *(C. U. 1932)*

We have, $\mu = \dfrac{\sin\dfrac{D_m + A}{2}}{\sin\dfrac{A}{2}} = \dfrac{\sin\dfrac{30+60}{2}}{\sin\dfrac{A}{2}} = \dfrac{\sin 45^\circ}{\sin 30^\circ} = 1{\cdot}4.$

Questions

Arts. 42, 43 & 44.

1. State the laws of refraction of light. Explain how they are experimentally verified. (C. U. 1911, '12, '14, '19 ; Pat. '43, '45.)

Art. 46

2. A candle flame is viewed through (*a*) a prism (*b*) a parallel-sided plate of glass. Explain, with the aid of neat diagrams, the apparent positions of the candle as seen by the eye. (See also Art. 61) (C. U. 1932)

Art. 50.

3. A ray of light passing from air to water falls at a given angle on the surface of the water, the refractive index of which μ is $\frac{4}{3}$. Show the path of the refracted ray by a geometrical construction.

4. A small object is viewed with a microscope through a slab of glass 8 cms. thick. If the refractive index of glass is 1·5, through what distance would the object be raised ?

[*Ans :* $2\frac{2}{3}$ cms.]

5. Explain the apparent raising of a picture stuck on the bottom of a cube of glass, as it appears to an eye looking down, as if it were in the glass. If the index of refraction is 1·6, how much does the picture appear to be raised to a perpendicular vision ? (C. U. '23, cf. '33, '46)

[**Hints.** $\mu = \dfrac{t}{t-x}$, where t is the thickness of the glass cube, and x the distance through which the picture appears to be raised.]

6. A body is viewed through a glass plate 10 cms. thick, the body being 2·5 cms. behind the plate. Where will the body appear to be ? ($\mu = 1{\cdot}5$) (M. U. 1928)

[See Q. 5. Here $x = 3{\cdot}3$ cms. nearly, *i.e.* the body appears to be nearer by 3·3 ; so the body will appear to be at a distance of $(2{\cdot}5 + 3{\cdot}3) = 5{\cdot}8$ cms.].

7. Define index of refraction. A rod is partially dipped in a basin of water. Explain by means of a diagram the appearance presented. (C. U. 1916)

8. A coin placed in a basin is hidden form view by the side of the vessel. When water is placed into the vessel the coin just comes into view. Explain the phenomenon by means of a diagram. (C. U. 1914)

9. Describe the effect of atmospheric refraction on the apparent positions of heavenly bodies. Also explain, with a sketch, how a totally reflecting prism can be used to deflect a beam of light 90° from its original course. (See also Art 52). (Pat. 1918)

Arts. 51 & 52.

10. Deduce from the laws of refraction the condition of total internal reflection of light. Describe some phenomena depending on total reflection. (C. U. 1912, '16, '17, '21, '22, '25, '28, '30 ; Pat. 1919, '27)

11. (*a*) State clearly what is meant by 'critical angle' (Pat. '34, '46). (*b*) Prove that a parallel beam of light incident on a prism emerges out of it also as a parallel beam. (Pat. 1934)

12. State clearly the elementary laws of reflection and refraction of light. Explain how total reflection occurs when a ray of light passes from one medium to another in which the speed is different. (C. U. 1936)

13. What is total reflection and in what circumstances does it occur ? (Pat. 1938, '42 ; All. '44)

14. Bubbles of air coming out through water in a glass vessel appear silvery to an observer standing by the sides. Explain this. (C. U. 1922, '23)

15. A right-angled isosceles glass prism is sometimes used in place of a plane mirror. Explain by the aid of a diagram how it can be used. Is it more advantageous ? If so, why ? (C. U. 1924 ; Pat. 1919, '25, '28)

16. Explain the phenomenon of total reflection. Why is it so called ? Describe a totally reflecting prism, and state its use. (Pat. 1932)

17. Explain the terms 'total reflection' and 'critical angle' and establish the relation between critical angle and refractive index. (Pat. 1944 ; C. U. '44, 46)

If the refractive index of Benzene is 1·5, what is the value of the critical angle ? (C. U. 1946)

[Ans. 41·8].

18. Explain clearly why a smoked ball on being introduced in a beaker of water appears silver white. (Pat. 1930 ; '45)

Also explain the use and construction of a periscope. (Pat. 1930 ; All. 1917)

Art. 53.

19. Define refractive index. How will you determine it for water ? Give full experimental details. (Pat. 1931 ; All. 1919)

20. You are given a block of glass, a piece of paper with a pencil mark, some lycopodium powder and a microscope capable of vertical motion. Explain how you would find out the refractive index of glass. (All. 1924)

21. Explain a method, using the phenomenon of total internal reflection, for finding the refractive index of a liquid. (C. U. 1940 ; All. '44)

22. What is meant by the critical angle for a given refracting medium ? Show how you would measure it. Hence determine the refractive index of the medium. What is total reflection and when does it take place ? (C.U. 1934)

Art. 54.

23. Describe and explain, with the help of a diagram, the phenomenon of mirage. (C. U. 1912, '16, '17, 21, '37)

24. Explain why the mirage is observed in deserts and over very cold water surfaces. (Pat. 1924)

Art. 56.

25. Draw a neat diagram, showing the path of a ray of light through a 60° prism ; the ray makes an angle of 25° with one of the faces, refractive index of the material being 1·5 with reference to air. (Dac. 1932)

Arts. 57, 58 & 59.

26. Prove that in the position of minimum deviation the ray passes symmetrically through a prism. If the prism has a refracting angle of 60° and refractive index $= \sqrt{2}$, calculate the angle of minimum deviation. (Dac. 1930)

[*Ans :* $29^\circ 40'$]

27. Show how the relation between the angle of incidence and the deviation caused in the case of a prism may be determined by the use of pins. (Pat. 1918, '36 : C. U. 1910)

28. What do you mean by the angle of deviation of a ray of light, and when is this angle minimum when the deviation is caused by a prism placed in the path ? (Pat. 1934 ; C. U. '45)

29. The refracting angle of a prism is 60° and the refractive index for sodium light is known to be 1·5. What would be the angle of minimum deviation for the prism for sodium light ? ($\sin 48^\circ 36' = 0·75$). (C. U. 1938)

[*Ans :* $37^\circ 12'$].

30. Explain with theory how you will calculate the refractive index of the material of a prism. (Pat. 1925 Cf. '42 ; C. U. '45)

Art. 60.

31. Explain what is meant by the minimum deviation of a ray passing through a prism. How can you determine it with the spectrometer ? (All. '46)

The refracting angle of a prism is 60°, and the minimum deviation produced in a pencil of monochromatic light is 40°. Find the refractive index of the prism for the light used. Given, $\sin 50^\circ = 0·706$. (See also Arts. 57 & 101) (C. U. 1930)

[*Ans :* $\mu = 1·412$]

32. Explain how you would determine the refractive index of a given glass prism.

Calculate the refractive index of a glass prism for sodium light when the refracting angle of the prism is $45^\circ 4'$ and the minimum deviation of a ray of sodium light passing through it is $26^\circ 40'$. [$\sin 35^\circ 52' = 0·586$ and $\sin 22^\circ 32' = 0·383$] (C. U. 1934)

[*Ans :* 1·53]

Art. 63.

33. Define critical angle. Show that if the angle of a prism be greater than twice the critical angle of glass of which it is made there will be no emergent ray. (Pat. 1929)

CHAPTER V

Lenses

64. Definition—

Lens—A **lens** is a transparent refracting medium bounded by two spherical surfaces, or by one spherical and another plane surface. A lens bounded by cylindrical surfaces is called a cylindrical lens. Lenses are divided into two classes :—

(1) **Convex** (or **converging**).—These lenses are thicker in the middle than at the edges.

Such a lens may have any one of the following three forms :

(*i*) *Double-convex* or *Bi-convex* [Fig. 73(*a*)]—of which both the surfaces are convex.

(*ii*) *Plano-convex*—one side plane and one side convex, Fig. 73(*b*).

(*iii*) *Concavo-convex*—one side concave and the other convex, Fig. 73(*c*)

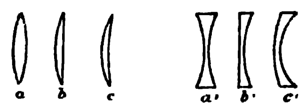


Fig. 73.

(2) **Concave (or diverging)** :—These lenses are thinner in the middle than at the edges. The three forms are :—

(*i*) *Double-concave* or *Bi-concave*, (Fig. 73*a'*), (*ii*) *Plano-concave*, (Fig. 73*b'*), and (*iii*) *Convexo-concave*, (Fig. 73*c'*).

Fig. 74 shows how (*a*) double-convex. (*b*) concavo-convex, (*c*) plano-

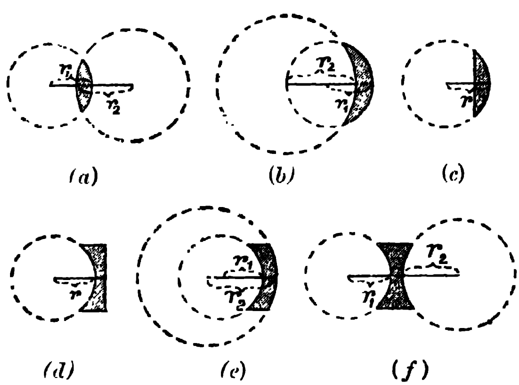
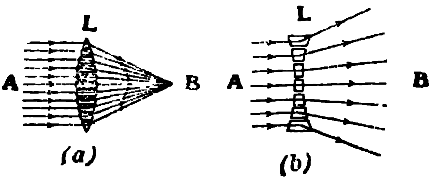


Fig. 74

convex, and (*d*) plano-concave, (*e*) convexo-concave, and (*f*) double-concave lenses are formed of two spherical surfaces, each having a centre and

its own radius of curvature. In plano-convex [Fig. 74(*c*)] and plano-concave [Fig. 74(*d*)] lenses, one surface is spherical and the other is plane, which can be considered also as a spherical surface of infinite radius.

65. A Lens acts like a number of Prisms.—The reason why canvex lenses are described as converging and concave as diverging will be understood by reference to Figs. 75 (*a*) and (*b*). In Fig. 75 (*a*), it will be seen that two sets of truncated prisms are arranged symmetrically about an axis AB with the base of each prism turned towards this line, and the angle of the prism becoming less and less as the axis AB is approached. The deviation in the case of a prism, does not depend on the distance between the two refracting surfaces, but upon the inclination of the surfaces, *i.e.* upon the angle of the prism (see Art. 56*b*). It has been seen that a prism bends rays towards its base, so the above arrangement will bend all rays towards the axis AB and thus will make the beam more convergent as the deviation gradually increases with the approach of the apex. This is the case for a **convex lens**.



Convex Fig. 75 Convex

Similarly (Fig. 75*b*) shows that the section of a **concave lens** may be considered as built up of several truncated prisms with their refracting angles turned towards the axis AB. So, in this case the rays after refraction will bend away from the axis and so the beam will be divergent.

In reality, however, the lenses have continuous curved surface.

66. Optical Centre of a Lens.—Let C and C' be the centres of curvature of the two surfaces of a convex lens (Fig. 76). Draw any radius CA to the first surface and through C' draw a parallel radius $C'B$ to the second surface. These two radii being normal to the surfaces, the two surfaces of the lens are parallel to each other at A and B. So a ray AOB, which is incident at A and cuts the axis at O, will emerge at B in the same direction, as in the case of refraction through a parallel plate (see Art. 46). The ray is laterally displaced, ***the displacement being very small for thin lenses.***

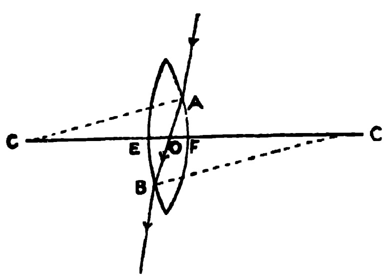


Fig. 76—Optical Centre

CA and $C'B$ being parallel, the triangles CAO and $C'BO$ are similar ; $\therefore \frac{CA}{C'B} = \frac{CO}{C'O}$ but $CA = CF$ and $C'B = C'E$;

$$\frac{CF}{C'E} = \frac{CO}{C'O} = \frac{CF - CO}{C'E - C'O} = \frac{OF}{OE}.$$

If $CF = r$ and $C'E = r'$, we have, $\frac{OF}{OE} = \frac{r}{r'}$, = a constant.

Thus O divides EF in a fixed ratio, and is, therefore, a fixed point. This point is called the **Optical centre** of the lens. It is thus found that any ray, which after passing through the lens emerges out undeviated, will pass through the point O. Thus, *if a ray of light passes through a lens in such a way that the direction of the emergent ray is parallel to the direction of the incident ray, the path of the ray inside the lens intersects the axis (CC', called the principal axis) at a fixed point, which is called the* **Optical centre** or the **centre of the lens.**

Remember that a ray passing through the optical centre is displaced but not deviated : and, in the case of thin lenses, the displacement is very small and can be neglected. *That is, in the case of a thin lens, a ray passing through the optical centre passes out straight.*

(**N.B** —The optical centre of a lens must be carefully distinguished from the centres of curvature of its faces).

To find the position of the Optical Centre :—

Since, $\frac{OF}{OE} = \frac{r}{r'}$, we have $\frac{OF}{OE + OF} = \frac{r}{r' + r}$.

That is, $OF = (OE + OF) \cdot \frac{r}{r' + r} = t \times \frac{r}{r' + r}$;

where t = thickness of the lens at the middle. Similarly,

$$OE = t \times \frac{r'}{r' + r}.$$

N.B. For a double convex or double concave lens, the optical centre is within the lens. For a plano-concave or plano-convex lens, it coincides with the pole of the curved surface. For a concavo-convex or convexo-concave lens, it is outside the lens.

66(a). Some Important Definitions.—

Principal Axis.—The line (CC' in Fig. 76) that joins the centres of curvature of the two bounding surfaces of a lens is termed the **principal axis** of the lens.

Principal Section.—The cut-away section of a lens through the principal axis is termed a **principal section.** The section of the lens shown in Fig. 76 is a principal section.

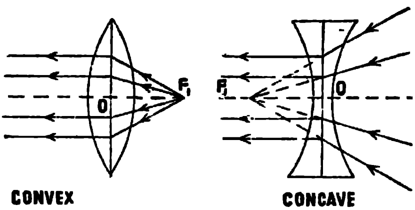


Fig. 76 (a)

Principal Focus.—A lens has two principal foci, the first principal focus and the second principal focus. The second principal focus is conventionally called the principal focus of a lens.

First principal focus is a point on the principal axis such that rays diverging from it (in case of a convergent lens) or tending to converge to it (in case of a divergent lens), are rendered parallel to the axis after refraction through the lens [Fig. 76 (a)].

Second principal focus is a point on the principal axis such that rays incident on the lens in a direction parallel to the axis either actually converges to it (in case of a convex lens) or appears to diverge from it (in case of a divergent lens) after refraction through the lens [Fig. 76(b)]. This second principal focus is conventionally referred to as the **principal focus** of a lens.

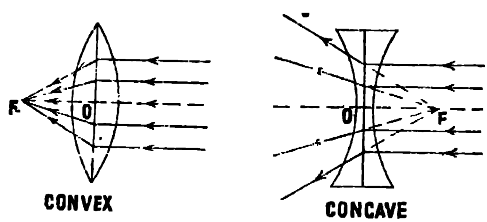


Fig. 76 (b).

Focal Length.—The distance of either of the two principal foci of a lens measured from the optical centre is the same if the lens is surrounded by the same medium on both the sides and is equal to the focal length of a lens. Conventionally, the distance of the second principal focus from the optical centre is called the focal length.

Focal Plane.—When a parallel beam of light falls on a lens in a direction not parallel to the principal axis of the lens, the beam is brought approximately to a focus at a point in a plane passing through the principal focus at right angles to the principal axis. Such a plane is called the focal plane of the lens. The images of very distant objects are formed on the focal plane of the lens.

Aperture of a lens is given by the diameter of the lens.

Thin lens.—A lens is said to be *thin* when the distance between its two surfaces is negligible in comparison with their radii of curvature. The lenses used for ordinary experiments nearly satisfy this condition. For such a lens the points E, O, and F (Fig. 76) may be regarded as coincident, and it may be said that the optical centre is the point in which the lens and the axis intersect.

The defect of a lens as ordinarily used is that when an image of an extended object is formed by it, the image in the region of its principal axis appears to be a true representation of the corresponding portion of the object, but for regions off the axis the image becomes definitely distorted. This effect is visible in the *photographs* of a large object taken by a *cheap camera.*

67. (I) Graphical Construction for Images.—The position of the image is traced by drawing the courses of the following rays :—

(a) *A ray falling on a lens in a direction parallel to the principal axis passes through, or appears to diverge from, the principal focus after refraction.*

(b) *A ray passing through the optical centre of the lens emerges out undeviated and undisplaced.*

The **point** where these two emergent rays converge, as in the case of a *real* image, or from which they appear to diverge, as in the case of a *virtual* image, is the **image** of the luminous point.

(II) (a) **Rules of Signs.**—These are the same as given for mirrors with the difference that the term *pole* in the case of mirrors should be exchanged for *optical centre* in the case of lenses.

So *the* **focal length** *of a* **convex lens** *is* **negative** *and that of a* **concave lens** *is* **positive.**

(b) **New Convention of Signs.**—Formerly it had been the practice (which is still followed in this book) to take the direction along the incident light as negative and that against it as positive ; according to which *a concave mirror* had a positive focal length and positive radius of curvature and a *convex mirror* a negative focal length and a negative radius of curvature. The general formula obtained for both the mirrors was $\frac{1}{v} + \frac{1}{u} \quad \frac{1}{f}$. It should be remembered that, in these cases, real image distances were taken as positive, and virtual image distances as negative.

In the case of **lenses**, however, the distances on the object side measured from the optical centre, *i.e.* the direction against which the incident light traversed, were taken as positive, while the distances on the other side, *i.e.* the direction along the incident light, were taken as negative, which meant that all real image distances were negative and virtual image distances positive ; and thus this had no logical agreement with mirrors. Again, according to this, the focal length of a convex lens, generally producing a real image, had to be taken as negative, while that of a concave lens which only gives virtual images, was regarded as positive. Besides this, there was another difficulty in as much as the opticians have long regarded the convex lenses as positive and concave lenses as negative.

To remedy all these difficulties a new convention of signs was recommended by the Committee of Physical Society in 1934 ; and in many text books this new convention of signs is being used, according to which **all real distances** (*i.e. distances actually traversed by the light either in coming from a real object or in going to a real image*) **are taken as positive** and **all virtual distances** (*i.e. distances traversed by the light only virtually either in appearing to go towards a virtual object or in coming from a virtual image*) **are taken as negative.** According to this, the general formula for lenses would be

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}\,. \quad \text{(Compare eq. (3) Art. 68).}$$

Thus the *focal length of a convex lens* which brings parallel light to a real focus *is positive* and that of *a concave lens is negative*. Hence a **convex lens** *has got a positive power* (see Art. 75) and is called a **positive lens**, while a **concave lens** *has got a negative power* and is called a **negative lens**. According to this, an object and its image may be on the same side of the lens, and yet the object distance may be positive and the image distance negative.

In the case of **spherical mirrors**, a *concave mirror* with real principal focus has a *positive focal length* and positive radius of curvature, while *a convex mirror* with virtual principal focus has a *negative focal length* and negative radius of curvature.

The advantages of the new system are : (*a*) the same formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ holds for both mirrors and lenses ; (*b*) it conforms with the practice all along followed by the opticians.

68. General Formula for Lenses.—

(1) Convex Lens

(a) Real Image. *(Object beyond the principal focus)* :—Let PQ be a luminous object placed vertically on the axis of a thin convex lens RO (Fig. 77) of which O is the optical centre, F the principal focus and FOF the principal axis. To get the image, draw a ray PR parallel to the axis, which after refraction passes through the principal focus F. Draw another ray PO through the optical centre of the lens, which passes out undeviated. The point P', where these rays intersect, is the image of P, and let $P'Q'$ be drawn perpendicular to the axis. Thus, $P'Q'$ *is the image of* PQ. This image is real and inverted.

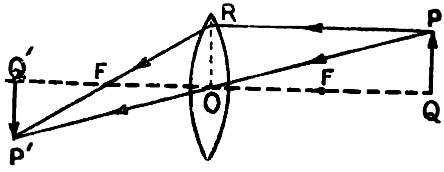


Fig. 77—Convex Lens

Here the triangles PQO and $P'Q'O$ are similar ; $\therefore \dfrac{PQ}{P'Q'} = \dfrac{OQ}{OQ'}$ ...(1)

Again, from the triangles $P'O'F$ and ROF, we have $\dfrac{P'Q'}{RO} = \dfrac{Q'F}{FO}$ ...(2)

Form (1) and (2), $\dfrac{PQ}{P'Q'} \times \dfrac{P'Q'}{RO} = \dfrac{OQ}{OQ'} \times \dfrac{Q'F}{FO}$

But $PQ = RO$; $\therefore 1 = \dfrac{OQ}{OQ'} \times \dfrac{Q'F}{FO}$.

Let $OQ = u$; $OQ' = v$; $FO = f$; Then, we have, by using proper signs, $1 = \dfrac{u\{(-v)-(-f)\}}{-v \times (-f)}$ $[\because Q'F = OQ' - OF = (-v) - (-f)]$;

or $1 = \dfrac{u \times (f-v)}{vf}$; *or* $uf - uv = vf$.

Dividing by uvf, $\frac{1}{v} - \frac{1}{f}$ 1 1 (3)

(b) Virtual Image. *(When the object distance is less than the focal length)* :—

From similar triangles POQ and $P'OQ'$ (Fig. 78),

$$\frac{P'Q'}{PQ} = \frac{OP'}{OP} \qquad (1)$$

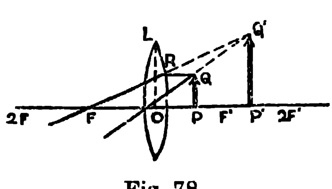


Fig. 78

Again, from similar triangles $P'Q'F$ and ROF, since $PQ = OR$,

$$\frac{P'Q'}{PQ} = \frac{P'Q'}{OR} = \frac{P'F}{OF} = \frac{P'O + OF}{OF} \quad ... \quad (2)$$

$\therefore \quad \dfrac{OP'}{OP} = \dfrac{P'O + OF}{OF}$, from eq. (1) ;

or $\dfrac{v}{u} = \dfrac{v+(-f)}{-f} = \dfrac{v-f}{-f}$; or $\quad uf - vf = uv$;

or dividing by uvf, $\quad \dfrac{\mathbf{1}}{\mathbf{v}} - \dfrac{\mathbf{1}}{\mathbf{u}} = \dfrac{\mathbf{1}}{\mathbf{f}}$ ... (3)

(2) Concave Lens

The method of finding the position of the image due to an object is the same in case of a concave lens as in the case of the convex lens.

The ray PR, incident parallel to the axis, emerges after refraction so as to appear to diverge from the principal focus F (Fig. 79). The ray PO, passing through the optical centre O, emerges out undeviated. The point P', where the paths of these two rays appear to intersect, is the image of P, and $P'Q'$ is the image of PQ. This image is evidently *virtual*, *erect* and *diminished*.

From the two triangles PQO, $P'Q'O$ (Fig. 79), we get

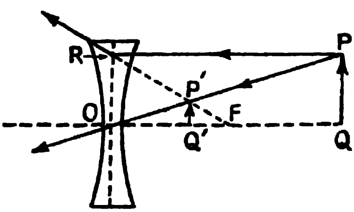


Fig. 79—Concave lens

$$\frac{PQ}{P'Q'} = \frac{OQ}{OQ'} \quad \ldots \quad \ldots (1)$$

Again, from the triangles ROF, $P'Q'F$,

we get, $\quad \dfrac{RO}{P'Q'} = \dfrac{OF}{Q'F} \quad \ldots \quad \ldots (2)$

But $RO = PQ$; $\therefore \dfrac{OQ}{OQ'} = \dfrac{OF}{Q'F}$ ...from (1).

Using the same letters u, v, f as before, we have,

$$\frac{u}{v} = \frac{f}{(f-v)} ; \quad or \quad vf = uf - uv ;$$

Dividing by uvf, $\quad \dfrac{1}{u} = \dfrac{1}{v} - \dfrac{1}{f}$; or $\quad \dfrac{\mathbf{1}}{\mathbf{v}} - \dfrac{\mathbf{1}}{\mathbf{u}} = \dfrac{\mathbf{1}}{\mathbf{f}}$. ... (3)

Conjugate Foci.—The path of light is reversible, so two points, on or near the principal axis of a lens, are said to be **conjugate foci**, when the image of a point source placed at one of them is formed at the other by refraction through the lens. The positions of the object and the image are therefore interchangeable and hence are called conjugate foci (see Art. 31). In Fig. 77, PQ and $P'Q'$ are conjugate foci.

69. (1) Magnification.—The magnifying power of a lens producing an image of an object is defined as the ratio of any **linear**

dimension of the image to the corresponding linear dimension of the object. Thus,

$$\text{Magnification } (\mathbf{m}) = \frac{\textbf{size of image}}{\textbf{size of object}} = \frac{P'Q'}{PQ} = \frac{OQ'}{OQ} = \frac{\mathbf{v}}{\mathbf{u}}.$$

It should be noted that in Fig. 77 the image is *inverted* and v and u have opposite signs, v being negative ; so for lenses, when v/u (*or* m) *is negative, the image is inverted.* Again, in Figs. 78 and 79, the image is erect and both v and u are positive ; so when v/u (or m) *is positive, the image is erect* (Here note that lenses and mirrors differ in this respect).

The magnification may be expressed in terms of u, v, and f as follows :—

(*a*) We have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Multiplying each term by v, we get

$$1 - \frac{v}{u} = \frac{v}{f} \; ; \; \therefore \; \mathbf{m} = \frac{v}{u} = 1 - \frac{v}{f} = \frac{f-v}{f} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

(*b*) Multiplying each term of the general equation by u, we get

$$\frac{u}{v} - 1 = \frac{u}{f} \; ; \quad or \quad \frac{u}{v} = \frac{u+f}{f} \; ; \; \therefore \; \mathbf{m} = \frac{v}{u} = \frac{f}{u+f} \quad \ldots \quad (2)$$

(2) **Remember the Signs of u, v, and f.**—

u	...	+	*always* .
v	...	−	*when the image and the object are on opposite sides of the lens ;*
	...	+	*when the object and the image are on the same side of the lens ;*
f	...	−	*for convex lens ;*
	...	+	*for concave lens.*

So, when working out problems, the following corrections of sign in the general equation are to be applied :—

(*i*) For a **Convex lens** producing real image, both v and f are **negative.**

So the general equation becomes $\frac{1}{-v} - \frac{1}{u} = -\frac{1}{f}$; *or* $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

(*ii*) For a **Convex lens** producing virtual image, $\frac{1}{v} - \frac{1}{u} = \frac{1}{-f}$.

(*iii*) For a **Concave lens**, u, v, f positive, so $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

69(a). Formula according to New Convention of Signs :—

(*i*) *For a* **convex lens** *producing real image*—

In deducing $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ both v and f were taken as negative. According to the new conventions, v and f should be both positive, for they are *real* distances traversed by the rays. Hence the formula will suit the new conventions if v and f, in the above equation, be made negative again.

That is, $-\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$; or $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

(*ii*) *For a* **convex lens** *producing virtual image.*—

In deducing the equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, v was taken as positive and f negative. According to new conventions, they are reverse in this case.

$\therefore \ -\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$; or $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

(*iii*) For a **concave lens**, which always produces virtual image.—

In deducing the equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, v was taken positive and f also positive. According to the new conventions, they being virtual distances are negative. Therefore, the formula should be given by, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

It is clear from the above that, according to the new conventions, the general formula connecting the conjugate foci should be $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, whereas, according to the old conventions, it is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Thus, according to the new conventions, the same general formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, applies to the cases of both lenses and mirrors and a uniformity in practice is attained. [Vide Art. 67 (*ii*)]

70. Nature, Position, and Size of the Image in Case of Lenses.—The position, size and nature of the image formed by a lens depend on the position of the object with respect to the lens. A convex lens forms sometimes a real and sometimes a virtual image according to the position of the object; but a concave lens always forms virtual and erect images. Typical cases are shown below by the following diagrams drawn according to the rules stated in Art. 67(1).

(1) **Convex Lens.**—

(a) *Object at Infinity.*—Parallel rays inclined at a small angle to the principal axis and proceeding from the object will converge in the focal plane of the lens, where the image is formed, which is *real*, *inverted* and *extremely diminished* [Fig. 80 (a)].

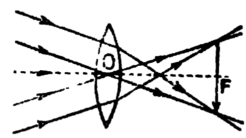


Fig. 80(a).

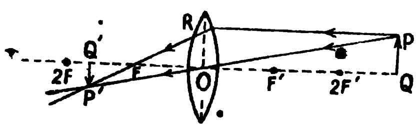


Fig. 80(b).

(b) *Object between Infinity and 2F.*—When the image of an object is drawn, according to the rules already stated, it is seen that the image is formed between F and $2F$ and is real, inverted and diminished [Fig. 80(b)]. This case is applied in an ordinary *photographic camera.*

(c) *Object between 2F and F.*—The image in this case is formed between $2F$ and infinity [Fig. 80 (c)], and is *real*, *inverted* and *enlarged*. This case is applied in the *magic lantern*, *enlarging camera*, etc.

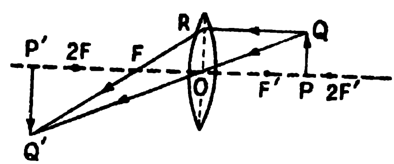


Fig. 80(c).

(d) *Object at 2F.*—In this case the image is also at $2F$ [Fig. 80(d)]. This is also obtained mathematically by putting $u = 2f$ in the general equation; whence $v = -2f$, which means that the image is formed on the other side at a distance equal to $2f$.

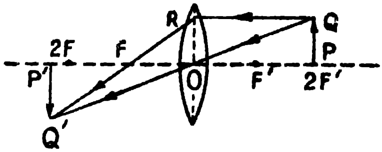


Fig. 80(d).

Again, magnification $m = \frac{-2f}{2f} = -1$, which means that the image is real and is equal in size to the object. This case is applied

in the case of a *terrestrial telescope* (Art. 81) fitted with an erecting eye-piece.

(*e*) *Object at F.*—This is the converse of the case (a). The image is formed at infinity, and is *real*, *inverted* and *infinitely enlarged* [Fig. 80(e)].

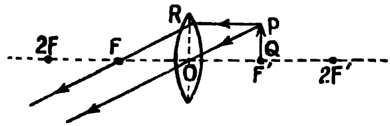


Fig. 80(e).

In the general equation,

if $u=f$, then $\frac{1}{v}-\frac{1}{f}=-\frac{1}{f}$.

or $\frac{1}{v}=0$; or $v=\infty$. Again, magnification $m=\frac{v}{u}=\infty$.

This case is applied in case of the *collimator* of a *Spectrometer* (Art. 100).

(*f*) *Object between F and the Lens.*—The two rays taken from Q diverge after refraction, and meet at Q' when produced backwards [Fig. 80 (f)].

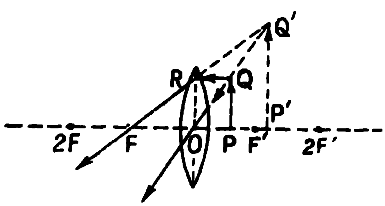


Fig. 80(f).

Draw $Q'P'$ perpendicular to the axis, which is the image of PQ. The image is *virtual*, *erect* and *enlarged*. This case is applied in *magnifying glasses* and *eye-pieces of telescopes* and *microscopes* (see Chapter VI).

Remember :—(*a*) For a **magnified real image** the object is to be placed between F and $2F$. For all other positions from infinity to $2F$, the image is diminished in size, though gradually increasing. It becomes equal in size when the object is at $2F$.

(*b*) For a **magnified virtual image** the object is to be placed between F and the lens. The size of the image diminishes as the object moves towards the pole, and when the object is on the pole, the size of the image which is virtual and erect becomes equal to the object.

(2) Concave Lens.—For all positions of a real object in this case the image is virtual, erect and diminished, though the size of the image slowly increases as the object is gradually brought nearer to the lens (see Fig 79).

71. Identification of Lenses :—To identify by means of the image formed whether a lens is convex or concave ; (*a*) point it towards a distant object and move it towards a white wall or paper screen behind it. If any image is obtained, the lens is convex, other-

wise it is concave ; (*b*) hold the lens *close* to any object, and a virtual, erect and enlarged image will be formed when the lens is convex, which may be viewed by means of an eye placed close to the lens. This is the principle of a magnifying glass or a simple microscope (see Art. 78). If the image be erect and diminished, the lens is concave.

Summary of Results.—

Figures	Position of Object	Image			Remarks
		Position	Size	Nature	
Convex Fig. 80(a)	At ∞	At F	Diminished	Real and inverted	Image and object change their positions.
Fig. 80(b)	Between $2F$ and ∞	Between F and $2F$	Diminished	Real and Inverted	,,
Fig. 80(c)	Between F and $2F$	Between $2F$ and ∞	Enlarged	,,	,,
Fig. 80(d)	At $2F$	At $2F$	Same size	,,	,,
Fig. 80(e)	At F	∞	Enlarged	,,	,,
Fig. 80(f)	Between F and Optical centre	On the same side as object	Enlarged	Virtual and Erect	Lens used as a magnifier.
Concave Lens	∞	At F	Diminished	Virtual and Erect	
Fig. 79	Between ∞ and Optical centre	Between F and Optical centre	,,	,,	

Notes.—(*i*) The different positions mentioned in the above table can also be obtained from the formula (Art. 68) as done in Art. 37(3) in the case of mirrors.

(*ii*) It should be remembered that *in the case of lenses* **2f is not equal to the radius of curvature,** as focal length does not only depend on the curvature of the surfaces but also on the refractive index of the material of the lens with respect to the medium in which the lens is placed.

72. Spherical Aberration in a Lens—In the formation of images in the case of lenses it has been assumed that all the rays coming from a point in any object are accurately brought to focus by the lens to one point. This is, however, not possible for a lens of wide aperture, where the rays striking the outer portions of the lens are refracted more than the rays which strike the central portion of the lens, and thus the marginal rays are brought to a focus nearer to the lens than that for the central rays The effects of spherical aberration are to make the image *curved* and *distorted* in shape. The defect may be reduced by using a *stop*, *i.e.* by taking only the central rays. The defect is also much minimised by using a plano-convex lens, the convex surface facing the incident rays.

Solution of Examples.—*Follow the instructions given in the cases of mirrors, remembering that the focal length of a convex lens is negative while that of a concave lens is positive.*

Examples.—*1. Two convex lenses of focal lengths 3 inches and 4 inches respectively are placed at a distance of 6 inches apart and a luminous object 1 inch high is situated on the common axis of the lenses at a distance of 4 inches in front of the lens of smaller focal length. Find the position, nature and size of the image.* (*Pat. 1927*)

General formula of the lens is, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

Here, for the lens of smaller focal length, $u = 4$; $f = -3$; $v = ?$

Substituting the values, we have $\frac{1}{v} - \frac{1}{4} = -\frac{1}{3}$, whence $v = 12$ inches.

The image serves as an object for the second lens. As the distance between the lenses is 6 inches, the object distance u' for the second lens $= v + 6$ $= -12 + 6 = -6$ inches. So, $\frac{1}{v'} - \frac{1}{-6} = -\frac{1}{4}$; whence $v' = -2\frac{2}{5}$ inches.

That is, the final image is formed at a distance of $2\frac{2}{5}$ inches beyond the second lens, and is real and inverted.

Magnification by the first lens $= \frac{v}{u} = \frac{12}{4} = 3$, and by the second $= \frac{\frac{12}{5}}{6} = \frac{2}{5}$.

Therefore, the total magnification $= 3 \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$.

So, the length of the image $\left(1 \times 1\frac{1}{5}\right) = 1\frac{1''}{5}$.

2. A convex and a concave lens each 10 inches in focal length are held co axially at a distance of 3 inches apart; find the position of the image if the object is at a distance of 15 inches beyond (a) the convex, and (b) the concave lens. (*Pat. 1928*).

We have, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$. (*a*) Here, $u=+15$ inches ; $f=-10$ inches ; $v=$?

So, $\frac{1}{v}-\frac{1}{15}=-\frac{1}{10}$; *or* $v=-30$ inches.

So the image is formed at a distance of 27 inches on the other side of the concave lens, as the distance between the lenses is 3 inches. This serves as an object with respect to the concave lens, the object distance being $u'=u+3$

$=-30+3=-27$. Again, $\frac{1}{v'}-\frac{1}{-27}=\frac{1}{10}$; whence $v'=+15\frac{15}{17}$ inches.

Thus the image is formed $15\frac{15}{17}''$ in front of the concave lens (on the same side as the object).

(*b*) Here, $x=+15$ inches ; $f=+10$ inches ; $v=$?

so, $\frac{1}{v}-\frac{1}{15}=\frac{1}{10}$; *or* $v=6$ inches.

So a virtual image is formed 6 inches in front of the concave lens. This serves as an object with respect to the convex lens, the object distance being $(6+3)$ inches.

So, $\frac{1}{v'}-\frac{1}{9}=-\frac{1}{10}$ (for $f=-10$ inches) ; *or* $v'=+90$ inches.

That is, a virtual image is formed at a distance of 90 inches in front of the convex lens.

3. A real image of an object placed in front of a concave lens of 5 cms. focal length is formed 30 cms. away from the lens. Where must the object be placed? Calculate how far the lens must be moved so that another image may be obtained on the screen.

Here $u=$? ; $v=-30$ cms. ; $f=-5$ cms.

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$; *or* $-\frac{1}{30}-\frac{1}{u}=-\frac{1}{5}$, whence $u=6$ cms.

To obtain the position of another image the lens must be moved into the second of the two conjugate positions for image and object. Thus the image distance v' must now become -6 cms., and the object distance $u'=+30$ cms.

To verify this, $\frac{1}{f}=\frac{1}{v'}-\frac{1}{u'}=-\frac{1}{6}-\frac{1}{30}=-\frac{6}{30}=-\frac{1}{5}$;

$\therefore f=-5$; and so is correct

4. An object is 60 cms. in front a lens, the image being 300 cms. on the other side of the lens. Calculate the displacement of the image when the object is moved 20 cms, (a) nearer to the lens, (b) away from the lens. (*C. U. 1929*).

We have, for lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Here $u = 60$ cms. ; $v = -300$ cms.; $f = ?$

$\therefore \quad -\frac{1}{300} - \frac{1}{60} = \frac{1}{f}$; whence $f = -50$ cms. $\therefore$ The lens is a convex one.

(*a*) Here $u = 60 - 20 = 40$ cms. $\therefore \quad \frac{1}{v} - \frac{1}{40} = -\frac{1}{50}$; whence $v = 200$ cms., (on the same side as the object). Hence the displacement of the image $= 300 + 200 = 500$ cms.

(*b*) Here $u = 60 + 20 = 80$ cms. $\therefore \quad \frac{1}{v} - \frac{1}{80} = -\frac{1}{50}$;

whence $v = -\frac{400}{3} = -133\frac{1}{3}$ cms. (on the other side of the lens). Hence displacement of the image $= 300 - 133\frac{1}{3} = 166\frac{2}{3}$ cms. towards the lens.

5. *Rays of light from a luminous object are brought to a focus at a point A. A convex lens of 12 in. focal length is then placed 12 in. from A, so as to intercept before they meet at A. If they now meet at B, find the distance AB. Where else could the lens be placed so that after passing through it, the rays might appear to diverge from B?* (*Pat. 1926*)

In the first case A serves as a virtual image with respect to the lens. Draw the diagram. Here $u = -12$; $v = ?$; $f = -12$;

Hence $$\frac{1}{v} - \frac{1}{-12} = \frac{1}{-12}\text{ ; whence } v = -6.$$

That is, the image is formed (*i.e.* the rays meet) at B on the same side as A at a distance of 6 inches from the lens. $\therefore \quad AB = 12 - 6 = 6$ in.

Now let the lens be placed at a distance of x inches from A on the side opposite to B.

Hence $u = x$ in. (*i.e.* the distance of A from the lens) ; $v = (x + 6)$ in. (*i.e.* the distance of B from the lens) ; $f = -12$.

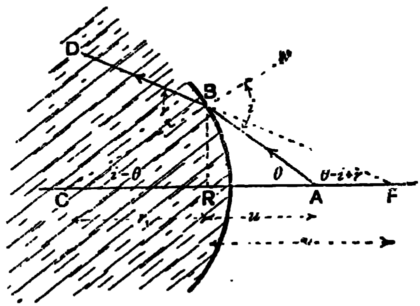
$$\therefore \quad \frac{1}{x+6} - \frac{1}{x} = -\frac{1}{12}\text{ ; or } \frac{x-(x+6)}{x(x+6)} = -\frac{1}{12};$$

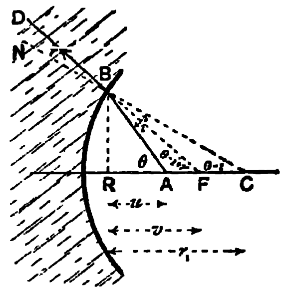
$$\text{or} \quad x^2 + 6x - 72 = 0\text{ ; or } (x-6)(x+12) = 0\text{ ; } \therefore \quad x = 6 \text{ or } -12.$$

The negative value of x is not admissible here, so the lens must be placed at a distance of 6 inches from A on the side opposite to B in order that the rays might appear to diverge from B.

73. Refraction at a Single Spherical Surface.—In Fig. 81 (a), the spherical surface is convex and in Fig. 81 (b), the spherical surface is concave, each of which has its centre at C. Join CB, then CB is normal to the spherical surface at B. Let A, in either diagram, be a

point source of light situated on the principal axis. Consider a single ray AB which, on entering the glass of refractive index μ, is refracted





(a) Fig. 81 (b)

along BD (in glass). The ray BD produced backwards meets the axis at F.

In Fig. 81(a), we have, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin ABN}{\sin CBD}$.

In Fig. 81(b), we have, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin ABC}{\sin NBD}$.

In Fig. 81(a), $\theta = \angle BAR = \angle ABF + \angle BFA = i - r + \angle BFA$

$\therefore \quad \angle BFA = \theta - i + r.$

Again, $i = \angle NBA = \angle BCR + \angle BAR = \angle BCR + \theta$;

$\therefore \quad \angle BCR = i - \theta$. Similarly in Fig. 81(b), $\angle BFR = \theta - i + r$; and $\angle BCR = \theta - i$.

Draw BR perpendicular to the axis ; then considering all the rays in the vicinity of the pole (as done in the case of lenses), the angle θ is small and R nearly coincides with the pole. As small angles may be represented by their tangents, we have from either diagram,

$$\theta = \frac{BR}{AR} = \frac{BR}{u} \qquad \ldots \qquad \ldots \qquad (1)$$

$$(\theta - i) = \frac{BR}{CR} = \frac{BR}{r_1} \qquad \ldots \qquad \ldots \qquad (2)$$

$$(\theta - i + r) = \frac{BR}{RF} = \frac{BR}{v} \qquad \ldots \qquad \ldots \qquad (3)$$

$\therefore$ From (1) and (2), $i = \dfrac{BR}{u} - \dfrac{BR}{r_1}$, and from (2) and (3),

$$r = \frac{BR}{v} - \frac{BR}{r_1}.$$

Remember that in Fig. 81(a), r_1 is negative according to the convention of signs. Again, when angles i and r are small, we may write

$$\frac{\sin i}{\sin r} = \frac{i}{r} = \mu; \quad or \quad i = \mu r.$$

Hence, substituting the values of i and r, we have from above

$$\frac{BR}{u} - \frac{BR}{r_1} = \mu\left(\frac{BR}{v} - \frac{BR}{r_1}\right); \text{ or } \frac{1}{u} - \frac{1}{r_1} = \mu\left(\frac{1}{v} - \frac{1}{r_1}\right);$$

which, when re-arranged, becomes $\dfrac{\mu}{v} - \dfrac{1}{u} = \dfrac{\mu - 1}{r_1}$ ... ... (4)

74 Refraction through a Double Surface (or Lens).—When a ray passes through a lens, it first crosses a surface, air to glass, and then emerges through a surface, glass to air. In Fig. 82, P is a luminous point on the axis of a lens, which, in this case, has got its radii r_1 and r_2 both positive. Let P_1 be the image of P formed by refraction at the first surface. So, for the ray from P entering the lens, the relation from the above formula [eq. (4)] is

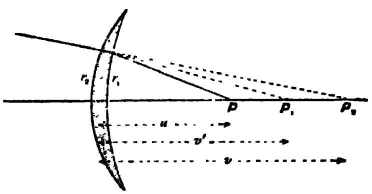


Fig. 82

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{r_1} \quad \ldots \quad \ldots \quad (5)$$

where r_1 is the radius of the first surface and v' is the image distance.

Now, this image P_1 formed by refraction at the first surface serves as the virtual object of the final image P_2 formed by the second surface of radius r_2. So in this case $u = v'$. Remembering that ${}_a\mu_g = \dfrac{1}{{}_g\mu_a}$,

we have, if v be the final image distance, $\dfrac{\frac{1}{\mu}}{v} - \dfrac{1}{v'} = \dfrac{\frac{1}{\mu} - 1}{r_2}$.

Multiplying each term by μ, we have $\dfrac{1}{v} - \dfrac{\mu}{v'} = \dfrac{1 - \mu}{r_2}$ ... (6)

Adding equations (1) and (2) for the two surfaces, we get

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \ldots \quad \ldots \quad (7)$$

If the object be at infinity, the rays from it are parallel, and $1/u=0$: but we know that in this case the image is at the principal focus, so $v=f$. Hence we get from eq. (7),

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \ldots \quad \ldots \quad (8)$$

N.B. According to the **new convention of signs** [See Art. 67(II*b*)] the *signs* attributed to the *radii of curvature* of lens surfaces in air are as follows :—The radius of curvature of the surface convex to the air, *i.e.* a surface which is convex to the incident light, is positive. and that which is concave to the incident light is negative. Thus eq. (4), Art. 73, becomes $\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{r}$, and r_2 in eq. 8, Art. 74, being a negative quantity the equation (8), Art. 74, becomes $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$.

Examples.—(*1*) *The plane side of a plano-convex lens is silvered and the lens then acts as a concave mirror of 30 cms. focal length. The refractive index of the lens is 1.5 : calculate the radius of curvature of the convex lens.* (*Pat. 1931 ; All. 1929*)

The lens is acting as a concave mirror of 30 cms. focal length. Therefore, if an object be placed at the centre of curvature of such a mirror, *i.e.* at a distance of 60 cms. from the lens, the image is also formed at the same place. Hence, here the focal length of the plano-convex lens is 60 cms. If the plane side of the lens is silvered, the rays are reflected back, and, being again refracted through the lens along the same path, meet at the focus.

The formula connecting the radii of curvature of the lens, the refractive index of the material of the lens, and its focal length is,

$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$. Here $\mu=1.5$; $r_1=?$ $r_2=\infty$ (being plane) ;

$f=-60$; $\quad \frac{1}{60}=(1.5-1)\times\frac{1}{r_1}$ whence $r_1=-30$ cms. That is, the radius of curvature of the lens is 30 cms.

(*2*) *Find the focal length of a convex lens having radii of curvature 4 and 6 cms. respectively* (*a*) *in air,* (*b*) *in water. Refractive index of material of the lens* $=1.5$.

(*a*) We have the equation $\quad \frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$.

Let the light strike the surface of radius 4 cms. first, then $r_1=-4$, and $r_2=+6$.

$$\therefore \frac{1}{f}=(1.5-1)\left(-\frac{1}{4}-\frac{1}{6}\right)=-\frac{1}{2}\times\frac{5}{12} ; \therefore f=-\frac{24}{5}=-4.8 \text{ cms.}$$

(*b*) Now, ${}_{air}\mu_{water}=\frac{4}{3}$, ${}_{water}\mu_{air}=\frac{3}{4}$, and ${}_{air}\mu_{glass}=\frac{3}{2}$.

$\therefore$ ${}_{\text{water}}\mu_{\text{glass}} = \frac{3}{2}\times\frac{3}{4}=\frac{9}{8}$; $\therefore$ $\frac{1}{f} = \left(\frac{9}{8}-1\right)\left(-\frac{1}{4}-\frac{1}{6}\right)$;

whence $f = -19{\cdot}2$ cms.

(3) *A sphere of glass, 6 inches in diameter, contains a small air bubble which appears, to an eye looking from the surface along the radius of the sphere, to be 2 inches below. What is its true distance from the surface ?* ($\mu = 3/2$)

The refraction takes place in a spherical surface, and we have,

$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu-1}{r}$, eq. (4), Art. 73. Here, $v=2$ inches ; $u=?$ $r=3$ inches ;

${}_{\text{air}}\mu_{\text{glass}} = \frac{3}{2}$; $\therefore$ ${}_{\text{glass}}\mu_{\text{air}} = \frac{2}{3}$. $\therefore$ $\frac{2}{3\times2} - \frac{1}{u} = \frac{2}{3\times3} - \frac{1}{3}$;

or $\frac{1}{u} = \frac{1}{3} - \frac{2}{9} + \frac{1}{3} = \frac{4}{9}$. $\therefore$ $u = 2{\cdot}25$ inches. That is, the true distance of the air bubble is $2{\cdot}25$ inches below the surface.

75. Power of a Lens.—The power of a lens means its power of convergence in the case of a convex lens or its power of divergence in the case of a concave lens. That is, it indicates to what degree the lens can converge or diverge (as the case may be) an incident beam of light. For a pencil incident parallel to the axis, the convergence or divergence produced by a lens will be greater, the shorter the focal length is. So the reciprocal of the focal length $\left(\frac{1}{f}\right)$ is taken to indicate the power of a lens. A lens having a focal length of one metre is said to possess **Unit Power,** called a **Dioptre.** According to the opticians, *the power of a convex lens is positive and that of a concave lens is negative.*

So to express the power of a lens in dioptres, express the focal length in metres, get the reciprocal and change the sign.

Thus, a concave lens of 100 cms. focal length has a power of $-1D$, and a convex lens of -50 cms. focal length has a power of $1/\frac{1}{2}=2D$.

[**Note** that according to the new *convention of signs* the change of sign is not necessary [see Art. 67 (II*b*)].

76. Two Thin Lenses in Contact.—Let two thin lenses of focal lengths f_1 and f_2 be in contact on the same axis. Let O be a point source of light on the axis and P' be its image formed after refraction through the first lens A (Fig. 83). Then if u and v_1 be the distances of O and P' respectively from the concave lens A,

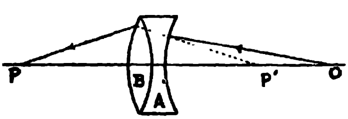


Fig. 83

we have, $\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$ . ... (1)

P' now serves as the object with respect to the second lens B (convex) which, therefore, produces an image at P. The lenses being thin, the distance of P' from B can be taken to be v_1. If v be the distance of P from the lens B, we have

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \dots \quad \dots \quad \dots \quad \dots (2)$$

Now considering the two lenses A and B together as equivalent to a single lens of focal length F, we have,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$, where u is the distance of the object O, and v that of the image P with respect to the combined lens.

But on adding (1) and (2),

we get, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$;

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

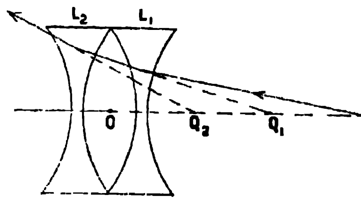


Fig. 83(a)

N. B. In Fig. 83 (a), L_1 and L_2 are two concave lenses in contact having focal lengths f_1 and f_2 respectively. Suppose for the object P, Q_1 is the virtual image formed by the lens L_1. This virtual image serves as object so far as the lens L_2 is concerned, by which the final virtual image Q_2 is produced. Proceeding as before, it can be shown that if F be the focal length of the combined lens, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.

That is, *the power of two thin lenses in contact is the sum of their powers.* Thus the combination of two lenses acts like a single lens of focal length F which is called the **equivalent lens.**

Example.—*What is the power of a lens which, when combined with a converging lens of focal length 25 cms, will produce a combination of power of 2 dioptres.*

The power of the converging lens is $\frac{100}{25} = 4$ dioptres ;

$\therefore \quad 2 = 4 + p$, where p is the power of the other lens ; $\therefore \quad p = -2$.

The negative sign shows that the lens is concave and has a focal length f, where $2 = \frac{100}{f}$; $\quad f = 50$ cms.

77. Practical Determination of Focal Length of a Lens.—

(a) Convex Lens :—

(1) Direct Method.—Point the lens towards a distant object, say a window at a large distance or, better still, the sun ; and move it towards a white wall or a paper screen held behind it. The distance from the centre of the lens to the screen gives the focal length of the lens.

(2) U-V Method.—A convex lens L (Fig. 84) is mounted on a suitable vertical stand. Place a lighted candle C on one side of the lens, and a white card-board screen S on the other side. Adjust the height of the candle so that the flame is on the axis of the lens. Move the screen backwards and forwards until a sharp inverted image of the flame is obtained on the screen. Measure the object and image distances from the centre of the lens with a metre scale, and find out the value of f by using the equation,

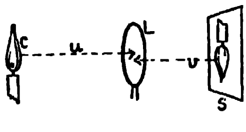


Fig. 84

$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f},$$ where v is negative in the present case.

Repeat this several times and calculate the mean value of f. Draw a graph with u and v. The curve will be a branch of a *hyperbola* (Fig. 85).

The experiment can also be performed by exchanging the positions of the candle flame and the screen by two pins fixed on two stands. Looking at the image of the object-pin from the other side of the lens the position of the image can be located by **avoiding parallax** between the second pin and the image of the first pin.

(3) Focal Length from Graph.—*(i)* The graph obtained by plotting u and v (using equal scales on the two axes) is shown in Fig. 85. This is a branch of a hyperbola. A line OP drawn from the origin O making an angle of 45° with the axis of X or Y will meet the curve at the point P. This is a straight line for which $u=v$. The point P will be equidistant from the two axes (X and Y). The value of this perpendicular distance (x or y) will be equal to $2f$ of the lens, from which f can be calculated. (That is done in the same way as in the case of concave mirror, Art. 39).

Note.—If, however, equal scales are not used on both the axes or if one of them does not begin from O, then take two points on the graph having equal values for both the X and Y co-ordinates ; join them and produce the straight line to cut the curve at a point P, the perpendicular distances of which from both the axes will each be equal to $2f$.

(ii) Another Method of obtaining the value of f from graph is to mark off the values of u along one axis and the values of v along the other axis. The corresponding points on the axes are then joined by straight lines, as done in Fig. 85, and, if construction is carried out accurately, all these straight lines should intersect in a single point R (Fig. 85) such that OR is inclined at 45° to the axis. The perpendicular distances of this point from each of the axes, *i.e.* the co-ordinates of the common point, give the focal length f (9·2 in Fig. 85).

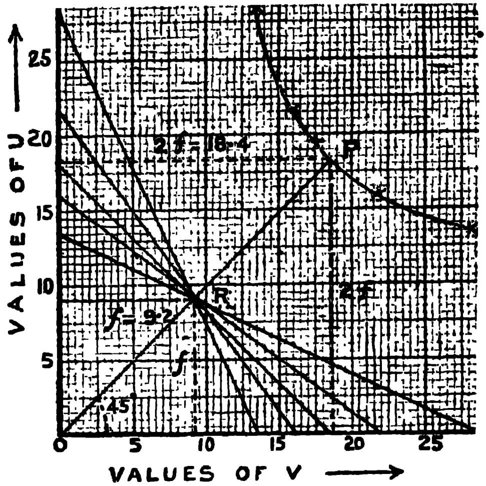


Fig. 85

Proof—Consider any of the triangles like 13·5,0,28·5 (Fig. 85). Area of the whole triangle = sum of the areas 13·5, R, O and 28·5, R, O.

The area (13·5, 0, 28·5) $= \frac{1}{2} uv$; area (13·5, R, O) $= \quad fv$; area (28·5, R, O) $= \frac{1}{2} fu$. $\therefore \frac{1}{2} uv = \frac{1}{2} fu + \frac{1}{2} fv$;

Or, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$; This is true.

$\therefore$ The co-ordinates of R are (f, f), and the point lies on the line joining u and v.

(iii) Draw another graph of $1/u$ against $1/v$. The graph will be a straight line as in Fig. 37. Find the value of f from the graph in the same way as done in the case of concave mirror where the graph cuts each axis at a distance equal to $1/f$.

(iv) Another graph may be drawn by plotting the values of u on the X-axis and $(v+u)$ on the Y-axis. The graph is like Fig. 68. It will be seen that $(v+u)$ is the distance between the object and its image, and from the graph it is evident that this distance diminishes rapidly as u increases, reaches a minimum value and then increases again. The value $(v+u)$ is minimum, *i.e.* the object and image are closest, when each is equal to $2f$ from the lens, or, in other words, their distance apart $(v+u)$ is $4f$ (*vide* displacement method (5), note (*ii*) below). So the values of the co-ordinates of the minimum point A' of

the graph are $2f$ and $4f$, that is OA (or u) $= 2f$ and OD (or $v+u$) $= 4f$ in Fig. 68. Thus f is found.

(v) Draw a graph with v/u (*i.e.* the magnification) along the Y-axis and u along the X-axis. This will be a curve like Fig. 85. From the point on the Y-axis, whose value is 1 (*i.e.* where $v/u = 1$), draw a straight line parallel to the X-axis. This will cut the curve at a point (like P in Fig. 85). From P drop a perpendicular on the X-axis, the value of the intercept (or u) will be equal to $2f$. For, the value of the Y-co-ordinate of the point $P = 1$; or $v/u = 1$; or $v = u$. But when $v = u$ we know that each $= 2f$, so $u = 2f$. Thus f is found.

(4) Plane Mirror Method.—Another simple method is to place a plane mirror upright behind the lens L mounted on a stand (Fig. 86). A pin P is clamped in front of the lens L with its head very nearly on the principal axis of the lens. The pin is then moved backwards and forwards in front of the lens until say, at PQ, there is no *parallax* between the pin and its inverted image $P'Q'$.

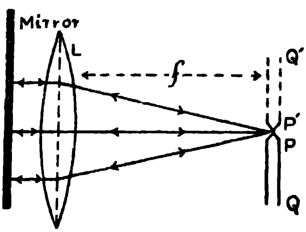


Fig. 86

The position PQ gives the focus of the lens. For, the rays diverging from the object at this position will be rendered parallel to the axis and after emergence from the lens will be incident normally on the mirror but will be reflected along the same path and after refraction by the lens will converge at the same position.

The distance of the pin PQ at this position from the optical centre of the lens will give the focal length of the lens. For an equi-convex lens, this distance is equal to the distance of the pin from the 1st surface of the lens plus half the thickness of the lens.

The distance of the lens from the mirror is immaterial. So the plane mirror may also be placed on a horizontal platform and the lens placed on it, and a pin attached horizontally to a stand may be adjusted in height until a real inverted image coincident with the object-pin is found.

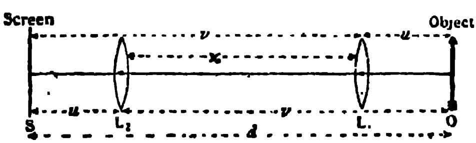


Fig. 87

(5) Displacement Method.—If in the $u-v$ method [Art. 77 (2)] *the screen and the object are placed at any fixed distance greater than 4 times the focal length of the lens* it will be found that there are two positions L_1, L_2, of the lens (Fig. 87) for each one of which a

sharp real (inverted) image may be obtained on the screen. If d be the ditstance between the object and the screen, and x the distance between the first and the second position of the lens, called the *displacement of the lens*, the focal length of the lens is given by the relation,

$$f=\frac{d^2-x^2}{4d}.$$

Proof :—We have, $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$, in case of a real image formed by a convex lens ; $\therefore\ uv=f(u+v)=fd$ ... ... (1)

In the second position, when the lens is shifted through x,

we have similarly, $(u+x)(v-x)=fd$;

or $uv+(v-u)\,x-x^2=fd$; *or* $(v-u)x-x^2=0$, since $uv=fd$, from (1) ; *or* $x=v-u$. But $(v-u)^2=(v+u)^2-4uv$;

i. e. $x^2=d^2-4fd$ ($\because$ $u+v=d$, and $uv=fd$.)

$$\therefore \quad \mathbf{f=\frac{d^2-x^2}{4d}}.$$

[**Note**.—(*i*) This method does not involve any error due to an incomplete knowledge of the position of the optical centre of the lens.

(*ii*) Since $x^2=d^2-4fd$, we have $x=\sqrt{d^2-4fd}$.

If x is to be real, $d^2>4fd$; *i. e.* $d>4f$.

Therefore, for the success of the displacement method, the object and the screen must be placed at a distance greater than 4 times the focal length of the lens.

(*iii*) It should be observed that a limiting position is found when the two image positions coincide, *i.e.* when $x=0$; then $f=\frac{d^2}{4d}=\frac{d}{4}$; *or* $d=4f$; in this case, there is only one position of the lens giving a real image.]

(6) The focal length can also be calculated from the formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \text{.....(eq. 4, Art. 74)}$$

by measuring the radii of curvature of the two faces of the lens by a spherometer, knowing the value of μ, the refractive index of the material of the lens and giving proper signs to r_1 and r_2.

(b) Concave lens :—

(1) Combination Method.—The image in the case of a concave lens is always virtual ; so it cannot be received on a screen. In order to get a real image, the given concave lens is combined in this method with a suitable convex lens, *i.e.* a lens of greater power, to make the whole system converging. After this, proceeding just as in the case of a convex lens, the focal length of the combined lens can be determined.

Now, knowing the focal length of the convex lens used in the combination, the focal length of the concave lens can be determined from the relation, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$, where f_1 is the focal length of the convex lens, and f_2 that of the concave lens. With proper signs it becomes,

$$-\frac{1}{F}=-\frac{1}{f_1}+\frac{1}{f_2}, \quad \text{or} \quad \frac{1}{f_2}=\frac{1}{f_1}-\frac{1}{F}.$$

[**Note.** (*i*) To get a real image with the combination, the focal length of the convex lens should be less than that of the concave lens. *i. e.* the power of the convex lens should be greater.

(*ii*) If P, P_1, and P_2 represent the powers of the combined lens, convex lens and the concave lens respectively, the above equation is given by $P=P_1+P_2$.]

(2) Auxiliary Lens Method.—When the power of the convex lens is less than that of the concave lens it will not be possible to receive an image on the screen when they are combined. In this case a sharp image Q_1 of the source P is first of all obtained on the screen by the convex lens L_1 (auxiliary lens) alone, or the position of the image is located by placing a pin at Q_1 and avoiding parallax as explained before. Keeping the positions of P and L_1 fixed, the given concave lens L_2 is then interposed between L_1 and Q_1, and a new position of image is obtained at a somewhat greater distance Q_2 (Fig. 88). This is due to the fact that the converging rays after passing through the lens L_1 fall on the lens L_2 by which they are a little diverged.

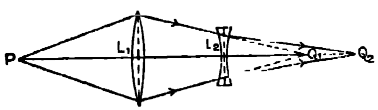


Fig. 88

Reversibly, rays of light from Q_2 will appear to diverge from Q_1 ; hence Q_1 is considered as the virtual image of Q_2 with reference to the lens L_2, where $L_2Q_2=u$, and $L_2Q_1=v$, both u and v being positive. Hence f is calculated from the general formula for a lens, which will be positive in value.

Note that the image of Q_1 will be obtained only when the distance $L_2 Q_1$ is less than the focal length of the concave lens L_2, otherwise the light, after passing through L_2, will diverge from a point behind L_2.

Questions.

Art. 65.

1. What is the difference in the behaviour of a lens and a prism ? (Pat. 1937)

Art. 66.

2. Draw neat diagrams to illustrate the following :—

(*a*) Formation of virtual image in a concave mirror ; (*b*) phenomenon of 'total reflection' in a prism ; (*c*) the position of 'optical centre', in the case of a double convex lens. (See also Arts. 37 to 52)

Art. 67.

3. What is meant by focal length of a convex lens ?

An electric light is distant 6 ft. from a wall. A convex lens gives a sharp image of the light on the wall, when it is held 2 ft. away from the light. A second sharp image is obtained on the wall, when the lens is held 2 ft. away from the wall. Determine the focal length of the lens and compare the magnifications produced in the two positions of the lens. (C. U. 1949)

[*Ans* : $f = \frac{4}{3}$ *ft.* $I_1 : I_2 = 4 : 1$]

4. Show how to find, by a geometrical construction, the position of an image formed by a (thin) double convex lens. (C. U. 1914, '15)

Arts 68 and 69

4. (a) Obtain a *formula* connecting together the position of an object and its image formed —real or virtual—by direct refraction through a convex lens.

Explain "conjugate Foci". (C. U. 1931, '35, '47 ; Pat. 1926, '46)

5. Establish for a lens the eq. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, where u, v, and f have the usual significance. Also show that, in a real image formation with a convex lens, the minimum distance between the object and its image is $4f$.

[For the second part see Art. 77 a (5)]

6. A rod 5 cms. long is held in front of a convex lens and forms an image 25 cms. long upon a screen (placed parallel to the rod) at a distance of 100 cms. from the lens. What is the focal length of the lens ? (C. U. 1918)

[*Ans* : $f = -25$ cms.]

7. An object 6 cms. high is placed at a distance of 40 cms. from a thin convex lens and an inverted image of height 4 cms. is formed on the other side of the lens. Find the focal length of the lens graphically. Verify your result by calculation. (Dac. 1934)

[*Ans* : 16 cms.]

7(a). The real image formed by a lens is twice the size of the object and 18 cms. from it. Find the focal length of the lens.

Deduce the distance between the object and the image when the image is half the size of the object. (C. U. 1947)

[*Ans* : $f=4$ cms., $D=18$ cms.]

8. A convex lens of focal length 12 cms. and a concave lens of focal length 10 cms. are placed co-axially at a distance of 10 cms from each other. An object, which is nearer to the convex lens on the common axis of the two lenses, is placed 48 cms. away from the convex lens. Find the position, magnification and nature of the final image formed. (Pat. 1942)

[*Ans* : The final image distance is -15 cms. with respect to the concave lens and away from the convex lens, *i.e.* it is nearer to the concave lens ; magnification $=5/6$; nature = real, inverted.]

9. An object is placed in front of a convex lens so that a real image of the same size is obtained. It is then moved 16 cms. nearer the lens, when the image, still real, is three times as large as the object. What is the focal length of the lens ? [L. M.]

[*Ans* : 24 cms.]

9(a). A convex lens is adjusted to form an image of a candle flame on a wall. It is then found that by moving the lens 20 cms. nearer to the wall a sharp image is again obtained exactly one quarter the length of the first. Find the focal length of the lens. (Pat. 1946)

Art. 69.

10. If an observer's eye be held close to a convex lens of 3 cms. focal length to view an object at a distance of 2·5 cms. from the lens, show that the magnifying power is 6. Illustrate your answer by a neat diagram. (C. U. 1931)

11. Find the size of the image on the squared paper provided, the size of the object (placed symmetrically with its centre on the axis) being 5 cms., and its distance 30 cms. from the lens. (The focal length of the lens = 10 cms.)

[*Ans* : Size of the image = 2·5 cms.] (C. U. 1914)

Art. 70.

12. A convex lens of focal length 10 cms. is made to approach a rod of length 5 cms. placed perpendicularly to the axis of a lens. Show by means of a typical diagram, drawn to scale (on the squared paper provided), the changes in the nature and size of the image. (C. U. 1912, '16, '17, '19 ; Pat. 1921)

13. Investigate with the help of diagrams, the conditions for the formation of (*a*) a real image, (*b*) a virtual image, by a plano-convex lens.

[**Hints.**—Proceed as in the case of double convex lens, for a plano-convex lens behaves like a double convex lens].

14. Describe with suitable diagrams the formation of image by a convex lens of an object placed at different distances from it. (Dac. 1933 ; Pat. '42)

15. An object is placed at a distance of $2f$ from a convex lens of focal length f. The rays, after traversing the lens, are reflected from a convex

mirror and again refracted by the lens, forming a real inverted image coincident with the object. If the distance between the lens and the mirror is a, show that the radius of curvature of the mirror is $2f-a$. (Dac. 1927).

[*See* Art. 39(1)*b*]

Art. 71.

16. You are given a lens through which you can look but which you are not allowed to handle. What tests would you apply in order to determine if it is concave or convex ? (Pat. 1929, '40).

Art 73.

17. Prove that for a concave refracting surface $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$,

where μ is the refractive index, and apply this formula to obtain a formula for the focal length of a lens. (All. 1921)

Art. 74.

18. Obtain a general formula for refraction at any single spherical surface. Hence prove the formula for a thin lens. (All. 1946)

19. The radii of curvature of the surfaces of a double convex lens are 20 cms. and 40 cms., and its focal length is 20 cms. What is the refractive index of glass ? (C. U. 1935).

[*Ans :* 1·667]

Art. 75.

20. What do you mean by the power of a lens ? (C. U. 1926)

Art. 77.

21. Draw a graph showing the relation between the position of object and image in a convex lens from the following observations.

$u = 18{\cdot}8,\ 20{\cdot}9,\ 22,\ 24,\ 26,\ 28,\ 30,\ 32.$

$v = 41{\cdot}5,\ 33{\cdot}5,\ 30,\ 27,\ 28{\cdot}7,\ 22,\ 21.$

Hence find the focal length of the lens. (Pat. 1921)

22. Draw a curve showing the relation between the distance of an object and that of its image as measured from a lens, as the distance of the object is progressively varied. Take, for simplicity, the case of a convex lens of negligible thickness. (Pat. 1923)

23. Describe two methods for determining the focal length of a thin convex lens. Give neat diagrams. (Pat. 1931, '36 ; C. U. '11, '17)

24. Explain clearly how you would experimentally determine the focal length of a concave lens. (C. U. 1933 ; Pat. '29).

25. A luminous source and a screen are placed at a fixed distance apart along a scale and a convex lens can be moved between them. Explain how you will utilise this arrangement to determine the focal length of the lens. If a and b be the sizes of the image for the two positions of the lens, show that the size of the object is equal to $\sqrt{ab}$. (All. 1925)

[**Hints.** *See* Art. 77a (5). Let s be the size of the object. Then in the first position $s/a = u/v$; *or* $s = a \times u/v$; and in the second point $s/b = v/u$; *or* $s = b \times v/u$: $\therefore s^2 = ab$; *or* $s = \sqrt{ab}$.]

26. A luminous object is kept at a fixed distance from a screen, and with the help of a convex lens an image is obtained. The lens is then moved and another image is obtained on the screen. (*a*) If the distance of the object from the screen be 9 in., calculate the focal length ; (*b*) if the sizes of the images be 2 and 8 inches, calculate the size of the object. (All. 1931)

[**Hints.** See Art. 77a (5). The size of the object $= \sqrt{2 \times 8} = 4$ in. Again $v/u = 8/4 = 2$ $\therefore v = 2u$. We have $v + u = 9$; *or*, $2u + u = 9$, *or* $u = 3$ in., and $v = 6$. Hence $f = 2$ in.]

CHAPTER VI

Optical Instruments : Defects of Vision

78. Simple Microscope.—If an object be placed between a convex lens and its principal focus, an erect, magnified and virtual image will be visible to an eye placed behind the lens. The convex lens, thus used, forms a simple microscope. This is popularly called a **Magnifying glass** or a **Reading glass** (Fig. 89).

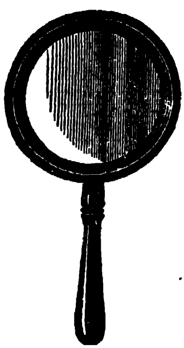


Fig. 89—Reading glass

An object becomes visible when it forms an image on the retina of the eye. The apparent size of the object is proportional to the size of the image formed on the retina, and the size of the image depends upon the angle subtended at the eye by the object. It should be clearly remembered that, *for the greatest advantage in vision, the angle subtended at the eye by the object should be as large as possible.* This angle increases as the object is brought nearer to the eye, and so the object appears to be larger. But as we cannot see objects clearly when too near the eye, there is obviously a limit to the distance up to which an object can be brought near the eye, if clear vision is to be obtained. This distance is known as the **least distance of distinct vision (D)**. For the normal eye it is about 25 cms.

In Fig. 90, OQ' is the least distance of distinct vision. So the eye without the lens cannot see clearly the object placed at Q. For clear vision the angle subtended at the eye by the object PQ is maximum when placed at Q'. If the object be brought nearer, the subtended angle would be larger, but the image would be indistinct as for clear vision OQ' is the least distance. This happens when the object is placed at Q. But it will be clear from Fig. 90, that by means of the lens, the image of PQ is formed at $P'Q'$, and that both the object and the image subtend the same angle at the centre of the lens, *i.e.* at the eye placed close to the lens. So the image appears both distinct and magnified. Thus the effect of a simple microscope, or a simple magnifying glass, is to enable an object to be brought very close to the eye and yet to be distinctly visible.

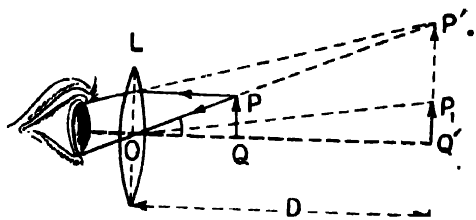


Fig. 90—Simple Microscope

Magnifying Power.—*It is defined as the ratio of the angle subtended at the eye by the image to that subtended by the object when placed at the least distance of distinct vision.*

For distinct vision the angle subtended at the eye by the image $= \angle P'OQ' = \tan \angle P'OQ'$ (the angle being small is taken as proportional to the tangent of the angle $= \dfrac{P'Q'}{OQ'} = \dfrac{\text{size of image}}{D}$ (Fig. 90). Similarly, the angle subtended by the object, when placed at the least distance of distinct vision $= \angle P_1OQ' = \tan \angle P_1OQ'$

$$= \frac{P_1Q'}{OQ'} = \frac{PQ}{OQ'} = \frac{\text{size of object}}{D}.$$

Therefore, the magnifying power, according to the definition,

$$\mathbf{m} = \frac{P'Q'}{OQ'} \div \frac{PQ}{OQ'} = \frac{P'Q'}{PQ} = \frac{\text{size of image}}{\text{size of object}} = \frac{v}{u}. \text{ But for a lens, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Multiplying every term by v, $\dfrac{v}{u} = 1 - \dfrac{v}{f}$. Since the lens is convex and the image is formed at the least distance of distinct vision, we have, $v = D$, and f is negative;

$$\therefore \quad \mathbf{m} = \frac{\mathbf{v}}{\mathbf{u}} = \mathbf{1} + \frac{\mathbf{D}}{\mathbf{f}}.$$

The magnification in the case of a simple microscope is limited, because a powerful lens must be thick and very thick lenses can not produce sharp images and then the images are also curved and distorted.

Best position where the eye is to be placed.—If the distance of the eye from the lens is a, the magnifying power will be given by,

$$m = \frac{v}{u} = 1 + \frac{v}{f} = 1 + \frac{D-a}{f}.$$

From this it is clear that the best position of the observing eye will be when m is maximum, *i.e.* $a = 0$. In other words, the eye should be placed as close to the lens as possible.

Achromatism of a Magnifying Glass.

With the magnifying glass different coloured images should be produced due to dispersion. But if the eye is placed very close to the magnifying glass, no chromatic error is produced. Since corresponding points of the image and the object and the centre of the lens (*e.g.*, P', P and O in Fig. 90.) lie in the same straight line, the visual angle subtended at the eye by the red image is the same as that subtended by the blue image. The same is the case for every other colour. Thus the images of different colours subtend the same visual angle at the eye and are of the same size. Therefore they superpose on each other on the retina and the image perceived is achromatic.

Field of view.—Through every optical instrument a definite **field of view** is seen, which is expressed as the angle subtended by the extreme portions of the field at the instrument.

Example.—*Find the focal length of the lens which would produce a magnification of 5 for an eye of which the least distance of distinct vision is 25 cms. What kind of lens is it?*

Here $v/u = 5$; $25/u = 5$; $\therefore u = 5$.

Then $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{25} - \frac{1}{5}$ whence $f = -6{\cdot}25$ cms.

The negative sign means that the lens is convex.

79. Compound Microscope.—This instrument is used for producing much greater magnification than is possible with a simple microscope. It consists of two convergent lens-systems O and O' (Fig. 91) placed co-axially in a tube at a distance from each other. The lens O is tur-

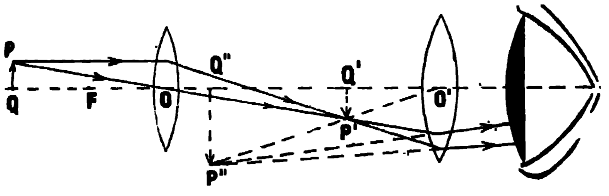


Fig. 91—Compound Microscope

ned towards the object and is called the **objective**, while the lens O' behind which the eye of the observer is placed is called the **eye-piece.** The objective is of short focal length and short aperture. The eye-piece is also of short focal length but of wider aperture. It can be slided within the main tube whereby its distance from the objective can be altered. The Microscope tube is fifted on a slider by which it can be moved as a whole towards or away from the object (PQ).

In an experiment the microscope tube is so adjusted that the object PQ is placed just beyond the principle focus (F) of the objective which thus produces a real, inverted and magnified image $P'Q'$ of the object. The eye-piece O' is then so adjusted that the image $P'Q'$ formed by the objective falls within the focal length of the eye-piece and the final image $P''Q''$, virtual, magnified and inverted (erect with respect to $P'Q'$ but inverted with respect to the object PQ) is formed at the least distance of distinct vision for an eye placed just beyond the eye-piece.

In an actual instrument, the objective as also the eye-piece is formed of a number of lenses so as to avoid the defects which result from the use of a single lens, namely the appearance of colour (chromatic aberration) and curvature and distortion of the image (spherical aberration). In high power microscopes used for biological purposes, there are usually two or three objectives of different magnifications, any of which can be used according to necessity with the same eye-piece.

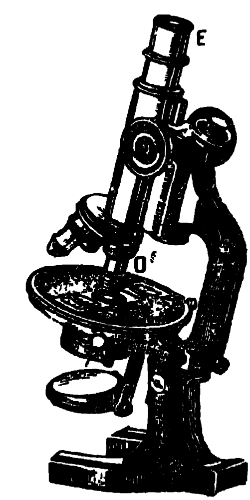


Fig. 92—Compound Microscope

The magnification in this case may be very large. The area of the image may be even a million times that of the object. The image is therefore considerably less intense and so there is an arrangement in high-power microscopes to strongly illuminate the object by light reflected from a mirror mounted *below* the stage O (Fig. 92) on which the object is placed.

For measurements between parts of the image compound microscopes are usually fitted with **cross-wires.** These consist of two very fine fibres (spider lines are best for this purpose) fixed in a metal ring in positions at right angles to each other. These cross-wires are placed where the image is produced by the objective so that the final image and the cross-wires are seen through the eye-piece simultaneously without any parallax. As a matter of fact in focussing an

object, the *cross-wires* are first focussed (*i.e.* the distance of the eye-piece from the cross-wires is altered until the cross-wires are placed at the least distance of distinct vision) and then putting the eye-piece (containing the arrangement of cross-wires) as far wide apart from the objective as possible in the microscope tube, the tube as a whole with the objective facing the object is adjusted in position until a distinct image coincident with the cross-wires (*i.e.* without any parallax with respect to the cross-wires) is obtained.

Magnifying power.—Magnification by the object-glass, $m_1 = \frac{P'Q'}{PQ} = \frac{v}{u}$ (without considering signs). The second magnification is due to the eye-piece and is obtained as in the case of a simple microscope by the following,

$\frac{P''Q''}{P'Q'} = m_2 = 1 + \frac{D}{f}$ (where f is the focal length of the eye-piece).

$\therefore$ The total magnification $\mathbf{m} = P''Q''/PQ = P''Q''/P'Q' \times P'Q'/PQ$ $= m_2 \times m_1 = \left(1 + \frac{\mathbf{D}}{\mathbf{f}}\right)\frac{\mathbf{v}}{\mathbf{u}}$.

Conditions for Large Magnifying Power.—The equation for magnification shows that m is large when,

(i) u is small, which means that the focal length of the objective should be small ; (ii) f is small, which means that the focal length of the eye-piece should also be small ; (iii) v is large, which means that the eye-piece and the objective should be separated as far as possible.

Examples.—(*1*) *If the focal lengths of object-glass and eye piece of a microscope are 2 cms. and 5 cms. respectively, and the distance between them is 20 cms, what is the distance of the object from the object glass when the image seen by the eye is 25 cms. from the eye piece ?*

Eye-piece : Here image for eye-piece is on the same side as the object. So v is positive, $v = +25$; $f = -5$; $u =$?

Then $\frac{1}{25} - \frac{1}{u} = \frac{1}{-5}$; whence $u = 25/6$.

Objective : $v' = 20 - \frac{25}{6} = \frac{95}{6}$ cms. Here $u' = -\frac{25}{6}$; $f' = -2$ cms. ; $u' =$?

$\frac{1}{-95/6} - \frac{1}{u'} = \frac{1}{-2}$; whence $u' = 2{\cdot}29$ cms.

(*2*) *The focal length of the object glass of a microscope is $\frac{1}{2}$ an inch, that of the eye-piece is 1 inch. Taking the least distance of distinct vision to be 12 inches, find the distance between the object-glass and eye-piece when the object viewed is $\frac{3}{4}$ of an inch from the object glass.* (*Pat. 1942*)

Objective : $u' = +\frac{3}{4}$; $f = -\frac{1}{2}$; $v' =$?

Then $\frac{1}{v'} - \frac{4}{3} = -2$; *or* $v' = -\frac{3}{2}$ in.

Eye-piece : $v = +12$; $f = -1$; $u =$? Then $\frac{1}{12} - \frac{1}{u} = -1$; or $u = \frac{12}{13}$ in.

$\therefore$ The distance between the objective and eye-piece $= v' + u = \frac{3}{2} + \frac{12}{13} = 2\frac{11}{26}$ in.

(3) *The objective of a compound microscope has a focal length of half an inch and is placed at a distance of $2\frac{1}{2}$ inches from the eye-piece. What must be the focal length of the eye-piece if the final image of an object, placed $\frac{3}{4}$ inch from the object-glass, is formed at the least distance of distinct vision, which is 10 inches.*

(*Pat. 1941*)

Objective : $u = +\frac{3}{4}$; $v =$? $f = -\frac{1}{2}$.

Then $\frac{1}{v} - \frac{4}{3} = -2$; whence $v = -\frac{3}{2}$,

i. e. the image is at a distance of $1\frac{1}{2}''$ behind the objective.

$\therefore$ The object distance u' of the eye-piece $= (2\frac{1}{2} - 1\frac{1}{2}) = 1''$.

Eye-piece : $u' = 1''$; $v' = 10$; $f =$?

Then $\frac{1}{10} - 1 = \frac{1}{f}$; whence $f = -\frac{10}{9} = -1\frac{1''}{9}$

80. Telescopes are instruments by which images of distant objects can be seen.

The Astronomical Telescope.—It consists of two convex lenses mounted co-axially in a tube (Fig. 93). Here the objective *O* has a **large focal length and large diameter** and the **eye-piece** *E* has a **short focal length**. The eye-piece should be large enough to cover the pupil of the eye, but it should not be very large, for the light which does not pass into the eye is lost, but, on the other hand, the objective has to collect sufficient light to form a bright image and so it must be of large diameter. As the instrument is meant for viewing distant objects, the rays falling on the objective may be considered to be parallel, and so a real, inverted and diminished image *PQ* is formed at the focal plane of the objective. The eye-piece, by a sliding adjustment, is so placed as to have the image *PQ* formed just within its focal length, and the final image *P′Q′*, virtual, magnified and erect with respect to *PQ* but inverted with respect to the object, is formed by the eye-piece at the least distance of distinct vision.

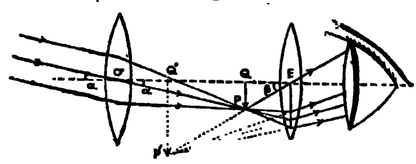


Fig. 93—Astronomical Telescope

To focus the telescope for infinity, the eye-piece is placed so as to have the image PQ formed at its *focal plane*. So the rays diverging from the image PQ, due to the objective, fall on the eye-piece and emerge parallel. An eye placed close to the eye-piece sees at infinity an image $P'Q'$ virtual, magnified and erect with respect to the first image PQ. Since the first image is inverted, the final image $P'Q'$ is also inverted with respect to the object. The *length* of the telescope is then equal to the *sum of the focal lengths* of the objective and the eye-piece. The instrument is, in this case, said to be adjusted for normal vision.

Frequently the instrument is used to have the final image of a *distant object* formed at the least distance of distinct vision, as in Fig. 93. In this case the eye-piece is *nearer* the objective (*i e.* it is pushed inside) than is the case for normal adjustment.

To focus the instrument on near objects, with the final image at infinity, the eye-piece is to be moved *away* from the objective.

Astronomical telescopes are usually fitted with **cross-wires** for purposes of measurements between the parts of the image. They are placed at the position where the real image is formed by the objective so that the final image and the cross-wires can be seen through the eye-piece simultaneously without parallax.

This telescope is used for viewing heavenly bodies such as the moon, stars, planets, etc., and hence is called an astronomical telescope.

Magnification.—The angle subtended by the object at the objective is α, and since the object is at a great distance we may consider α to be the angle subtended by the object also at the eye piece. The angle subtended by the image at the eye-piece is β. Hence magnification is given by,

$$\frac{\text{angle subtended by image}}{\text{angle subtended by object}} = \frac{\beta}{\alpha} = \frac{\angle PEQ}{\angle POQ}$$

$$= \frac{\tan PEQ}{\tan POQ} \text{ (as the angles are small)}$$

$$= \frac{PQ/EQ}{PQ/OQ} = \frac{OQ}{EQ} = \frac{\text{Focal length of objective}}{\text{Focal length of eye-piece}}, \text{ in case of normal vision.}$$

Hence, **magnification** $= \dfrac{F}{f}$, where F and f are the focal lengths of the objective and the eye-piece respectively.

This is the case when the object is at infinity, and the image PQ due to the objective is formed at the focal plane of the eye-piece.

The magnification is, however, increased by focussing such that the final image $P'Q'$ is at the least distance of distinct vision.

Note—(a) It is obvious from the expression for *magnifying power* that *for large magnification,* the objective should be of long focus and the eye-piece should be of short focus.

It should be noted that the principle of this instrument is similar to that of the compound microscope, but to meet the difference in purpose, modifications are made. Both these instruments consist of at least two convex lenses of suitable focal lengths.

(b) An image, when highly magnified, is proportionately reduced in brilliancy. So in order that the telescope should be able to grasp as much light as possible from the object, object glasses of large diameter are used and this is the reason why astronomical telescopes are made large. *Moreover if the diameter is large, the resolving power will be large, i.e. finer details of the object will be observed.*

Examples—(*1*) *In a simple form of astronomical telescope the focal length of the object-glass is 30 inches, that of the eye-piece is 2 inches. Calculate the magnifying power when the final image of a distant object is seen* (*i*) *a long way off,* (*ii*) *at a distance of 12 inches. Find the distance between the two lenses in each case.*

(*i*) Since the final image is seen a long way off, the image due to the objective is formed at the principal focus of the eye-piece. Hence, in this case,

$$\text{magnifying power} = \frac{F}{f} = \frac{30}{2} = 15.$$

The distance between the lenses $= 30 + 2 = 32$ inches.

(*ii*) In this case the distance of the final image due to the eye-piece is 12 inches. We have, in the case of a lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \quad \text{Here } v = +12\ ;\ f = -2\ ;\ u = ?$$

or

$$\frac{1}{12} - \frac{1}{u} = -\frac{1}{2}\ ;\ \text{whence } u = \frac{12''}{7}.$$

Hence the magnifying power $= \dfrac{F}{u} = 30/\tfrac{12}{7} = 17{\cdot}5.$

Since $12/7''$ is the distance of the image, due to the objective, from the eye-piece, the distance between the two lenses $= 30 + 12/7 = 30 + 1{\cdot}71 = 31{\cdot}71$ inches.

2. A telescope is formed of two convex lenses of 10 cms. and 1 cm. respectively. If the telescope is focussed on a scale one metre from the objective and the final image formed 25 cms. from the eye, calculate the magnification produced. (*Pat. 1944*)

In order to form a final (virtual) image at a distance of 25 cms., the object (*i.e.* the real image formed by the objective) must be at a distance, say, u from the eye-piece, when we have,

$$\frac{1}{25} - \frac{1}{u} = -\frac{1}{1}\ ;\ \text{whence } u = \frac{25}{26}.$$

Again, the distance v (OQ in Fig. 93) is given by,

$$\frac{1}{v} - \frac{1}{100} = -\frac{1}{10};\ or\ \frac{1}{v} = -\frac{9}{100};\ or\ v = -\frac{100}{9}$$

$$\therefore \text{ Total magnification, } \frac{v}{u} = \frac{100}{9}\bigg/\frac{25}{26} = 11\frac{5}{9}.$$

81. The Terrestrial Telescope.—An inverted image produced by an astronomical telescope is of little use for observing terrestrial objects For example, erect image is esential if the instrument is to be used by navigators, surveyers, etc.

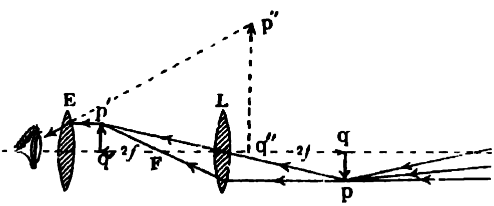


Fig. 94—Terrestrial Telescope

This difficulty of the astronomical telescope is obviated in the terrestrial telescope (Fig. 94) by the addition of a convex lens in the eye-piece so as to convert the inverted image into an erect image, though the length of the instrument is thereby inconveniently increased. Some light is reflected and lost at each glass surface, so the image becomes fainter by the use of the extra lens.

Due to the objective (which is not shown in the diagram), a real, inverted image pq is formed (Fig 94). A convex lens L, which is the erecting lens, is interposed between the objective and the eye-piece in such a way that the distance of the image pq from the auxiliary lens L is twice the focal length ($2f$) of the lens L. Thus another real, inverted image $p'q'$, equal in size to pq, is formed at the same distance $2f$ on the other side of the lens L. This image being inverted with respect to pq is erect with respect to the object. The eye-piece E is so adjusted that the image $p'q'$ is formed just within its focal length, which is looked at through the eye-piece E, and a virtual magnified image $p''q''$ is seen at the least distance of distinct vision.

Instead of one lens L, a combination of two lenses A and B (Fig. 95), called an erecting eye-piece, is also sometimes used. The first lens A of the combination is placed at a distance of its focal length from the image PQ and the rays emerge as parallel beams and fall on B which brings them to a real focus at $P'Q'$.

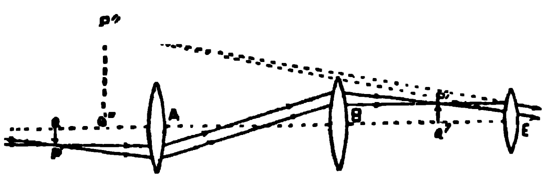


Fig. 95—Terrestrial Telescope

Thus a real, erect image is obtained just within the focal length of the lens E, and so a virtual, magnified image is finally obtained. The lenses A and B are placed apart by a distance equal to twice their focal length.

82. Galileo's Telescope.—The disadvantage of the great length of a terrestrial telescope, described in Article 81, is avoided to some extent in the Galileo's telescope, which is the earliest form of telescope. It consists of a **convergent objective** O of **large focal length** and a **concave lens** O' as the **eye-piece**. A real, inverted image $P'Q'$ of the object would have been formed at the focal plane of the objective, but the concave lens intercepts the path of the rays before they could reach the focus, such that the rays emerge from it as a practically parallel pencil, and the eye sees a *virtual, erect* and *magnified image* $P''Q''$ at infinity (Fig. 96).

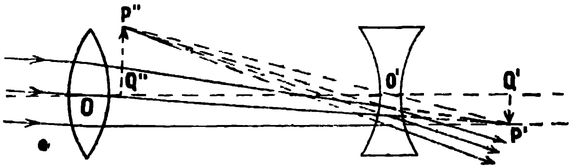


Fig. 96—Galilean Telescope

The eye should be placed as close to the eye-piece as possible, for the rays being divergent from the eye-piece, the field of view diminishes as the eye recedes from it. As the rays emerge as a parallel pencil due to the interception of the eye-piece at O', $O'Q'$ is the focal length of the eye-piece. Similarly, OQ' is the focal length of the objective O.

Magnification :—

As in the case of the astronomical telescope, the magnification is given by,

$$m = \frac{\angle P''O'Q''}{\angle P'OQ'} = \frac{\angle P'O'Q'}{\angle P'OQ'} = \frac{\tan P'O'Q'}{\tan P'OQ'} \text{ (the angles being small)}$$

$$= \frac{P'Q'}{O'Q'} \div \frac{P'Q'}{O'Q'} = \frac{OQ'}{O'Q'} = \frac{\text{Focal length of objective}}{\text{Focal length of eye-piece}}$$

$$= \frac{F}{f}, \text{ (when the eye-piece is focussed for infinity).}$$

Astronomical and Galilean Telescopes Compared.—

(*a*) Since the focus of the eye-piece coincides with the focus of the objective, the *distance* between the two lenses in a Galilean telescope is equal to the *difference* between their focal lengths, *i.e.* $(F-f)$; in an astronomical telescope this distance is equal to the *sum* of their focal lengths, *i.e.* $(F+f)$. A Galilean telescope is thus shorter in length, which is a distinct advantage over the other types as it causes less loss of light.

(*b*) In a Galilean telescope the image formed by the eye-piece is erect and so it is best adapted for viewing terrestrial objects. In an astronomical telescope the final image is inverted, so this type can be used only for astronomical work.

(*c*) The field of view and the magnification of an astronomical telescope are much greater than those in the Galilean type. In the Galilean telescope, the eye-piece being a divergent lens, the image is practically formed by those rays only which pass near about the centre, the marginal rays being mostly lost, and for this reason, the final image is faint, *i.e.* the field of view is limited.

(*d*) Cross-wires can not be fitted in the Galilean telescope, for the eye piece being a divergent lens there is no position where the cross-wires may be placed such that the final image formed by the eye-piece and the image of the crosswires will be coincident in position. The focal plane of the objective is outside the instrument and no cross-wires can be placed there if the eye is to be placed immediately behind the eye-piece.

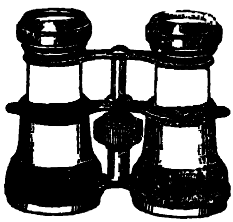


Fig. 97—Opera-Glasses

Opera-Glasses—The ordinary *Opera-Glasses* (Fig. 97) usually consist of a pair of Galilean telescopes mounted side by side with their axes parallel. The magnification in this case is small owing to the shortness of the tubes. The distance of the two telescopes may be altered to suit the two eyes of the observer.

Example. *In an opera-glass the focal length of the objective is 4 inches, that of eye-piece $1\frac{1}{2}$ in. What will be the magnifying power and also the distance between the objective and the eye-piece when focussing a distant object?* (*All. 1926*)

The magnifying power is $\frac{F}{f}$, where F is the focal length of the objective and f that of the eye-piece. $\therefore$ The magnifying power $= 4 \div \frac{3}{2} = \frac{8}{3}$.

In this case, the distance between the two $= F - f = 4 - \frac{3}{2} = \frac{5}{2} = 2\frac{1}{2}$ in.

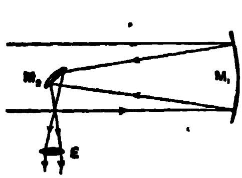


Fig. 98—Reflecting Telescope

83. Reflecting Telescope.—In Newton's reflecting telescope (Fig. 98) a parallel beam of light from a distant object falls on a concave mirror M_1 and, after reflection, a real, inverted and diminished image would have been formed in its focal plane, but before reaching it the rays are intercepted by a small plane mirror M_2 inclined at 45° to the axis of the instrument by which the image is shifted in a side tube, where it is viewed by means of the eye-piece E. The eye-piece is so placed that the image formed by the mirror is in its focal plane and thus a

virtual and magnified image is seen at infinity. The 100-inch reflector at the Mount Wilson observatory has the mirror made of glass silvered on its front surface. The largest reflector of this type is set up at the California Institute of Technology where a 200-inch Pyrex mirror coated with Aluminium has been used.

Advantages of a Reflecting Telescope.—*(a)* In a refracting telescope, objectives of large aperture are used in order to have a large amount of light, but due to the large aperture an appreciable portion of the light is absorbed by the lens. To avoid this, large concave mirrors are used where the loss of light by reflection is much less and so the image obtained is much brighter.

(*b*) There is no chromatic aberration (Art. 105) due to refraction and so the image is quite sharp and free from colour defects.

(*c*) It is more difficult to make large lenses used in telescopes than to make reflecting mirrors.

(*d*). By using a parabolic mirror spherical aberration may be prevented.

84. **Prism Binoculars (or Field Glasses)**—These are essentially terrestrial telescopes in a compact form containing *two totally reflecting prisms*. It will be seen from the diagram [Fig. 99(a)] that rays from the objective first enter a prism and are internally reflected by it. They are again internally reflected by a second prism after retraversing the length of the instrument and finally pass through an erecting eye piece. Thus the rays from the objective travel thrice the length of the tube by means of the prisms before reaching the erecting eye-piece. That is, the distance between the objective and the eye-piece is increased without increasing the length of the instrument. So an objective of larger focal length can be used in this instrument. In the figure, the image formed by the first prism which is placed with the refracting edge vertical is inverted *laterally*, but not *vertically*, and by the second prism which is placed with its refracting edge horizontal, *i.e.* at right angles to that of the first prism, the image is inverted *vertically*, but not *laterally*. So the final image is upright and the correct way round. Thus the purpose of the terrestrial telescope is served in a short length. This is the best form of binocular. These intstruments are termed binoculars (Lat. *bini*, two together + *oclus*, eye) as they are constructed in pairs, one for each eye.

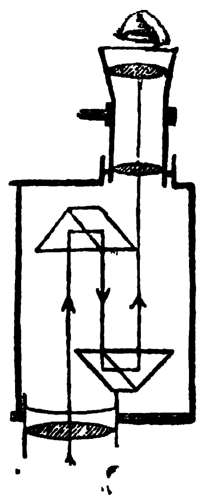


Fig. 99(a)—Prism Binocular

Advantages—Here by using an objective of greater focal length a greater magnification is possible than in the case of ordinary opera-glasses, and by the arrangement of prisms the objectives are placed farther apart than in opera-glasses by which a wider field and an increased stereoscopic effect are obtained. Fig. 99 (b) illustrates a modern form of prism binocular.

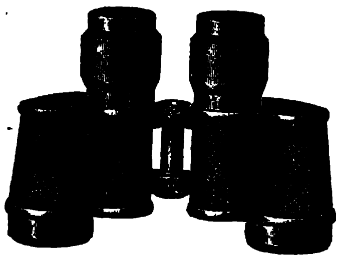


Fig. 99 (b)—Prism Binocular

85. Stereoscope.—A photograph of an object always gives a *flat* appearance because it is taken from the same angle. But if two pictures of an object are taken simultaneously by two cameras whose lenses are separated in the same way as our eyes, *i.e.* from slightly different angles, and then if the two pictures are mounted side by side and looked through lenses or tubes so that one eye looks at one picture only, they will form in the brain a **stereoscopic view**, *i.e.* the impression of one picture with *depth* (as the object is actually seen). This is the action of the stereoscope.

The action will be understood from Fig. 100. Here AB and $A'B'$ are two stereoscopic pictures of the same object, *i.e.* photographs of the same object taken by a double camera having two objectives placed side by side at the same angular distance as the eye. The two pictures are correctly mounted on a card-board which is introduced into the stereoscope. This is a wooden box having two compartments in which the two pictures are to be housed. On the same side of the box, there are two holes, one for each compartment, in which two converging half lenses L_1, L_2 are fixed behind which the eyes of the observer are placed. The positions of the lenses are so arranged that rays coming from corresponding points P_1 and P_2 of the two stereoscopic pictures, which enter the eyes after refraction through the lenses, form images which appear to combine into a single image at P.

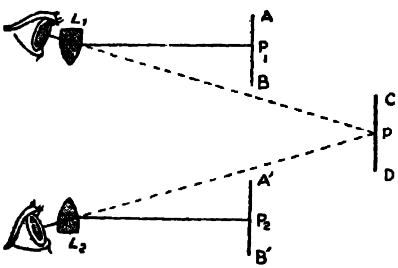


Fig. 100—Stereoscope

As the two photographs of the same object are taken side by side at the same angular distance as the two eyes, the two images seen by the eyes appear to combine and give the impression of only one object in relief, *i.e.* an object as actually seen.

86. Magic Lantern (Optical Lantern).—By means of this apparatus a real, magnified image of a transparent object, usually a lantern slide, a film, or a drawing on glass placed inside a closed box is projected on a screen so that it can be displayed to a large gathering.

The following must be its essential parts (Fig. 101) :—

(i) A Powerful Source (A).—If it is a carbon-arc lamp, the positive carbon on which the crater is formed is taken to be horizontal and the other carbon-rod vertical in order that the full light produced in the crater may pass through the apparatus. Lime light, acetylene burner or any other suitable source may be used if electric light is not available. A suitable reflector may be placed at the back in order to turn the back-rays to the forward direction.

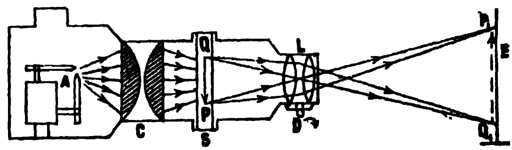


Fig. 101—Optical Lantern

(ii) The Condenser (C).—It consists of two plano-convex lenses placed with their curved surfaces facing each other. The convergent lens-system turns the divergent rays from the source into a convergent beam directed towards the transparent object PQ. It concentrates the rays to illuminate the object properly and is therefore called a condenser.

(iii) The Slide :—It is a photograph on a glass plate of suitable size or a film or a drawing on glass. It is put in a wooden frame which is inserted in a groove *S* in the lantern. It can be changed by drawing out the wooden carrier. This constitutes the object.

(iv) The Focussing Lens (L) :—It is a convergent combination of lenses corrected for the chromatic and spherical defects. Its position with respect to the slide can be adjusted by means of a rack and pinion arrangement *D*. The action of the lens can be stopped by means of a cap which can be fitted on it.

(v) A Vertical Screen (E) :—It should be preferably white.

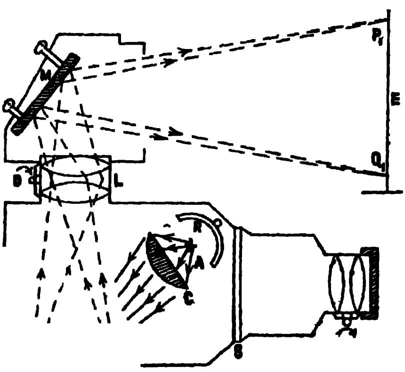
Action :—By the action of the condenser *C* the diverging rays from the source *A* are turned into a convergent beam directed towards the object PQ which is thereby uniformly and strongly illuminated. The slide is put in an inverted fashion. The position of the slide is beyond the principal focus of the **focussing lens** *L* and so an inverted, magnified and real image P_1Q_1 of the inverted object PQ

(*i.e.* an erect image) is produced on the distant screen *E*. The focussing lens *L* is then slightly adjusted (*i.e.* moved forward or backward) by means of the rack and pinion arrangement *D* until the image formed is made very distinct and of the desired size.

Lantern slides are usually about 3″ square and the size of the image on the screen may be as large as 8′ square. Thus the light passing through the slide is spread over an area 1000 times as great. Hence it is obvious that unless a powerful source of light is used the image will not be distinct.

N. B. As the image thrown on the screen is inverted, so the lantern slide is to be inserted in an inverted position in order to get an erect image.

87. Epi-diascope :—An epi-diascope (Fig. 102) combines the principles of a magic lantern and an episcope. The **episcope** is a convenient instrument for projecting the images of opaque objects like ordinary diagrams, maps, pictures, and photographs on a distant screen. Light from a strong gas-filled lamp, or arc-lamp *A*, having a concave reflector *R* at the back is concentrated by the condenser *C* (vide Art. 86) on to a map or a diagram in a book placed on a table, called object table. Light from the object PQ then travels upwards through a focussing lens system *L* on to a mirror *M* with a silvered surface from which it is reflected and finally focussed on a screen *E*.



EPI-DIASCOPE

Fig. 102—Epi-diascope

The mirror *M* is placed at 45° with respect to the horizontal. The object PQ is beyond the principal focu of the convergent focussing lens *L* and so a real magnified image is produced on the screen *E*. By adjusting the rack and pinion arrangement *D*, the distance between *L* and PQ may be altered until the image P_1Q_1 is made distinct and of the proper size.

When an episcope is also fitted to project lantern slides, like an ordinary magic lantern, it is called an **epi-diascope**.

88. Photographic Camera.—It is an apparatus by which a permanent image of an object can be taken on a photographic plate or film. Its essential parts are ;—

(i) A Light-tight Box BA (Fig. 103).—It is made of folded black cloth, leather or paper so that it can be extended or shortened in length according to necessity. The inside must be black or painted black for stopping internal reflections. The box is usually fitted on a tripod stand by means of which it can be set at any desired height or tilted in position. The base of the box is provided with a rack and pinion arrangement (not shown in the figure) by which the back of the box at which the plate is placed can be moved in or out.

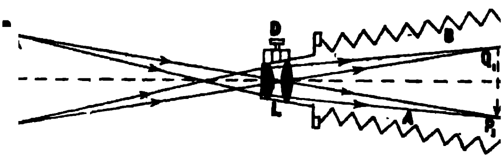


Fig. 103—Photographic Camera

(ii) The Photographic Objective.—It is a combination of lenses equivalent to a converging lens and is corrected for chromatic and spherical errors. Its design depends on the purpose for which the camera is used. There is a rack and pinion arrangement D by which the objective can be moved forward and backward for purposes of focussing.

(iii) The Iris Diaphragm.—It is an adjustable shutter by which the aperture of the lens can be regulated to permit variation in the intensity of the image and improve its definition.

In order to obtain a clearly defined image only the middle portion of the lens should be used ; the other parts are covered up by means of an adjustable diaphragm, called a **stop**, by photographers. By using stops with holes of different sizes the requisite amount of light may be allowed to reach the film. The smaller the diameter of the stop the better is the definition of the image, though a longer exposure will be necessary. The diameter of the stop is always expressed as a fraction of the focal length of the lens ; thus, the fractions $f/16$ or $f/8$ marked on the outside of the adjustable diaphragm of a camera means that the diameter of the stop is $\frac{1}{16}$ or $\frac{1}{8}$ of its focal length.

(iv) The Shutter.—It permits the time of exposure to be varied. In modern cameras, there is automatic arrangement for varying the time of exposure from 1/10th to 1/100th second. Such exposures are called ***instantaneous***. There is provision for time exposure also whereby the plate can be exposed to light for any length of time according to the judgment of the photographer.

(v) **The Screen** (E) :—It is a ground glass plate on which the focussing is first done and is subsequently replaced by the plate.

(vi) **The Slide** :—This is a flat light-tight box for housing the sensitive plate. It has a movable shutter which can be drawn out when the plate is to be exposed.

(vii) **The Plate** :—It is a plate of glass (or celluloid) on which there is a thin layer of an emulsion of one or more halides of silver in gelatin.

How a Photograph is taken :—The camera is set up at the proper height in front of the object, the shutter of the lens is opened and by varying the distance of the camera from the object, an image is roughly formed on the ground glass screen. Then by adjusting the distance between the objective and the screen (by altering the position of the screen or the position of the objective or both, by the help of rack and pinion arrangements) an image of the proper size is sharply focussed on the screen. The aperture of the diaphragm is then adjusted for the proper illumination and definition of the image. The camera is then loaded with the plate in the slide. The plate is then exposed to light by drawing out the shutter of the plate. The time of exposure is a matter of art which can be learned only by experience. It depends on the intensity of light and the size of the aperture.

No visible image, however, appears on the plate on being simply exposed to light. The shutter of the slide-carrier is then closed and the slide is removed to a dark room where the plate is kept immersed in a chemical solution, called the **Developer**. The silver salts in those parts where they have been acted on by the incident rays are here gradually reduced by the developer to the metallic state. When the picture is satisfactorily revealed, the plate is carefully washed with water so that even the last trace of the developer may be washed out. The plate is next washed in a solution of "**Hypo**", called the fixer solution. Until this is done, the plate is not free from the action of light. The Hypo (hypo-sulphite of soda) solution washes away the silver salts from the parts of the plate not affected by light. By repeated washing in water, the last trace of the hypo is removed, after which the plate is dried up in a current of air. The plate thus developed and *fixed* is called the **Negative**, since the bright parts of the image are depicted black on the plate and vice-versa. To print the positive (called the photograph) the film-side of a sensitised paper, similarly coated as the plate, is held in contact with the negative in a suitably constructed frame and light is passed through the negative to act on the film for an appropriate period. The image on the paper is then *developed* and

fixed as before. The paper is afterwards thoroughly washed and dried. This is the photograph. The method is called **contact printing.**

89. Pin-hole Camera and Lens Camera Compared—The image formed by a pin-hole camera is never perfectly sharp. The smallar the hole the sharper is the image, but the amount of light received on the screen is very small for photographic purpose. Another defect of the pin-hole camera is that the images formed by it are equally sharp, *i.e.* there is little difference in sharpness, though the distance of the objects are changed, but the images formed by a lens camera are much sharper if the object is correctly focussed, but for other distances of the object blurred images are formed.

In photography the amount of chemical action taking place on any small area of the plate will depend upon the amount of light falling on it, which again depends upon (*a*) the intensity of light from the source: (*b*) the area of the hole; (*c*) time of exposure. Keeping (*a*) and (*c*) constant for both, the area of the hole, *i.e.* the front surface of the lens in a lens camera can be made large compared with the pin-hole, so that with a very short exposure the same quantity of light will reach a small area on the plate. This is *a great advantage of the lens camera*; but for still objects with long time exposure there is no special advantage with a lens camera.

90. Eye and Vision.—The eye (Fig. 104) is nearly spherical in shape, and, within limits, is capable of turning in its socket. It is nature's optical instrument.

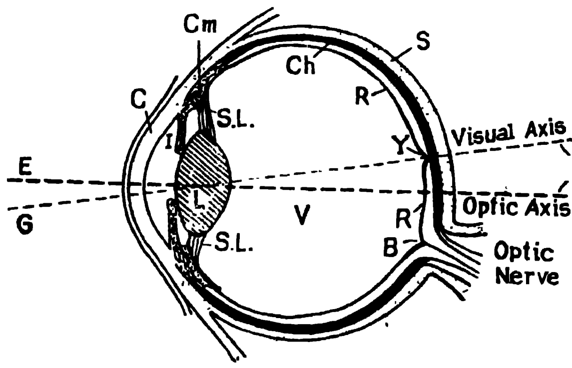


Fig. 104—The Human Eye

A human being has two similar eyes and has the advantage of binocular vision (vide Art. 92). The eye-globe is complex in structure. Its exterior is formed by a white membrane S, called the **sclera.** The front portion C of the sclera is transparent and more convex than the rest. This part is known as the **cornea.** Internal to the sclera is a dark brown membrance (*Ch*) which

is known as the **choroid.** The anterior continuation of the choroid is a seat of muscles known as the **ciliary muscles** (*Cm*). From this ciliary body a diaphragm circular in form hangs and is perforated in its centre by an aperture whose size can vary by an involuntary action of the ciliary body. Its function is to regulate the size of the aperture. Its colour gives the colour of an eye. This diaphragm *I* is called the **Iris.** Behind the iris is suspended the focussing lens *L* of the eye. It hangs from the inner surface of the ciliary muscle by means of a few fibres, called the **suspensory ligaments** (*S. L.*). The lens is bi-convex (more convex at the back), transparent and is composed of different layers of different refractive indices. The space in between the cornea in front and the iris is called the **anterior chamber** while the space between the iris and the suspensory ligaments is called the **posterior chamber,** and they are filled up by a salt solution known as the **aqueous humour.** On the internal back-portion of the choroid is stretched a delicate purple-red membrane having a network of minute structures, called the **rods** and **cones,** which receive the light waves, and the optic-nerve-fibres which carry the light sensations to the visual centre of the brain. This light sensitive membrane *R* is called the **retina.** On the inner surface of the retina there is a region (*Y*) about 1 to 2 mm. in diameter, called the **yellow spot** or the **macula lutea.** In the centre of the yellow spot there is a small depression, called the **fovea centralis.** The imaginary line that passes through the optic centre of the eye-lens and the fovea centralis is called the **visual axis** of the eye The yellow spot is the region of the highest optical sensitivity in the retina. The imaginary line that passes through the centre of the cornea and the optic centre of the eye-lens is called the **optic axis** of the eye. The optic axis meets the retina at the posterior pole of the eye. The optic nerve-fibres enter the retina about 3 mm. below the posterior pole of the eye. The spot where the optic nerves enter the retina is called the **optic disc.** This disc *B* has no sensitivity and hence is called the **blind spot.** The space between the retina and the lens is known as the **vitreous chamber** and is filled up with a transparent, colourless and gelatinous mass known as the **vitreous humour** (*V*).

Action of the Eye.—Rays from an external object entering the eye suffer refraction mainly at three surfaces, the outer surface of the cornea and the two surfaces of the lens. Then again there is the continuous refraction experienced by the rays in passing from layer to layer of the lens and finally the rays are brought to a focus on the retina. The image produced by the eye-lens (convex) on the retina is real, diminished and inverted. *That we see it erect is due to a process of mental interpretation only, to which all observers are habituated.*

Photographic Camera and the Eye compared.

Photographic Camera (Fig. 103)	Eye (Fig. 104)
1. A light-tight box.	1. The sperical eye-ball formed of a fairly hard substance called the *sclerotica*
2. A converging lens or a combination of lenses by which a real, inverted and diminished image is formed on a photographic plate. The focussing is done by adjusting the length of the box.	2. The *cornea, aqueous humour and the crystalline lens* together form a similar image on the **retina**. The focussing is done by altering the curvature of the lens by means of what are called *ciliary muscles*. This power of the eye to focus the image by adjustment of the lens is called **accommodation**.
3. The image is received after developing the sensitised plate by chemical means.	3. The image impressions received on the *retina* are conveyed to the brain through the *optic nerves*.
4. An adjustable *diaphragm* or stop regulates the amount of light entering the camera.	4. The amount of light entering the crystalline lens is regulated by the *iris I*, which is a diaphragm with a circular aperture, called the **pupil** near to its centre.
5. A *shutter* in front of the lens shuts out light whenever required.	5. The *eye-lids* can shut out the light for longer or shorter periods at will.

[**Note** It has already been said that the images formed on the retina are always inverted though we see them erect. This, however, cannot be clearly explained. The explanation is that our mind has learnt by touch and muscular sense from our infancy that an inverted image on the retina means the presence of an erect object. Thus it is only a mental interpretation.]

Power of Accommodation.—An eye is said to be normal when in a state of full relaxation it can focus on the retina objects at an infinite distance. When such an eye looks at a near object, the image should be formed beyond the retina, but the eye by virtue of an inherent power (which acts involuntarily), called its **power of accommodation**, can form the image of the object on the retina. According to Helmholtz, the mechanism of accommodation is that the ciliary muscles automatically contract, drawing forward the choroid and relaxing the suspensory ligaments ; this diminishes the tension of the lens-capsule and allows the inherent elasticity of the lens to increase its convexity.

Chiefly the anterior surface of the lens changes in curvature. The degree of accommodation obviously will vary with the distance of the object under view.

Thus **accommodation** *is that property of the eye-lens by which its effective focal length is automatically altered to suit the act of viewing distant or near objects.*

Far Point of the Eye.—The most distant position up to which the eye can see when fully relaxed is termed the far point. For the normal eye it should be at infinity. But actually the far point is only a distant point (which varies from person to person) for an eye.

Near Point of the Eye.—For every eye there is a limit to its power of accommodation. This power ceases when the object is brought upto a certain minimum distance from the eye. The nearest position upto which a small object can be distinctly seen, employing the maximum amount of *accommodation*, is referred to as the near point of the eye. It can be determined for an eye by noting the shortest distance at which a man can read the smallest test-type with the other eye closed.

Least distance of Distinct Vision.—It is the distance of the *near point* from the eye. For a normal eye it is about 10 inches (*i.e.* 25 cms.)

91. Defects of Vision.

The **common defects** of vision are the following :—**(a) Long-sight** (or Hypermetropia) ; **(b) Presbyopia** ; **(c) Short-sight** (or Myopia) ; **(d) Astigmatism.**

(a) Long-sight (or Hypermetropia)—For the normal eye, the distance of the near-point is about 10 inches (25 cms.), and for a long-sighted person it is greater. **A long-sighted eye cannot see near objects distinctly,** but if the accommodation is normal there is no difficulty about distant objects. Rays from the normal near-point are brought to a focus behind the retina (Fig. 105). The **causes** are (*a*) the eye-ball is too small, or (*b*) the focal length of the lens has become increased.

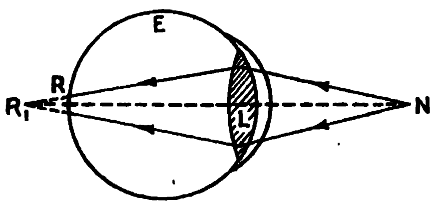


Fig. 105—A Long-sighted Eye

The remedy is to *interpose* a **convex lens** so that the focal length of the combination is shortened and by that the image is formed on the retina (Fig. 106). As a rule, parallel rays from distant objects are focussed on the retina. In order that the rays from the normal near-point N can be focussed on the retina, the converging lens L_1 of the spectacles should be of such power as to convert the rays from the object appear to come from the near point N_1 of the defective eye.

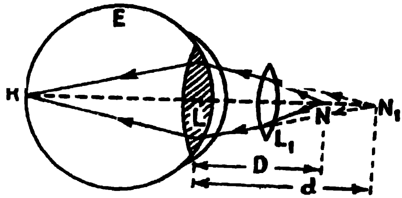


Fig. 106—A Corrected Long-sighted Eye

Let v = distance of retina from the lens, f_e = focal length of the eye, d = distance of the near point of the long-sighted person found by trial,

we have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Hence $\frac{1}{v} - \frac{1}{d} = \frac{1}{f_e}$ ... (1)

Suppose a lens of focal length f_1 should be interposed to bring the near-point of the defective eye to a position 25 cms. from the eye. So we have,

$$\frac{1}{v} - \frac{1}{25} = \frac{1}{f_e} + \frac{1}{f}\ ;\quad or\ \frac{1}{v} - \frac{1}{25} = \frac{1}{v} - \frac{1}{d} + \frac{1}{f_1} \quad \text{... from (1)}$$

or $\frac{1}{f_1} = \frac{1}{d} - \frac{1}{25}$. As $d > 25$, $\frac{1}{d} < \frac{1}{25}$, and, therefore, f_1 is $-ve$.

This gives the focal length of the spectacle lens required, and the negative sign of f_1 shows that the **lens must be convex.**

(b) **Presbyopia.**—This is another form of long-sight which is due to old age. This is called *Presbyopia* and sometimes called **Far-sight.** The crystalline lenses of the eyes lose elasticity gradually with age, and the accommodating power of the ciliary muscles decreases. Thus a short-sighted eye in childhood tends to become normal in after years, but the defect of long-sightedness is sure to increase gradually.

Examples.—*1. A long-sighted man can see clearly at any distance more than 10 ft. What kind of spectacle lens should be used to enable him to read print placed 10″ from his eye ?*

The distance of the near-point of the man 10 ft. or 120″. So, we have,

if v = distance of retina from the lens of the eye, $\frac{1}{v} - \frac{1}{120} = \frac{1}{f_e}$. Again, if

f = focal length of the spectacle lens, $\frac{1}{v} - \frac{1}{10} = \frac{1}{f_e} + \frac{1}{f} = \frac{1}{v} - \frac{1}{120} + \frac{1}{f}$.

or $\frac{1}{f} = \frac{1}{120} - \frac{1}{10} = -\frac{11}{120}$; $\therefore f = -10{\cdot}9''$ approx.

Hence, the spectacle lens should be convex and of focal length $10{\cdot}9''$ approximately.

2. *A long-sighted person has a minimum distance of 50 cms. What kind of lens must be used in order to reduce this distance to 25 cms.? What should be the focal length of the lens?*

Here $u = 50$ cms. Hence $\frac{1}{v} - \frac{1}{50} = \frac{1}{f_e}$, where f_e is the focal length of the eye lens. Again if f_1 is the focal length of the spectacle lens,

$$\frac{1}{v} - \frac{1}{25} = \frac{1}{f_e} + \frac{1}{f_1} = \frac{1}{v} - \frac{1}{50} + \frac{1}{f_1}; \qquad \frac{1}{f_1} = \frac{1}{50} - \frac{1}{25} = -\frac{1}{50};$$

$$\therefore f_1 = -50 \text{ cms.}$$

Thus the spectacle lens should be convex and of focal length 50 cms.

(c) **Short-sight** (or Myopia).—**A short-sighted person cannot see distant objects distinctly.** The rays from a distant object are brought to a focus in front of the retina R (Fig. 107); so the far-point is nearer than infinity, but as to near-vision there is no difficulty if the accommodation is normal, though the near-point N_1 may be much closer than 25 cms. The **causes** of the defect may be ;—(*i*) the eye-ball is too elongated, (*ii*) the focal length of the lens of the eye is too short.

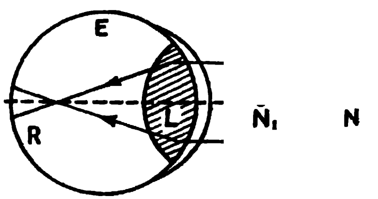


Fig. 107—A Short-sighted Eye

Let d = the distance of the farthest point F upto which the short-sighted person can see distinctly (Fig. 108). This point in the case of a normal eye should be at infinity.

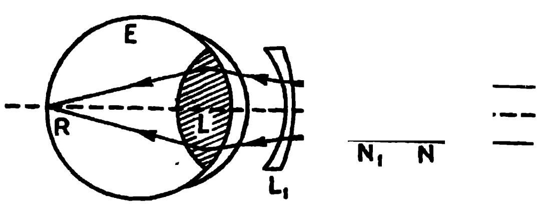


Fig. 108—A Corrected Short-sighted Eye

It is necessary to *interpose* a **concave lens** such that parallel rays from distant objects will, after refraction through the lens, appear to diverge from F, the far-point of the defective eye, and will thus be focussed on the retina by the lens L of the eye.

As before, we have, $\frac{1}{v}-\frac{1}{d}=\frac{1}{f_e}$ ... ... (1)

By interposing the lens, the farthest point would be at infinity.

So $\frac{1}{v}-\frac{1}{\infty}=\frac{1}{f_e}+\frac{1}{f}=\frac{1}{v}-\frac{1}{d}+\frac{1}{f}$ ... ... from (1)

or $\frac{1}{f}=\frac{1}{d}\left(\because \frac{1}{\infty}=0\right)$; $\therefore f=d$.

Hence the focal length of the spectacle lens should be equal to the distance of the farthest point upto which the short-sighted person can see distinctly, and the positive sign of f shows that the **lens must be concave.**

Examples.—*1. A short-sighted man, who can read clearly when the print is not more than 3 inches from his eye, requires spectacles to enable him to see a distant view. What kind of lenses does he need and what must be the focal length? Draw as accurately as you can the path of a ray of light from a distant object through the lens of the man's eye (a) without the spectacles, (b) with the spectacles.* (*C. U. 1909*).

Here, the farthest point = 3 inches. So, $\frac{1}{v}-\frac{1}{3}=\frac{1}{f_c}$.

But, if f be the focal length of the lens to be used, we have,

$\frac{1}{v}-\frac{1}{\infty}=\frac{1}{f_e}+\frac{1}{f}=\frac{1}{v}-\frac{1}{3}+\frac{1}{f}$; or $\frac{1}{f}=\frac{1}{3}$; $\therefore f=3$ inches.

The man will evidently require a concave lens of focal length 3 inches. (For diagrams see Figs. 107 & 108).

2. A short-sighted man can read printed matter distinctly when it is held at 15 cms. from his eyes; find the focal length of the glasses which he must use if he wishes to read with ease a book at a distance of 60 cms. (*C. U. 1932*)

$\frac{1}{v}-\frac{1}{15}=\frac{1}{f_e}$, and $\frac{1}{v}-\frac{1}{60}=\frac{1}{f_e}+\frac{1}{f}=\frac{1}{v}-\frac{1}{15}+\frac{1}{f}$;

$\therefore \frac{1}{f}=\frac{1}{15}-\frac{1}{60}=\frac{3}{60}=\frac{1}{20}$; $\therefore f=+20$ cms.

So the glasses of the spectacles must be concave of focal length 20 cms.

(d) **Astigmatism** (Gk. *a*, without + *stigmatos*, a point of focus).—This defect of eyes is usually due to irregularity in the curvature of the vertical and horizontal sections of the cornea; the curve generally is more pronounced in the vertical than in the horizontal with the result that horizontal and vertical lines at the same distance will not be in focus at the same time. Such an eye, when looking at a network, may be able to see clearly, for instance, the horizontal wires, while vertical wires may be indistinct or curved. A **Cylindrical or Sphero-cylindrical lens** is used to remove this defect. The defect may differ in degree in the two eyes.

92. The Advantages of Two Eyes (Binocular Vision).—As with two ears we hear only one sound, so, by our two eyes, when fixed on any object, we see only one object. Though each of the eyes forms an image on its own retina, the brain translates as a whole the different images. One advantage of having two eyes is *the power of estimating distance* correctly. It may have been noticed that the threading of a needle is very difficult when one eye is kept closed. To illustrate it further, suspend a small curtain ring at some distance with its plane in the line of sight. Now hold a long stiff wire, bent at one end, and try to insert the bent end through the ring keeping one eye closed. It will be found more difficult to do it when one eye is closed than when both eyes are open. This difference is due to the fact that the simultaneous vision of two eyes gives a marked facility in estimating distance.

Further, due to the difference in position of the two eyes with respect to an object, the right eye sees more of the right side and the left eye sees more of the left side. When the object is viewed simultaneously by both the eyes, the two images which are slightly different overlap on each other. The resultant gives a combined idea of *depth* and *solidity* (see Art. 85).

93 Persistence of Vision : Cinematography.—Visual impressions, however momentary, received on the retina of the eye persist for about one-tenth of a second even after the stimulus is removed ; so if a series of impressions at intervals of more than ten per second falls on the retina, the eye will not be able to distinguish between them and we get a continuous impression, and this is the reason why Newton's disc (Art. 106), when rapidly rotated, appears greyish white which results from the overlapping of all the colours on the disc.

Again, if there be a picture of a bird on one side of a piece of card-board, and a picture of a cage just on the opposite side, then on rapidly revolving the card-board the two impressions blend and the bird will appear to be inside the cage due to persistence of vision. For the same reason the red end of a burning splinter, when whirled round, gives the impression of a continuous red circle.

(a) Cinematography.—Cinematography depends essentially upon the principle of persistence of vision. If photographs are taken of a moving object at intervals of about $\frac{1}{20}$ of a second, then the discontinuous pictures, when projected on a screen at the same rate, will fuse together and produce an illusion of continuous motion.

In a cinema-film there are numerous pictures of some object in different succeeding positions taken by means of a special camera (motion picture camera). These pictures are rapidly moved

before a projection lantern the arrangements of which are similar to those of the optical lantern (Fig. 101). The pictures on the film come in front of the condenser in the position occupied by the slide of the optical lantern. About twenty pictures on the film pass in front of the condenser per second, and as each of these arrives in position, the film stops for $\frac{1}{20}$th of a second by a clock-work arrangement and passes on to the next picture after a brief dark interval. The images of these rapidly moving pictures are thrown on a screen with an instant between each pair during which the film is not illuminated, but as the pictures follow one another very rapidly they represent some continuous incident or story due to persistence of vision, as the impression of one picture does not vanish before the second is received ; and this goes on one after another.

Cinema tricks.—Sometimes curious tricks are seen to be performed on the cinema. Many natural phenomena are seen to be accelerated and sometimes retarded. The seed of a plant takes a long time to grow, but the photographs of its growth can be taken at intervals over a comparatively long period of time on a single film, and when it is thrown on the screen at the ordinary rate, we get the impression of very rapid growth. On the other hand, snaps of such things as running, high jumping, strokes in cricket or tennis can be taken at 160 to 200 per second and then thrown on the screen at the ordinary rate of 16 per second producing what is called a slow-motion-picture.

(b) Heliograph and Heliostat.—A **heliograph** simply consists of a plane mirror suitably mounted in order to reflect sun-light from one place to another several miles away. This instrument is used for the transmission of messages. The mirror is tilted in order to cover or uncover it, according to necessity, so that the observer at a distant station may note the duration and regularity of the flashes of light (according to a given code) from which the message is to be constructed.

The **heliostat** is only a heliograph mirror by which a reflected beam can be sent to a particular direction all day long suitably mounting the mirror on a frame driven by clock-work.

Questions.

Art. 78

1. *(a)* Explain how a single convex lens may be used as a magnifier. Trace the path of the rays by which an object would be seen in such a case.

(C.U. 1918 ; cf. Pat. 1918 ; cf. All. '46)

(*b*) Show that the magnifying power of a reading lens is $\left(1+\frac{D}{f}\right)$ where

D is the least distance of distinct vision, a the distance between the eye and the lens, and f the focal length of the lens. What conclusions do you draw about the best position of the eye ? (P. U. 1919 ; Del. U. 1942 ; All. 1946)

(c) Explain the use of a convex lens as a magnifying glass. How is its magnifying power defined ? (Pat. 1947)

Art. 79.

2. You are given two convex lenses of focal lengths of 60 cms. and 3 cms. respectively. How do you arrange them to form (a) a compound microscope (b) a telescope ? Draw diagrams and explain the arrangement. (C.U. 1922 ; '45)

3. Describe a compound microscope. Explain by means of a diagram how the magnification is produced.

(Del. U. 1942 ; C. U. 1915, '21, '39, '43 ; All. 1916, '22, '32 ; Pat. '41 ; Dac. '32 ; P. U. 1912)

4. A compound microscope is adjusted for viewing the distinct image of an object. If the distance of the object from the object glass is now slightly increased, explain what re-adjustment of the instrument would be necessary for obtaining a distinct image again. Will the magnification be the same as before ?

[**Hints.**—If the distance of the object PQ from the lens is increased, then the image $P'Q'$ is shifted towards the object glass. Hence to obtain a distinct image, the eye-piece will have to be shifted towards the object glass (see Fig. 91).

Magnification $= \frac{v}{u} = \left(1 + \frac{D}{f}\right)$; here u will be increased, also v will be diminished, so magnification will be much diminished, though D and f will remain the same.]

4 (a). Two convex lenses of focal lengths 1 cm. and 6 cms. respectively are arranged to form a microscope. A small object is placed 1·2 cms. from the object glass. If the image seen appears to be 25 cms. from the eye-piece, what is the distance between the object glass and the eye-piece ? (Pat. '47)

[*Ans* : 10·83 cms.]

Arts. 80, 81 & 82

5. Give a brief description of (a) the astronomical telescope and (b) a compound microscope, showing by sketch how the image is formed in each case. (C. U. 1912, '14, '19, '20, '23, '25, '39 ; Pat. 1928, '39, '42, '45.)

6. You are given two lenses of focal lengths 20 cms. and 2 cms. Explain how you will arrange them to form a telescope. Draw a neat diagram to show the paths of rays when a distant object is viewed through it. What will be its length and magnifying power ? (Pat. 1931)

[*Ans :* Length $= 20 + 2 = 22$ cms. ; $m = \frac{20}{2} = 10$]

7. The focal lengths of the objective and the eye-piece of an astronomical telescope are 10 in. and 1 in. respectively. The telescope is focussed on an object 5 ft. from the objective, the final image being formed 10 in. from the

eye of the observer. Calculate the length of the telescope and the magnification produced by it. (Pat. 1942)

[*Ans :* Length $= \left(12 + \frac{10}{11}\right)''$; magnification $= 2.2$]

8. Describe the construction of a celestial telescope. What modification will make it suitable for terrestrial purpose ? (see Art. 81) (Pat. 1930)

Explain by means of a neat diagram how the magnification in a telescope is produced. (Dac. 1928 ; C. U. '32)

9. Trace the paths of rays through a Galilean telescope directed towards a distant object and adjusted for normal vision.
(C. U. 1926, '33, '40 ; All. 1926 ; Pat. 1927 ; Dac. '34 ; Del. U. 1948)

10. Describe the construction of a simple telescope which will give erect images of distant objects. Why are images in cheap telescopes usually coloured. (Dac. 1930)

11. In what way is an opera-glass different from an astronomical telescope ? (C. U. 1932)

Art. 83

12. Explain the principle and give details of construction of a reflecting telescope. What advantages does it possess over the refracting type ? (Pat. 1922)

13. Explain how a telescope can be made from a concave mirror and a convex lens. Illustrate your answer with diagrams showing the paths of rays. (Dac. 1932)

Art. 84

14. Explain how two lenses are arranged to form a binocular. Illustrate your answer by a diagram.

Art. 86

15. What are the essential parts of a photographic camera or a magic lantern ? State the utility of the different parts. (see Art. 88). (Pat. 1932 ; cf. All. '45)

Art. 87

16. Describe the construction of an epi-diascope indicating the function of each of its component parts. Also trace the courses of rays through the instrument.

Art. 88

17. Describe a photographic camera, and explain how you would take a photograph with its help. (C.U. 1934)

Art. 90

18. Describe the optical action of the human eye with the help of a neat sketch. (C. U. 1949 ; P. U. 1923 ; Pat. 1943)

What is meant by "accommodation", and how is it effected ? (C. U. 1949)

19. Compare and contrast the optical arrangements of the human eye and those of a photographic camera. (Pat. 1937)

Art. 91

20. What spectacles are required by a person who cannot see clearly objects at distances greater than 10 cms. ?

[*Ans* : Man short-sighed ; a concave lens of $f=10$ cms. required.]

20(a). Explain "Dioptre" and "Myopic Eye". (Pat. 1946)

21. Why does a short-sighted person use a concave lens ? (Dac. 1930)

The focal length of such a lens is 6 inches and a small object is placed 18 inches from the lens ; draw a figure showing the paths of rays by which the image is formed, and determine its positions. What do you mean by the power of a lens ? (C. U. 1926)

[*Ans* : The position of the image is 4·5 inches in front of the lens.]

22. What are the two principal defects of vision ? Explain how they are rectified with the help of spectacles. (C. U. 1932, '44, '48 ; Pat. '36 ; P. U. 1920)

23. A student with defective eye-sight can• see clearly nothing that is farther from his eyes than 50 cms. What is the number, kind and focal length of the correcting lens that will enable him to see easily and clearly distant objects. (Pat. 1927)

[*Ans* : Concave lens, $f=50$ cms., or Power $= -2$D]

23(a). Find the lens needed by an eye whose minimum distance of distinct vision is 8 ft., if a book at a distance of 16 inches is to be seen clearly.

[*Ans* : Convex lens, $f=8/5$ ft.] (C. U. 1949)

24. A long-sighted person can see distinctly only objects which are at a distance of 50 cms. or more ; find the power of the spectacles which will enable him to see distinctly objects at a distance of 25 cms. (Pat. 1914)

[*Ans* : $f= -50$ cms., or Powder $=2$ Dioptres].

25. Give the focal length and type of lens required to enable a person to read a book at a distance of 10″, if he cannot see objects distinctly at a distance less than 30″. (Cal. '47).

[*Ans* : 15 inches ; Convex lens].

Art. 92

26. Explain the advantages of a pair of eyes over a single eye. (Pat. 1943)

Art. 93

27. Explain the principle of a Cinematograph. (Dac. 1942)

CHAPTER VII

Dispersion of Light

94. Dispersion.—When a beam of white light incident at a face is allowed to pass through a prism, the beam emerging at the other face is not only deviated towards the base of the prism but is also broken into different colours. This was first observed by Newton. Sun-light is allowed to enter into a darkened room through a narrow slit P, (Fig. 109). The light is allowed to fall on the face AB of a glass prism ABC placed with its refracting edge parallel to the slit. The emergent light is then allowed to fall on a white screen S where a coloured patch is obtained. The colour of one end of the patch is **red** and the other end **violet**. Besides these, beginning with violet there are other five colours—**indigo, blue, green, yellow** and **orange.**

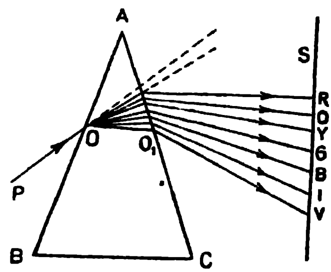


Fig. 109—
Dispersion of Sunlight

Such phenomenon of breaking-up of white light into several component colours is known as **dispersion**, and the coloured band is called a **spectrum.** In the **solar spectrum,** there are, however, a larger number of different tints, each of which shades off gradually into the next, though in general **seven principal colours,** violet, indigo, blue, green, yellow, orange and red (*vibgyor*), spoken of as the colours of the spectrum, are distinguished.

If, now, by making small pin-holes in the screen, each of the constituents be separated and allowed to be deviated by another prism, it will be found that *violet is the most deviated and red the least* ; that different colours occupy unequal spaces in the spectrum, the violet occupying the greatest and orange the least ; and that each of these colours is simple and cannot be broken up again into any other colour. Such light which can be decomposed into several colours is called *compound*, while the light of a single colour is called *simple or* **monochromatic.**

Newton's conclusions.—

(1) **White light** *is not simple,* but it **is a combination of seven colours.**

(2) *The colours can be separated by passing white light through a prism.*

(3) The different colours are deviated to different extents. This is what is expressed by saying that the different colours have different degrees of **refrangibility** ; *violet being the most refrangible and red the least.*

(4) The deviation of the yellow is intermediate between the deviation of the two extreme colours, violet and red. So yellow is called the mean colour.

95 Impure and Pure Spectrum —If a single ray of white light can be isolated and allowed to pass through a prism, it will be split up into its separate coloured constituents, and a pure spectrum may be obtained ; but in practice a single ray cannot be isolated, and even if a small pencil is taken each ray of that pencil being split up will produce a spectrum of its own on the screen with the result that the constituent colours will overlap on each other to some extent. Such a spectrum, in which the constituent colours are partially superposed on each other, is called an **impure spectrum** ; and the spectrum in which the colours do not overlap on each other but are separated distinctly into elementary coloured bands is called a **pure spectrum.**

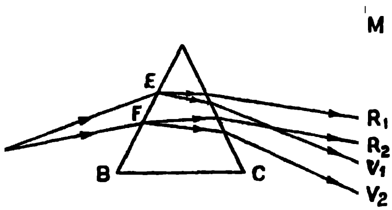


Fig. 110—Impure Spectrum

95(a). Production of Pure Spectrum.—It is obvious from Fig. 110 that the spectrum produced by a broad beam will be impure as there will be much overlapping of colours. The spectra R_1V_1, R_2V_2 are formed by the two extreme rays of the incident pencil. The spectra due to other rays will be formed between them. The widths of R_1R_2, V_1V_2 will depend on the breadth of the incident pencil ; so to obtain a pure spectrum with no overlapping, the incident beam should come through **a narrow slit.**

In a pure spectrum the differently coloured rays should be brought to separate foci on the screen. It may be pssible for a convex lens to bring the differently coloured rays to separate foci if they are in parallel groups after emerging from the prism, *i e.* if all the red rays, all the yellow and so on, come as parallel beams. For this, the incident rays should be parallel Moreover, all the different spactra will not be in focus unless all the beams forming the coloured images have got almost the same deviation. For this reason the best result is obtained

when the prism is set in such a way that the mean colour undergoes minimum deviation or, in other words, the prism is placed in the **position of minimum deviation** for the mean rays, *i.e.* yellow rays. The other rays will then be nearly at minimum deviation. Therefore, to obtain a pure spectrum, the following conditions should be satisfied :—

95(b). Conditions for Pure Spectrum.—

(*i*) *The slit should be narrow.*

(*ii*) *The prism should be placed in the position of the minimum deviation for the mean rays.*

(*iii*) *A convex lens should be placed between the prism and the screen to bring the emergent rays to focus, and another between the slit and the prism to make the incident rays parallel.*

(*iv*) *The refracting edge of the prism should be parallel to the slit.*

95(c). Arrangement for Pure Spectrum.—

A source of white light illuminates the narrow vertical slit S (Fig. 111), which is placed at the principal focus of a convex lens in order to have the rays rendered parallel. The prism is placed with its refracting edge vertical in the position of minimum deviation for the yellow rays, and, hence, approximately for all rays. A second lens brings the differently coloured rays to their different foci RV on the screen.

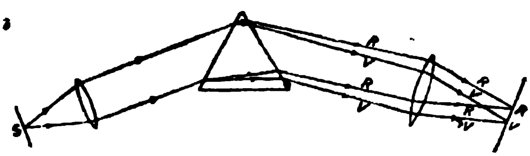


Fig. 111—Pure Spectrum

95(d). Other Methods for the Production of Pure Spectrum.— The method mentioned above is the best method for the production of a pure spectrum, but a pure spectrum can also be obtained by using a single lens instead of two.

(**i**) S is a narrow slit strongly illuminated by white light and the prism ABC (Fig. 112) is placed in the position of minimum deviation for the yellow rays, when rays of any particular colour will appear to diverge from a point from the same side of, and at almost the same distance from, the prism as the slit S. Thus the red rays appear to diverge from the point R', the violet rays from V', and all other rays from points intermediate between R' and

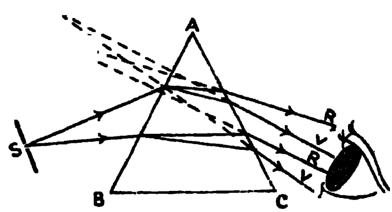


Fig. 112—Virtual Spectrum

V'. Thus, by this arrangement an observer looking in the direction of the emergent rays will see a **pure virtual spectrum** in which the order of the colours is reversed, that is, the red light appears to diverge from R', a virtual image of the slit S, and similarly the violet light from V' (Fig. 112); so the violet is seen towards the edge and the red towards the base of the prism.

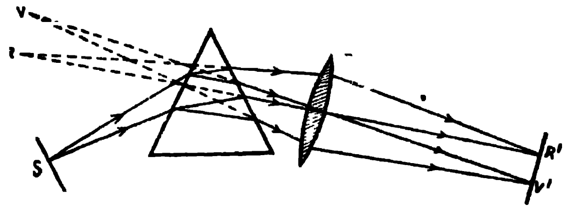


Fig. 113—Production of Pure Spectrum

If now a convex lens L is placed between the prism and the screen at a distance from the slit greater than its focal length, each of the coloured constituents lying between R' and V' (Fig. 112) will form a separate real image on the screen, and so a real image of the virtual spectrum can be projected on the screen (Fig. 113). Thus a real pure spectrum $R'V'$ is obtained in which different colours will occupy separate positions in order of their refrangibilities, red being towards the edge and violet towards the base of the prism (Fig. 113).

(ii) A convex lens L is placed between the slit and the screen and adjusted to form a well defined real image S' of the narrow slit S strongly illuminated by white light (Fig. 114). The prism is then introduced between the lens and the screen with its refracting edge parallel to the slit and set in the position of minimum deviation for the mean rays, say yellow, due to which all the rays of the same colour will be deviated approximately by the same amount and will be brought to one focus. Different colours being differently refrangible separate positions of the different images will be obtained and thus a real pure spectrum RV will be projected on the screen (Fig. 114).

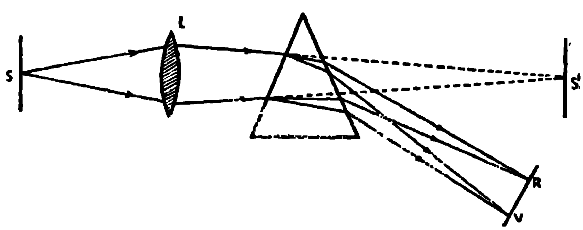


Fig. 114—Production of Pure Spectrum.

96. The Study of Spectrum.—It should be noted that the visible portion of the solar spectrum forms only a small part of the total

spectrum which extends on both ends of the **visible spectrum.** There is an invisible radiation beyond the red end of the spectrum, known as **Infra-red** radiation, which we have already called heat radiation, and also there is another invisible type of radiation beyond the violet, known as **Ultra-violet** radiation. There are also other waves smaller than the ultra-violet waves, such as **X-rays** and **Gamma-rays**. and also waves greater than the infra-red waves, such as **Wireless waves.** The effects of the different parts of the spectrum are also different.

Effects of the Different parts of Spectrum :—

(a) **Luminous effect.**—The different parts of the spectrum are not equally luminious. *The luminous effect is greatest at the yellow part and diminishes as the red or the violet part is approached*, as shown by the intensity curve in Fig. 115. For this reason we can read a book more easily in yellow light than in red or violet.

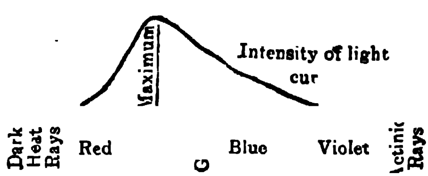


Fig. 115

(b) **Heating effect.**—By holding a linear *thermopile* (Art. 53, Part VII), or a *delicate thermometer* with blackened bulb at different parts of the spectrum it can be proved that the *heating effect diminishes from the red to the violet end of the spectrum.* By using a prism of *rock salt, which is transparent to heat radiation,* the increase of the heating effect can be detected up to some distance in the invisible portion of the spectrum beyond the red end of the visible spectrum where it is maximum, and the heat rays of diminishing intensity may be detected for a distance about seven times the length of the visible spectrum. This portion of the invisible spectrum extending beyond the red of the visible spectrum is known as the **infra-red spectrum** ; and the rays are called **infra-red rays.** As glass absorbs these rays, prisms and lenses of **rock salt** are used for studying the infra-red spectrum.

In the invisible part of the spectrum beyond the red, the heating effect is great, and so these long waves are called radiant heat waves, but it must be *remembered* that all waves carry energy, and, therefore, may produce heat, though in different degrees, when they are absorbed.

(c) **Chemical effect.**—The chemical effects can be shown by the decomposition of certain salts by the action of the rays of different colours of the spectrum. *This action increases from the red rays to the violet rays and extends to a considerable distance beyond the visible spectrum. This portion of the invisible spectrum beyond the violet is*

called the **ultra-violet spectrum**, and the rays forming it are called **ultra-violet** rays. As glass absorbs ultra-violet rays, prisms of **quartz** (also called rock-crystal), or of fluorspar, are used for studying this portion of the spectrum. The rays of this part on account of their chemical action in decomposing salts of silver are also known as **actinic rays**.

The ordinary photographic plates which contain silver salts are greatly affected by ultra-violet rays. Red rays have very little action on the photographic plate.

97. Further study of Infra-red and Ultra-violet Spectra.—It has already been stated that both infra-red and ultra-violet portions of the spectrum are invisible and they are to be studied by their heating or chemical effects. The presence of the infra-red portion was discovered by William Hershell in 1800 and the ultra-violet portion by Ritter in 1801:

The ultra-violet radiation from the sun has a great beneficial effect on our health, though the excess of it is dangerous. This is used for many curative and sterilizing purposes. Many chemical changes brought about by light are due to this ultra-violet rays, the exposure to which increases the vitamin content in some food-stuffs. They cause fluorescence in some substances which may be used to distinguish real diamonds from artificial ones. This radiation is, however, absorbed by the ozone present in the atmosphere and also by clouds and the smoke particles present in the air. It is also absorbed by ordinary glass ; so people working indoor lose much of the beneficial effects of the sunshine passing through the glass window. It has already been said that ultra-violet rays are most effective in photography. So photographs taken on a cloudy day would require longer exposure, as such rays from the sun are greatly absorbed by the clouds, but the infra-red radiation, on the other hand, near about the visible spectrum penetrates through the cloud and fog readily, so photographs taken on a cloudy day by means of *specially prepared photographic plates*, which are made sensitive to infra-red rays, would give very clear pictures. It should be noted, however, that the infra-red radiation, which is far from the visible part of the spectrum like that chiefly emitted by cold bodies such as the earth, is absorbed by clouds and fogs as ordinary light is.

98. Different kinds of Spectra.—Spectra may be divided into two classes,—(1) *Emission spectra* ; (2) *Absorption spectra.*

(1) Emission Spectra may be divided into two subdivisions ; (*a*) *continuous spectra* ; (*b*) *line spectra.*

(i) Continuous Spectra.—The spectrum given by an incandescent *solid* gives the colours continuously from red to violet without any break or gap, depending on the temperature of the solid, and so the spectrum is called *continuous.*

Liquids and gases under great pressure also give continuous spectra.

Examples.—The spectra of lime-light, a luminous coal-gas flame, incandescent electric lamp, electric arc, etc., are continuous.

(ii) Line Spectra.—The spectrum obtained from an incandescent gas or a vaporised substance in flames is not continuous, but consists of a number of bright lines separated from one another by dark spaces, each elementary substance giving its characteristic line or lines whether in combination or not. Thus the line spectrum is a property of the atom.

Example.—The spectrum of incandescent **sodium vapour** produced by adding a little common salt to a (non-luminous) Bunsen flame, when examied by a spectroscope; gives two deep bright lines in the position occupied by the yellow part of the white light spectrum, called *the* **D lines** *of the spectrum,* which are characteristic of the metal, sodium. But the wave lengths of these two lines differ only very slightly, so ordinarily yellow light given by sodium vapour is called monochromatic. Similarly, the spectrum of **hydrogen gas** produced by passing electrical discharges through it consists of several lines of which three lines, one in the red, one in the green, and a third in the violet portion of the spectrum, are prominent. The spectrum of **lithium salts** gives a bright *red* line. The salts of **potassium** have got *two* prominent lines in the *red* and one in the extreme violet and those of **iron** have got *a large number of bright lines* in different parts of the spectrum. Each elementary substance has its own characteristic lines.

(iii) Fluted (or Band) Spectrum.—The line spectrum is characteristic of atoms. Under certain circumstances, a molecule can also be made to emit light characteristic of the molecule, depending on the method of excitation. Such a spectrum is characterised by a number of broad luminous bands, each being sharply defined at one edge and gradually *shading off* at the other. On careful examination it has been found that a large number of bright lines are closely packed at the bright end while the spacing of lines is more and more wide at the faint end. Band spectra of a gas may be obtained by enclosing the gas in a Geissler tube at a low pressure and then passing an electric discharge at a comparatively low voltage. When in the solid state, the band sectra of a substance is generally obtained by filling the hole drilled in a pure carbon rod with its powder and then using this rod as the positive electrode of a carbon-arc.

(2) **Absorption Spectra.**—If in the path of white light some transparent substance be interposed which absorbs some constituent rays, then the spectrum of the transmitted light will be found wanting in the same colours. Such a spectrum is known as *absorption spectrum.* Such spectra may be divided into two subdivisions ; (*i*) *Dark line spectra*, (*ii*) *Dark-band spectra.*

(i) **Dark-line Spectra.**—*If white light from a hot source be passed through a space filled with a cooler vapour, the vapour will absorb from the white light just those constituents which the vapour itself emits when heated to incandescence ;* so the resultant spectrum is like a continuous spectrum crossed by a number of dark lines due to the absorption of some of the rays during their passage through the vapour.

Examples.—(*a*) Solar spectrum, *i.e.* the spectrum obtained by *sun light*, in an example of this class. Here a continuous spectrum is crossed by a large number of dark lines.

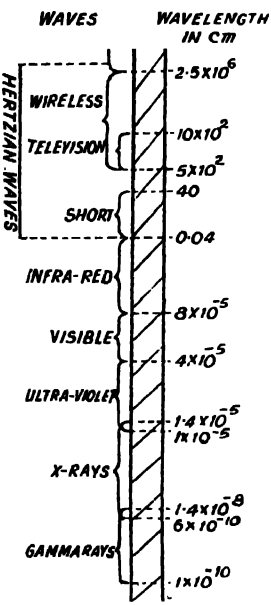


Fig. 116

(*b*) View the spectrum of an *electric arc.* It is a continuous spectrum. Now interpose between the arc and the slit sodium vapour by heating sodium (common salt) in a non-luminous Bunsen flame. *A dark line appears in the yellow part of the white light spectrum.*

This shows that vapours and gases, which produce bright-line spectra when emitting light, must, when absorbing, produce dark-line spectra.

(ii) **Dark band Spectra.**—Instead of interposing sodium vapour, as in example (*b*) given above, if the spectrum be viewed through a piece of red glass, then only red colour of the spectrum will be visible, because red absorbs all the colours except the red. Similar effects will be produced by using other coloured glasses. A dilute solution of potassium permanganate absorbs the middle region of the spectrum. Hence in the spectrum of white light passing through such absorbing media, *dark bands*, or absorption bands, as they are termed, are present due to some portion being absorbed.

99. Some Wave-lengths of Ether Waves.—Of all known ether waves only a small portion can produce the sensation of light. The wave-length of the longest of these waves is 80×10^{-6} cm. These waves produce the sensation of **red light.** The wave-length of the shortest

waves 40×10^{-6} cm. [see Art. 102(b), Part II]. These produce the sensation of **violet** light. Wave lengths are measured in units, called **Angstrom units** (A. U.), one Angstrom unit being equal to 10^{-8} cm. Therefore the wave length of red waves is 8000 A. U, and that of the violet waves 4000 A.U. So the wave-length of the **visible portion** of the spectrum ranges from 8000 to 4000 A. U. The wave-length of **ultra-violet** rays ranges from 4000 to 1000, of **X-rays** from 1400 to 0·06; **infra-red** rays from 8,000 to 4,000,000 A.U.; longer waves are generally called the Hertzian waves. Hence the following types of radiation are all similar and are arranged in order :—Hertzian waves, infra-red, visible light, ultra-violet, X-rays, gamma rays (Fig. 116).

99(a). Spectra of the Sun and Stars : Fraunhofer Lines—If a solar spectrum be carefully examined, it will be observed that the whole length of the spectrum is crossed by a large number of dark lines. Fraunhofer was the first to notice this, and he made a systematic study of these lines. He named these lines by the letters of the alphabet *ABCDEFGH* : of which *A*, *B* and *C* are in the red, *D* in the yellow, and so on. These lines are known as **Fraunhofer lines.**

Kirchhoff's law.—It was not until 1861 that Bunsen and Kirchhoff first gave an explanation of the Fraunhofer lines. The sun is assumed to consist of a white-hot solid (or liquid) at the centre, known as the **photosphere**, surrounded by a comparatively cooler atmosphere, called the **chromosphere**, in which practically vapours of all the terrestrial elements like oxygen, hydrogen, calcium, sodium, etc., are present. It has already been stated that the vapour of an element absorbs those light waves which it would itself emit if it were incandescent. So, according to Bunsen and Kirchhoff, the white light emitted by the sun, in its passage through the cooler envelope, containing vapours of different elements, is robbed of those rays that can be produced by the elements when incandescent. Hence **the presence of the dark lines in the solar spectrum indicates the presence of some elements in the atmosphere of the sun.** The lines appear dark by contrast with other portions of the spectrum and are not really due to absence of light As evidence of the correctness of this, it may be cited that during a solar eclipse, when the sunlight is cut off by the moon's disc, the solar spectrum becomes *reversed*, the dark lines appearing bright in the absence of the more luminous spectrum.

Hence the law is, "*The vapour of an element at a lower temperature selectively absorbs the light which it will itself emit when at a higher temperature.*" This may be verified as in example (*b*) under **dark line** spectra in **Art. 98, 2**(i).

The spectra of most of the **fixed stars** are like the **solar spectrum**, *i.e.* a spectrum of dark lines on a bright background. There are certain heavenly bodies, known as the **nebulae**, which give an **emission spectrum** of a small number of bright lines. This shows that such bodies must be wholly gaseous, and those gases are probably at a very low pressure.

100. Spectrometer.—A spectroscope is a compact apparatus for producing a pure spectrum and also for observing spectra of various kinds. A spectroscope when provided with a suitable scale for measurement is called a **spectrometer** (Fig 117).

Parts—This instrument consists of a *collimator*, a *telescope*, and a *prism-table*. The collimator C is a telescopic metal tube having an adjustable slit S fitted at the outer end of a tube which can be moved in and out of the collimator tube. This has a convergent lens-system at the other end, which is turned towards the prism-table. The telescope T is also a metal tube having a convergent lens-system at each end, the object-glass being one which is turned towards the prism and the other is the eye-piece beyond which the eye is placed.

The telescope can be moved round the prism-table which, as also the telescope, can be clamped in any position. The telescope and the collimator should be such that their axes can be arranged to be in the same straight line passing through the centre of the prism-table. *The slit of the collimator should be at the principal focus of the lens,* so that the rays passing through the lens are rendered parallel before falling on the face of the prism placed on the table *in the position of minimum deviation in order that the rays of different colours will appear to be distinct and separate.* The rays refracted through the prism are received by the telescope which is already focussed for parallel rays and a **pure spectrum** will be seen through it. The axis of the prism-table and the axes of rotation of the telescope and collimator should be the same.

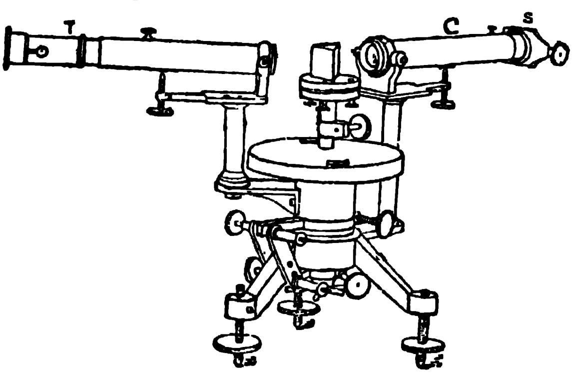


Fig. 117—Spectrometer

Adjustments.—

(1) *Telescope and Eye-piece.*—Turn the telescope towards a white surface, say a white wall, and adjust the eye-piece until the cross-

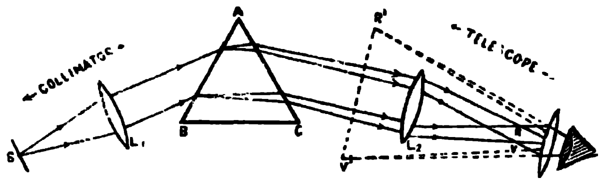


Fig. 118

wires are clearly visible. Focus the telescope on a distant object and it is then said to be focussed for parallel rays (Fig. 118).

(2) *Collimator* and *Telescope.*—Arrange a sodium flame in front of the slit *S* by heating in a Bunsen burner a strip of asbestos soaked in a solution of common salt. Turn the telescope to view the slit through the collimator after making the telescope co-axial with the collimator. Now the slit is drawn in and out till a sharp image of it is seen through the telescope, which was already focussed for parallel rays. So the rays emerging out of the collimator are parallel, that is, both the telescope and the collimator are now focussed for parallel rays.

(3) *Prism-table.*—Now place the prism on the prism-table and adjust its height properly.

101. Use of Spectrometer.—The spectrometer is used for (*a*) determining the refractive index of the material of a prism by the method of minimum deviation, and for (*b*) studying the different kinds of spectra of different sources (See Art. 103).

102. Experiments with the Spectrometer.—(1) **Determination of the Angle of a Prism.**—After the above adjustments of the spectrometer place the prism in such a way that the parallel beam from the collimator falls on the angle to be measured. Now keeping the prism fixed, the telescope is turned on either side of the two faces of the prism to receive the reflected image of the slit on the cross-wires. Accurate readings from the scale and vernier for the two positions of the telescope are taken, the difference of which will give the angle between the two reflected beams, which is twice the angle of the prism.

The principle of this method is the same as that explained in Art. 59 (see Fig. 70).

(2) **Determination of the Angle of Minimium Deviation** :—With the spectrometer set up as in the last experiment obtain through the telescope a refracted image of the slit. Now rotate the prism-table when the image will be found to move in a particular direction and then for a certain angle of incidence it will stop and then turn back in the opposite direction. This is the position of minimum deviation. Bring the telescope to receive the image on the cross wires, and read the scale and the vernier for this position. Then remove the prism and take the direct reading of the slit with the telescope facing the collimator. The difference of these two readings is the angle of minimum deviation [vide also Art. 58(b)].

(3) **Determination of μ of the Material of a Prism** :—Knowing the value of the angle of the prism and the angle of minimum deviation for sodium light, the refractive index of the material of the prism for sodium light can be calculated from the formula,

$$\mu = \operatorname{Sin}\frac{A+D_m}{2} \Big/ \operatorname{Sin}\frac{A}{2} \quad \text{(see Art. 57).}$$

103. Spectrum Analysis.—Each element gives its own peculiar spectrum ; for example, sodium gives two yellow lines ; lithium a red line ; hydrogen three red lines, one green line, and one violet line, etc. By the characteristic spectrum of each element, it may be detected even in minute quantities. A mixture gives a spectra of its components. The indentification of substances by observation of their spectra is known as **spectrum analysis**. This method has given a great deal of information about the nature of the stars and nebulæ. The study of spectra has also been of great service in many chemical investigations.

104. Dispersive Power.—When white light is passed through a prism, the composite colours are deviated to different extents and so an angular separation, which is called *dispersion*, takes place between the colours, which will increase with the refractive index of the prism. The power of spreading up of the differently coloured rays by different transparent materials—*i.e.* the **dispersive powers** of different substances—are different, and *the dispersive power of the material of any prism with respect to any two colours is measured by the ratio of the difference between the deviations of those two colours to the deviation of the mean ray between them.* Thus, if A is the angle of a given *thin prism* for which the deviations suffered by the red, violet and mean ray are respectively given by δ_r, δ_v, δ, and if μ_r, μ_v and μ be the respective refractive indices corresponding to those colours, we have,

$$\delta_r = (\mu_r - 1)\,A \;;\; \delta_v = (\mu_v - 1)\,A \;;\; \delta = (\mu - 1)\,A.$$

$$\therefore \quad \text{Dispersive power} = \frac{\delta_v - \delta_r}{\delta} = \frac{\mu_v - \mu_r}{\mu - 1}.$$

Prisms made of different kinds of glass have got different dispersive powers, *i.e.* they can separate the colours to different extents The dispersive power of *flint* glass, which is a silicate of lead and potassium, is much greater than that of *crown* glass, which is a silicate of sodium and calcium. For a given angle of deviation the dispersion produced by a carbon disulphide prism is even greater. So for projecting a long spectrum a *carbon disulphide prism* is often used.

From the above it is clear that it is possible to combine a crown glass prism and a flint glass prism of different angles (*i.e.* whose refractive indices are different), placed with their refracting angles turned in opposite directions, in such a way that rays of light passing through them will be (*a*) *dispersed without deviation*, or (*b*) *deviated without dispersion*.

(a) Direct-vision Spectroscope : Dispersion without Deviation.—The principle of dispersion without deviation is employed to construct the *Direct-vision* (or *Pocket*) *Spectroscope* where usually three crown glass and two flint glass prisms are combined to give dispersion, but they are so placed and the refracting angles of the prisms are so chosen that the combination allows light to pass through it without undergoing any deviation.

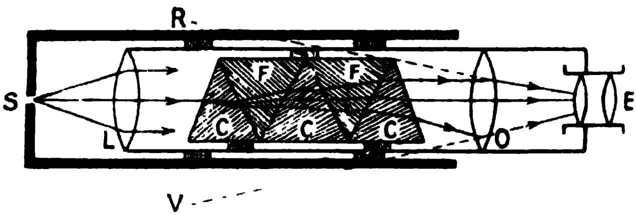


Fig. 119—Direct-vision Spectroscope.

In Fig. 119, three crown and two flint glass prisms are cemented together as shown in the figure, the crown and flint glass prisms being placed alternately with their refracting edges turned in opposite directions and mounted inside a metal tube having a lens L at one end and a lens O at the other end. This tube slides in another tube having an adjustable slit S through which light is admitted. The width of the slit S is regulated and the inner tube adjusted so that the slit S is placed at the principal focus of the lens L and thus a beam of parallel rays which is made to pass through the combined prism suffers dispersion without any deviation. The spectrum RV thus produced

is viewed by the eye-piece E. In Fig. 119 the paths of the different colours arising only from the central ray coming from S are shown.

(b) Deviation without Dispersion.—Similarly as above, the refracting angles of a crown glass prism and a flint glass prism can be so chosen that when two such prisms are combined together with their refracting edges turned in opposite directions, there will be no angular separation, *i.e.* *no dispersion*, though the combination will deviate the beam of white light as a whole. This is called an **achromatic** combination of prisms.

105. Chromatic Aberration.—A convex lens may be supposed to be built up of several prisms (see Art. 65), and it produces dispersion like a prism due to which violet rays being most refrangible are brought to a focus at F_V (Fig. 120) nearer to the lens, and the red rays being least refrangible are focussed at F_R at a greater distance. The rays of the intermediate colours are focussed between F_R and F_V. Due to this the real image on a screen is found to be fringed with colours of the spectrum. If the screen be placed at F_V, the outer edge of the image will be coloured red, and if placed at F_R, the outer edge will be coloured violet. This effect of dispersion of light by a single lens is called **chromatic aberration** of the lens.

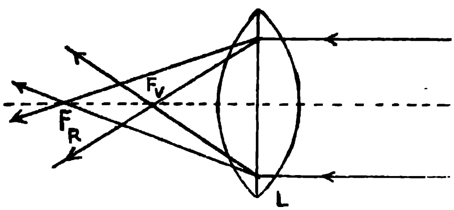


Fig. 120—Chromatic Aberration.

Achromatic Lens.—As two prisms, one of *crown glass* and another of *flint glass*, can be combined to obtain deviation without dispersion [Art. 104(b)], so, by combining a convex or converging lens of crown glass with a concave or diverging lens of less power (*i.e.* longer focal length) made of flint glass (which has a higher dispersive power), the dispersion due to the crown glass lens may be neutralised by that due to the flint glass lens ; but the deviation produced by the convex lens is only partially neutralised by the deviation in the opposite direction produced by the concave lens so that the combination still acts as a converging lens. Such a combination of two lenses in which chromatic aberration is reduced to a minimum is called an **achromatic lens.** It should be remembered, however, that such a combination of two lenses will not be achromatic for all colours of the spectrum but will be only achromatic for two colours.

106. Recombination of Colours.—**(a)** A pencil of white light is admitted through a vertical nrrrow slit S which is placed at the

principal focus of an achromatic lens L (Fig. 121). The emergent parallel beam falls on the prism P_1 and is dispersed. P_2 is an exactly similar prism placed with its refracting edge opposite to that of the first so that the two prisms constitute a parallel-faced slab of glass

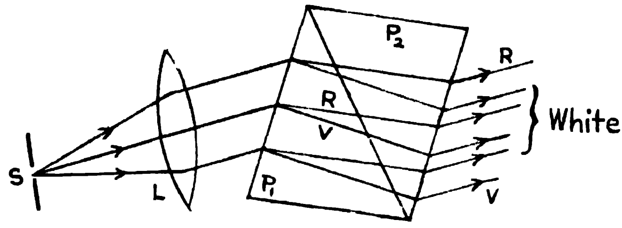


Fig. 121

only. The beam dispersed by the prism P_1 enters into P_2 and comes out as a parallel beam This light received on a screen appears as white, the reason being the neutralisation of the dispersion, or the angular separation, between the rays from one prism by the other. The incident pencil is only a little displaced laterally. The borders of the white patch will be tinged with red and violet colours respectively owing to the dispersion of the extreme rays.

(b) **Newton's Disc.**—The same effect can be produced also by the *Newton's Disc* (Fig. 122), which is a circular card-board disc divided usually into four quadrants each of which is painted with the different colours of the spectrum in the proportion in which they are present in white light. When the disc is rapidly rotated by means of a whirling table, the impressions produced by different colours overlap and the disc appears greyish white. The recomposition is due to what is called the "persistence of vision" (see Art. 93).

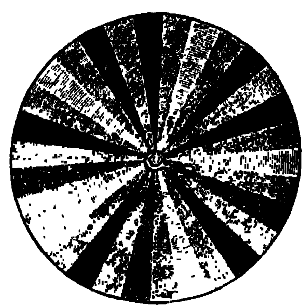


Fig. 122—Newton's disc.

(c) The different colours produced by dispersion of a composite light through a prism may also be recombined by the use of tiny plane mirrors placed suitably so as to reflect each of the dispersed colours to the same spot on a screen. The resultant effect will resemble the original colour.

107. Colours of Bodies.—(a) It has been verified by experiments that coloured bodies, whether opaque or transparent, have got no colour of their own. Their colours are determined by (*i*) the nature of the

incident light; (*ii*) the proportion of it *absorbed* by them, and (*iii*) the sensation of the colours produced in the eye by the colours not absorbed.

Incident Light.—Sunlight is white because all the different constituents of white light are present in it in necessary proportion, but all the so called artificial white lights are not really white. They more or less lack in some constituents of white light. For example, the light from an electric lamp contains much more red-orange and less blue-violet; that from a gas lamp is reddish yellow and deficient in blue constituent. So a blue suit looks darker in artificial light. Light from an electric arc is almost white as daylight. So the colours of bodies may be much changed if the incident light is itself coloured.

(1) **Opaque Bodies.**—Of the light incident on an opaque body a portion is reflected at the surface, some may penetrate a little distance within it and then return in part, and some may be completely absorbed. The colour of an opaque body then depends upon the *nature of the incident light* and also the light *absorbed* by it. So the body appears coloured with the constituents reflected by it. Thus in white light a red flower appears *red* because it absorbs all the constituents except the red which is reflected by it. A body appears *white* as it reflects all the constituents of white light absorbing nothing, and a body is *black* when it absorbs all the constituents reflecting nothing. Thus the colour of the reflected light is not due to something *added* to the incident light but something usually *subtracted* from it.

By passing the object along *different parts of a spectrum*, the theory of colour may be verified. A *white flower* appears white in white light but it will appear red in red light, green in green light, and so on. A *red flower* will appear bright red in the red part of the spectrum but black in other parts as it absorbs all the colours except the red which it reflects. Seldom we get a body having a pure colour, so the reflected colour is not always a pure colour, but may be a mixture of their adjacent colours; hence a body when held in the spectrum may appear bright in one portion, but not totally black in the adjacent portions, as it may reflect these colours also to some extent.

(2) **Transparent Bodies.**—When white light is incident on a transparent body, it absorbs some constituents and transmits the rest to which its colour is due. A piece of red glass appears red because it absorbs practically all the colours except the red which it transmits. Again, if the object be held in the light which is itself coloured other than that of the body, it will appear black. So a piece of red sealing-wax will appear red through red glass, but a blue or green object will appear black because the red light coming through the red glass is ab-

sorbed by the green or the blue object, and the observer receives no light.

The colours of many coloured glasses are not, however, *pure* ; yellow glass transmits yellow but green and orange as well, and blue glass allows indigo and green besides blue. So the combination of these two glasses will allow the light common to both of these, *i.e.* green.

Even a good transparent body like water, glass, etc., absorbs some light, which may not be noticeable in thin layers, but the effect is marked in thicker layers. Ordinary deep water looks greenish, but when the depth is very great it may appear black.

(b) **Colours of Powders.**—The colours of many substances in the powdered state look lighter because the incident light is repeatedly reflected from many particles in different layers and so is unable to penetrate far below to be absorbed. If, however, the powder is very fine, practically no absorption will take place and the powdered mass will appear white due to the diffused light reflected in all directions.

108. (a) **Primary and Complementary Colours : Mixture of Spectral Colours.**—It was shown by Newton that all the seven spectral colours into which white light was spilt up could not be further analysed, *i.e.* those colours were **pure** But there are three spectral colours, **red, green**, and **blue**, by mixing which in right proportions all other colours of whatever shade may be obtained. They are, therefore, called the **primary colours**. Any two spectrum colours, which together give white light, are called **complementary colours.** Thus bluish green and red, yellow and blue, greenish yellow and violet are complementary.

If *red* and *green* light are mixed, the resulting colour is *yellow*, which the eye cannot distinguish from a certain spectrum yellow ; but a spectroscope will at once show the difference ; for the spectrum yellow seen through a spectroscope will remain yellow, whilst the yellow formed by the mixture, when examined by a spectroscope, will be separated into its red and green components.

It is clear from this that **wave lengths determine colour but colour does not necessarily determine wave length**.

(b) **Colours of Paints or Pigments.**—The mixing of coloured paints or **pigments** is not the same as that of mixing of two coloured lights or spectrum colours. For example, yellow rays of the spectrum mixed with blue rays produce white light, but the mixture of yellow and blue *pigments* appears green. The colours of pigments depend upon the particular colour or colours each absorbs. This is because the yellow particles of the pigment absorb all except yellow, and blue

and green particles of the pigment absorb all except green and blue. Hence the mixture reflects the green rays alone which are not absorbed.

To sum up we may say that in the case of mixing spectral colours we have the effects of superposition or *addition*, while in the case of mixing pigments the effect is of absorption or *subtraction* of colours.

109. Retinal Fatigue.—When we look *intently* on a bright object for some time and then suddenly turn our eyes on to a white surface we do not see the object in its actual colours, but the shape of the object is seen in dark outline. This phenomenon is called **Retinal fatigue.** This is due to the fact that the nerves of the retina being excited by strong light become insensitive for some time to less powerful source. The colours of the temporary illusive image—called the *after image*—which is seen due to the retinal fatigue are complementary to the actual colours of the object Thus when we gaze intently on a red patch printed on a white screen for some time and then suddenly look at a white surface we appear to see a greenish blue patch as greenish blue is complementary to red. Similarly when we gaze at a blue patch painted on a red background. it will appear temporarily to be red on a blue background after suddenly turning our eyes on to a white screen.

110 (a) The Colour of the Sky—It may be noticed that of the sea-waves of various sizes striking at the sides of a ship lying at anchor, only the smaller waves are thrown back (*i.e.* reflected) from the sides of the vessel, while the larger waves pass right on without being hampered in their progress.

Somewhat similarly when light waves of different sizes (*i e.* wave lengths) start from the sun and pass on through the atmosphere they encounter innumerable dust and air particles which can easily scatter the smaller waves (*i.e.* blue and violet waves), but they are not large enough to stop the larger waves (*i e.* red waves). So the red waves pass right on like the larger sea waves passing the ship, while the blue and the violet waves become scattered. For this reason the sky appears blue.

(b) The Colour of the Sunset.—The sun at midday is nearest to us, but as it sinks lower towards the horizon the rays have got to travel through a greater depth of the atmosphere, and so gradually more and more of the smaller waves (*i.e.* violet, blue, etc.) are scattered or reflected, and finally the larger (*i.e.* red) rays predominate and produce the beautiful colour of the sunset.

111. Rainbow.—Rainbow is a beautiful solar spectrum formed when the sun shines on rain-drops or fountain spray. The result is due

to reflection, refraction, and dispersion of the rays by raindrops and an observer standing with his back to the sun sees a circular arc of spectrum colours, the red being at the top or the outer edge, and violet at the bottom or the inner edge. This is called a **Primary Rainbow.** Sometimes another arc is seen outside the first with the order of the colours reversed, *i.e.* violet on the top and red at the bottom. This is known as a **Secondary Rainbow**.

Primary Rainbow.—The primary bow is produced by rays of light which have undergone two refractions (not total reflection) in the raindrops. In order to understand the formation of the primary bow, consider light from the sun falling in parallel rays upon a spherical raindrop. One of the rays is shown in Fig. 123 incident at A. It will be refracted at A and, on reaching the surface of the drop at B, some of the light will pass out, but the rest will be reflected and will reach C, where some will emerge along CE, a fraction being reflected at C inside the drop. Here the angle of deviation of the ray incident at A,

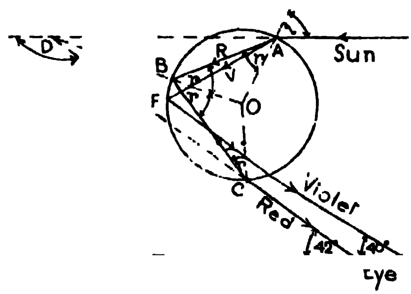


Fig. 123—A Ray of Light from the Sun suffering Reflection in a Raindrop

$$D = \pi - \angle ADC = \pi - 2\angle ADO$$
$$= \pi - 2[\pi - (\angle ABD + \angle DAB)]$$
$$= \pi - 2[\pi - (\pi - r) - (i - r)] = \pi + 2i - 4r.$$

Rays will be incident on the drop at all angles between 0° and 90° : those falling normally ($i = 0°$) will pass through the centre of the drop and will be reflected back along the same part for which D will be 180°. If a graph be plotted between D and i of any ray, the curve will be like that shown in Fig. 68, and the minimum value of D for any ray can be obtained from the graph. The minimum value of D, however, will be different for different colours. For red this value is 138°, the corresponding angle of incidence being 61°, and violet being more refrangible than red the minimum value of D for violet is 140°, the corresponding angle of incidence being less than 61°.

It will be noticed from Fig. 68 that for rapid changes of i in the neighbourhood of the lowest point on the curve, D changes slowly, and consequently the emergent parallel rays will be more closely packed ; so when the rays traverse the drop in such a way that the deviation is a minimum, then they become sufficiently concentrated

in one direction to be seen by the eye. Each ray of the beam from the sun suffers a deviation which is different for different colours and so dispersion would take place resulting in the formation of a spectrum. It has been seen that in a bow the emergent red rays make an angle of 42° (*i.e.* 180° − 138°), and the violet rays an angle of 40° (*i.e.* 180° − 140°), with the line parallel to the original direction, *i.e.* the line joining the sun and the eye of the observer.

Fig. 124 shows an observer at *E* with his back to the sun facing the raindrops A_1, A_2, etc. Hence if with the eye *E* as the apex a cone having a **semi-vertical angle** of 42° (*i.e.* 180° − 138°) with its axis *EK* parallel to the sun's ray is drawn, all the red rays emerging from drops lying on the bounding surface of this cone will travel towards the eye *E* of the observer; and since these rays will have suffered minimum deviation they will be easily seen. Now the violet being more refrangible, that is, the refractive index of water for violet light being greater than that for red, the angle of minimum deviation (140°) is greater for violet rays than for red, and so the semi-vertical angle of the corresponding cone will be less for the violet than for the red rays. It has been found that the emergent violet rays are inclined to the same line *EK* at an angle of 40°, and all the drops lying on the bounding surface of this cone will send violet rays to the eye. The other colours occupy intermediate positions between these. Thus the primary bow consists of a coloured arc, red on the outside and violet on the inside.

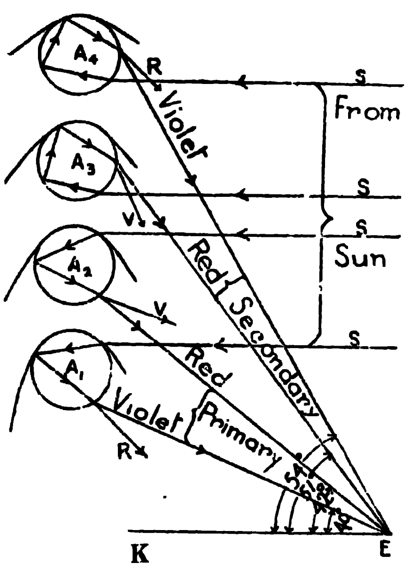


Fig. 124—The Rainbow

Secondary Bow—Here the sun's rays suffer two reflections and two refractions, as shown at A_3, and A_4 in Fig. 124, and the violet light emerges making an angle of 54° and the red an angle of 51° with the line *EK* from the sun to the observer. Thus the colours in the secondary bow are reversed, violet being on the outside and red inside. The secondary bow is much fainter than the primary bow.

112. Transformation of Absorbed Radiation.—Bodies are ge-

nerally rendered hot when light waves fall on them, which means that smaller light waves are absorbed by the bodies and are transformed into longer heat waves. There are also many substances which absorb light rays of one wave-length and emit light of another wave-length. This is shown by the following phenomena.—

(a) **Fluorescence**—Some substances when exposed to light of one kind become luminous and continue to emit light of another kind. These substances absorb waves of a particular colour, say violet, and emit waves of another colour, say blue. This phenomenon is called **Fluorescence** and the substances are called **fluorescent**. This effect is only confined to surface layers and lasts as long as the radiations fall on the fluorescent substance. This effect was first noticed in Fluorspar (calcium fluoride) and hence it is termed *fluorescence*. It should be noted that it is not a case of reflection (though the effect is confined to the surface only) as the colour of the emitted light is changed. If a solution of chlorophyll (the *green* colouring matter of plants) in alcohol is exposed to white light in a dark room, the emitted light is brilliant *red*. Ordinary paraffin oil, or a solution of quinine sulphate, exposed to sunlight, presents a *bluish* appearance.

Again certain substances exposed to invisible ultra-violet radiations convert them into radiations of longer waves and become visible.

Luminous dials of watches and clocks are coated with a compound of radium mixed with crystalline zinc sulphide. The greenish-yellow light which comes out is the result of the impact of the α-rays (Part. VII) given out from the radium on the zinc sulphide.

The fluorescent screens used in X-ray work are coated with barium platinocyanide. These screens will fluoresce *yellow* under the action of X-rays and give out a *greenish* glow in the violet and ultra-violet parts of the solar spectrum. In studying the phenomenon of fluorescence quartz lenses and prisms are to be used instead of those of glass.

(b) **Phosphorescence**.—Certain substances, such as diamond, calcium sulphide, etc., when exposed to sunlight for sometime, retain their fluorescent state *even after the light is completely cut off*. This phenomenon is called **phosphorescence**. This term is applied to the glowing of phosphorus seen in the dark which is really due to slow oxidation.

The glow-worm and many marine forms of life are self-luminious and this is different from phosphorescence. The phosphorescence of decaying substances, such as rotten fish, etc., seen at night, is due to some bacteria giving out light.

Phosphorescent paints can be purchased which absorb light during

day-time and give it out at night. These paints contain calcium sulphide and barium sulphide.

The **difference between phosphorescence and fluorescence** lies in the fact that the former effect persists for some time while the latter ceases as soon as the exciting light is cut off.

It has been found that violet and ultra-violet rays are most effective in exciting phosphorescence and also fluorescence.

(c) **Calorescence.**—Some substances which can absorb long heat waves and transform them into smaller light waves are called **calorescent** substances. If light from an arc-lamp be passed through a solution of iodine in carbon bisulphide to cut off the light rays, and then the heat radiations be focussed on a thin sheet of platinum, the platinum sheet becomes highly luminous, *i.e.* gives out visible light radiations of smaller wave lengths (first shown by Tyndall).

Questions

Art. 94.

1 Explain clearly why a prism is chosen for producing spectrum. Draw a diagram of the arrangement for producing a pure spectrum and add notes on the different parts. (*See Art. 95*) (Pat. 1932)

Art. 95

2. Describe an arrangement of apparatus by which a pure spectrum may be produced on a screen. (C. U. 1911, '13, '14, '17, '22, '28, '31, '39, '45, '47, '49 ; Pat. 1920, '26, '28, '30, '31, '36 ; All. 1916, '22, '23, '31 ; Dac. '30, '32, '34).

Explain the function of each part of the apparatus. Draw a careful diagram of your arrangement, showing the order of the coloured rays on the screen. (Cal. '47)

3. What is spectrum ? Distinguish between a real and a virtual spectrum, a pure and an impure spectrum. (Pat. 1941)

4. What is a pure spectrum and how it can be produced ? Describe experiments to show that the radiation from arc lamp extends beyond red at one end, and beyond violet at the other end of the spectrum. (Pat. 1931) (*See also Art. 96*)

[**Hints.**—In the *Infra-red part* of the spectrum (*i.e.* beyond red) the heat radiation can be detected by a *thermopile* or an *ether thermoscope.* The radiation in the *untra-violet portion* (beyond violet) can be observed by the aid of *photography.* A photographic plate exposed to the ultra-violet rays (termed actinic) will be found to be affected on development.]

5. Describe a spectrometer. Explain why it is necessary to place the prism in the minimum deviation position. (*See* also *Art. 100*) (C. U. 1937)

5(a). Explain how a pure spectrum may be formed on a screen by means of a prism, a slit, and two convex lenses. Illustrate your answer with a neat diagram, showing the course of the rays.

What differences are there between the spectra of the light from an ordinary electric lamp, a sodium flame, and the sun? (Pat. 1948)

Art 98.

6. Discuss in general terms the spectrum produced by the following :—

(*a*) When the sun is used as a source of light, (*b*) When the light is produced by an incandescent solid, (*c*) When the flame of a Bunsen burner is coloured by sodium light, (*d*) When light is produced by a luminous gas flame. (C. U. 1923, '25, '28, '32, '45 ; Pat. '28, '36 ; cf. Dac. '29, '35)

7. A clean platinum wire is gradually heated in a non-luminous Bunsen flame and observed through a spectroscope ; state what you observe. (C. U. 1933)

8. Describe the various forms of spectra that may be obtained illustrating each type by an example. (C. U. 1916 : All. '23)

9. Objects which appear variously coloured in white light are illuminated by sodium flame. Describe and explain the effects observed. (*See* Art. 107) (C. U. 1909, '11, '24)

10. A gas absorbs from the incident light just the rays it itself emits. How would you verify this experimentally? State the importance of the principle. (Pat. 1924, '29)

11. Describe briefly the nature of the observed spectrum when the source of light used is : (*a*) an iron arc, (*b*) a white hot carbon rod with a glass cell containing dilute solution of permanganate of potash in front. (Pat. 1946)

In what ways does the solar spectrum differ from that produced by an arc-lamp? How do you account for these differences? (Pat. 1938)

Art 99.

12. What are Fraunhofer lines in a solar spectrum? How has their origin been explained? (Pat. 1946)

13. Describe a solar spectrum, and give a general explanation of the dark lines in the spectrum. (C. U. 1913 : All. '22)

Arts. 100 & 101.

14. Describe and explain the use of a spectroscope. (C. U. 1911, '16, '18 : cf '46)

15. Describe the constituent parts of a spectroscope and their functions. State how would you fit it up and show the path of monochromatic light through it. (C. U. 1935 ; cf Pat. '34, '46)

16. How do you obtain a pure spectrum with the help of a prism spectrometer? Give details of adjustments required. (All. 1945)

17. Describe any compact apparatus that may be in use to obtain a pure spectrum. (Pat. 1930 ; C. U. '44)

Art. 102.

18. Describe the different parts of a spectrometer. How will you proceed to use the instrument to find the refractive index of a prism ?
(See Art. 100) (Del. U. 1938, C. U. 1937, All. 1946)

Art. 103.

19. What is monochromatic light ? How would you verify whether a given light is monochromatic or not ? (All. 1945)

(See Art. 94)

Art. 104

20. Distinguish between dispersion and deviation. Describe a contrivance by which you can get dispersion without deviation.
(C. U. 1922, '25, cf '46 ; All. 1921, '44 ; Pat. 1936)

21. Give a short description, with a neat diagram, of the direct-vision spectroscope. (C. U. 1933 ; All. '44)

Art. 106.

22. Describe any two methods of re-compounding, to form white light, the various kinds of light obtained in a spectrum. (C. U. 1946, '49)

Arts. 107 & 108.

23. Describe and explain the appearance of (*a*) a red flower, (*b*) a green flower, (*c*) a piece of white paper, and (*d*) a black object, when they are moved from one end of the spectrum of white light to the other. (Pat. 1933)

24. Why do white objects appear blue and yellow objects black when seen through a thick blue glass ? Describe some experiments to show that your explanation is correct. (Dac. 1932)

25. What is the colour of an object due to ? Explain why (*a*) a mixture of ordinary blue and yellow pigments appear green, and (*b*) when dark blue crystals are grounded into fine powder, the colour of the latter appears to be light blue. (Pat. 1940 ; C. U. '41)

26. Blue and yellow sectors on a rotating disc give white while blue and yellow glasses combined transmit deep green or none at all. Explain.
(Pat. 1927)

27. Write short notes on the following :—(*a*) complementary colours, (*b*) phosphorescence, (*c*) Fraunhofer lines. (*See Arts. 99, 108 and 112*).
(C. U. 1922)

28. Why do ordinary blue and yellow pigments appear green when mixed ? Objects which appear variously coloured in white light are illuminated by sodium flame. Describe and explain the effects observed. (C. U. 1919 ; '44)

Art. 109.

29. A man gazes intently, for a time, at a red square painted on a piece of white cardboard. He then looks at a white screen and appears to see a square of different colour. What colour does he see ? Explain the phenomenon.

The experiment is repeated with a blue square painted on a red back ground. Describe and explain what the observer notices on looking at a white screen immediately afterwards. (C. U. 1941)

Art. 111.

30. Write short notes on : (a) Rainbow, (b) Fraunhofer lines, (c) Achromatic combination, (d) Line spectra. *(See also Arts. 98, 99, 105).* (All. 1944, '46)

Art. 112

31. What is the difference between phosphorescence and fluorescence ? (All. 1921 ; cf. C. U. '35)

CHAPTER VIII

Velocity of Light : Theories of Light.

113. Velocity of Light (Romer's Method).—The velocity of light was first determined by a Danish astronomer, named Romer, in 1676 by observing the eclipses of one of Jupiter's satellites. The planet Jupiter has got nine satellites revolving round it just as the moon revolves round the earth. The satellites become eclipsed when in each revolution they pass into the shadow of the planet thrown by the sun. The interval between two consecutive eclipses of the innermost satellite of Jupiter is 42 hours 48 seconds, which is the time required for one complete revolution of the satellite round Jupiter.

Romer with his astronomical telescope kept the innermost satellite under observation and found from a stop-clock the time interval between two successive eclipses when the Earth and Jupiter were in **conjunction** (nearest to each other), i.e. they were at E_1 and J_1 (Fig. 125) with the sun S in the same straight line having the earth in the middle. He noted that during succeeding months as the earth moved away from the Jupiter, the time interval between successive eclipses gradually increased until about 6 months later it became maximum when the earth and Jupiter were in

opposition (farthest from each other), *i.e.* they were at E_2 and J_2 with the sun S in between. As time advanced and the earth moved nearer to Jupiter, again the time interval between successive eclipses was noted now to decrease gradually until about 6 months later, when the earth moved to the next position of *conjunction*, as seen at E_3, it again became equal to that at the first position of conjunction. The variation of the interval between successive eclipses was explained by Romer by pointing for the first time to the fact that light takes a definite time to travel from one place to another.

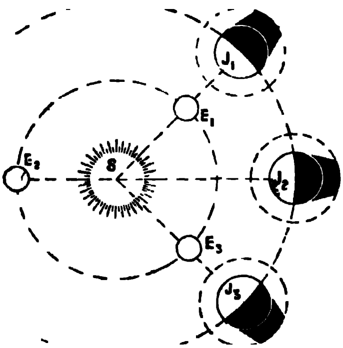


Fig. 125—Romer's method

Let θ be the actual interval between two successive eclipses. If n eclipses take place during the time the earth moved from the first position (E_1) of conjunction to the position of opposition (E_2), the actual interval between the occurrence of the first and the nth eclipse is $(n-1)\theta$. The interval (T_1) which will be observed from the earth's surface will, however, be given by,

$$T_1 = (n-1)\theta + \frac{E_2J_2}{v} - \frac{E_1J_1}{v}, \quad \text{(where } v \text{ is the velocity of light)}$$

because an eclipse will be observed on the earth after an interval equal to $\dfrac{\text{distance between Earth and Jupiter}}{\text{velocity of light}}$, after its actual occurrence. If n eclipses again occur as the earth moved from the position (E_2) of opposition to the second position (E_3) of conjunction, the time interval (T_2) between the first and the nth eclipses as observed from the earth will be,

$$T_2 = (n-1)\theta + \frac{E_3J_3}{v} - \frac{E_2J_2}{v}.$$

But $E_2J_2 - E_1J_1 = D = E_2J_2 - E_3J_3$, where D = diameter of the earth's orbit round the sun.

$$\therefore \quad T_1 = (n-1)\,\theta + \frac{D}{v}, \text{ and } T_2 = (n-1)\,\theta - \frac{D}{v}.$$

$$\therefore \quad T_1 - T_2 = \frac{2D}{v}, \text{ whence } v = \frac{2D}{T_1 - T_2}.$$

Romer found $(T_1 - T_2)$ to be 33 min. 12 secs. ; D was taken by him

to be 191×10^6 miles. His value of velocity of light in air was thus,

$$= \frac{2 \times 191 \times 10^6}{33{\cdot}2 \times 60} = 192{,}000 \text{ miles/sec.}$$

The accuracy of Romer's method was limited by the accuracy with which the diameter of the earth's orbit was known in Romer's time.

Sources of Errors :—

1. The earth's orbit is actually elliptical, with the sun at one focus, while Romer assumed it to be circular.

2. As there was no precision chronometer in Romer's time, it is unlikely that the observed difference of 33·2 min. in a continuous record of time for about a year is free from doubts.

114. Fizeau's Method.—Fizeau was the first to measure the velocity of light in 1849 over terrestrial distances. The principle of his method was to send a light signal to a distant station and back, and calculate the velocity by noting the small interval of time taken. The diagram of his apparatus in a simplified form is given in Fig. 126.

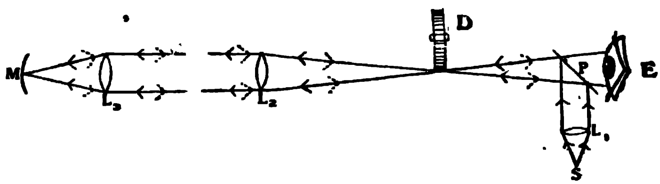


Fig. 126—Fizeau's method

A beam of light from a bright source S passes through a convex lens L_1, and is brought to a focus at the plane of the toothed wheel D, after being reflected by a plate of glass P placed at an angle of 45°. The toothed wheel rotates in a plane at right angles to the beam of light reflected from the glass plate P. The wheel has a large number of teeth equally spaced along its circumference, the opening between any two teeth being equal to the space occupied by a tooth. As the wheel is rotated, light is alternately intercepted by a tooth and allowed to pass through an opening. The plane of the disc is at the principal focus of a convex lens L_2, so that rays emerge from L_2 as a parallel beam, which, after traversing a distance of a few miles, arrives at the lens L_3 and is brought to a focus at the surface of a concave mirror M whose centre of curvature is at the centre of the lens L_3. Then the rays are reflected back and retraverse their path as a parallel beam, and if the path of light is not obstructed, they will be reflected by P to S.

some being transmitted to the eye E through P. The observer at E will then see the image of S.

The wheel D can be rotated with great velocity about its axle. It is possible by rotating the wheel, and so adjusting its speed, that when a beam of light after passing through the opening between two teeth of the wheel comes back to the wheel after reflection from the mirror M, it finds a tooth in place of the opening and thus gets stopped. The result of this will be that no image of the source will be seen. If the speed be doubled, the light will pass through one opening and will fall on the next opening after reflection and the image will be visible. If light travels with a finite velocity, then the speed of rotation of the wheel will affect the image seen, and if light travels instantaneously, however great the speed of rotation may be, the image will always be visible. Fizeau found that the speed of rotation affected the visibility of the image, so light travelled with a finite velocity. If the speed be so increased that more than 10 images are seen in one second, then due to persistence of vision a continuous image will be seen. But Fizeau so adjusted the speed that light travelled from the plane of the wheel to M and back again in the time taken by the next tooth to come into the position of the opening, and thus light was cut off from the observer at E.

Calculations.—If light travels with velocity v and if it takes time T to travel from the plane of the wheel to the mirror M (a distance equal to x, suppose) and back, $v = 2x/T$.

Let n be the revolutions of the wheel made per second. Then the angle turned through by the wheel per second, *i.e.* the angular velocity of the wheel $= 2\pi n$.

The time taken by the wheel to turn through an angle θ which is subtended by an opening at the axle of the wheel $= \theta/2\pi n$.

When the liqht was cut off for the first time for a particular speed of the wheel,

$$T = \theta/2\pi n \; ; \quad \therefore \quad v = \frac{2x}{\theta/2\pi n} = 4\pi n\, x/\theta.$$

If m = number of teeth on the wheel and if an opening is equal to one tooth,

$$2m\theta = 2\pi \; ; \; \theta = \pi/m.$$

$$\therefore \quad v = 4\pi n x/\theta = \frac{4\pi n x}{\pi/m} = 4mnx.$$

In Fizeau's experiment, the wheel had 720 teeth and made 12·6 revolutions per second when the image of the source was eclipsed. So

the time for one revolution of the wheel was 1/12·6. The teeth and openings were of equal length and so they together numbered (2×720). Therefore the time taken by a tooth to occupy the position of the preceding opening

$$= \frac{1}{2 \times 720 \times 12 \cdot 6} \text{ second.}$$

The distance between the plane of the wheel and the concave mirror M was 8633 metres, so light travelling from the plane of the wheel to M and back traversed a distance of (2×8633) metres. Therefore the velocity of light, according to this experiment,

$$= \frac{2 \times 8633}{\frac{1}{2 \times 720 \times 12 \cdot 6}} = 3 \cdot 15 \times 10^{10} \text{ cms. per sec.}$$

Advantages and Disadvantages :—

Advantages.—(1) It is a terrestrial method and the time of observation is small. (2) All the quantities involved in the calculation are measurable and no assumptions are necessary. In these respects this method is superior to Romer's method.

Disadvantages.—(1) Primary difficulty was in regard to the production and measurement of a uniform high speed of rotation of the wheel. Moreover, it was difficult to determine the speed at which the exact extinction of image took place.

(2) The light when not passing through an opening was reflected by the intercepting teeth back into the field of view causing a general illumination of the field of view. This made the image less distinct.

(3) A large open-air space, about 4 miles in Fizeau's experiment, is necessary and so this experiment cannot be done in the laboratory. Moreover, due to absorption of light in the open-air space and loss of light by reflection from the glass-plate P, the image received by the eye (E) is faint.

114(a). Value of Velocity Light.—

The values $v = (3 \cdot 004 \pm 0 \cdot 003) \times 10^{10}$ cms./sec. by M. Cornu, and $v = (2 \cdot 9986 \pm 0 \cdot 0003) \times 10^{10}$ cms./sec. by Michelson and Newcomb make the greatest claims to accuracy. The most generally accepted value, however, is 3×10^{10} cms./sec.

114(b). Light-year.—Distances between stars are enormously large. So in Astronomy such distances are measured in terms of a very large unit, called the "*light-year*". It is equal to the distance over which light travels in one year's time. Therefore one light-year

is equal to $365 \times 24 \times 60 \times 60 \times 3 \times 10^8$ metres $= 94608 \times 10^{11}$ metres $= 94{\cdot}608 \times 10^{11}$ km.

115. Theories of light.—To account for the various facts concerning light, *viz.* it is invisible, it travels in straight lines, it can be reflected and refracted, etc., two theories have been put forward.—(1) the *Corpuscular* or *Emission theory*, (2) the *Undulatory* or *Wave theory*.

(1) Corpuscular Theory.—According to the corpuscular theory a luminous body is supposed to emit tiny particles in all directions, called *Light Corpuscles*, very much like shots fired from a gun. These corpuscles travel in straight lines with an enormous velocity (186,000 miles per sec.) through gases, transparent solids and liquids, and through vacuum ; and the impacts of these corpuscles on the retina of our eyes produce the sensation of light. This theory was advanced by Sir Isaac Newton. *The rectilinear propagation of light* is readily explained by this theory and *the reflection of light* was explained by Newton by supposing that the reflecting surface exerted a repulsive force at right angles. This force gradually neutralized the component of velocity perpendicular to the surface of the approaching particles without altering the component parallel to the surface. In explaining *refraction*, Newton had to suppose that the velocity of light in an optically-denser medium is greater than that in air, but later on this was proved to be false ; but still the corpuscular theory was generally accepted up to about 1800.

(2) Wave Theory.—The wave theory was first formulated by the great Dutch Physicist Huygens in 1678 according to which light, like sound, is a form of wave-motion, but, unlike sound, light travels with perfect readiness through the best vacuum possible and even through the intermolecular spaces of all matter. But as every wave-motion requires a medium for its propagation Huygens assumed such a medium to exist and called it the **ether**, which pervades all space and penetrates matter. According to the wave-theory every point of a luminous body sends out waves in all directions which travel with the same velocity through the ether. Some of these waves falling on the eyes produce the sensation of sight. As when water is disturbed by throwing a stone into it, water waves are produced by the up-and-down movement (*i.e.* transverse movement) of the water particles, so ether waves start by the transverse movement of *ether* particles produced by the vibratory motion of the ultimate minute particles of which a body is composed, *i.e.* ether particles vibrate perpendicularly to the direction of propagation of the waves. To be able to transmit the waves, the ether must be continuous, elastic and possess density. The elasticity must be very high like that of a solid and the density

extremely small as that of an extremely rarefied gas. We have seen that light and heat are similar forms of energy and that a body which emits light also emits heat. Both light and heat energy are carried by *ether waves* generated by the vibrations of the internal structure of an atom, that is, by the vibrations of the **electrons** (Art. 7, Part VI), of which every atom of a body is composed. The *displacement curve* of a light wave is shown in Fig. 127. It should be remembered that water waves, or sound waves which are waves in air, are fairly large, whereas

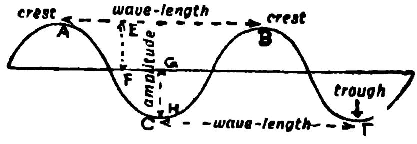


Fig. 127

the wave length—*i e. the distance of the highest point (crest), or the lowest point (trough), of a wave to the highest or lowest point of the next wave* (see Fig. 127)—of light waves is indescribably small. The wave theory can easily explain most of the common optical phenomena and so it has generally been accepted.

Propagation of Waves.—According to the wave theory every point of a luminous body is a centre of disturbance from which the waves spread out in all directions through ether. *If at any instant an imaginary surface is drawn tangentially through all the particles which are in the same state of vibration, i.e. in the same phase, the surface is known as* a **wave-front**. The propagation of light in a medium in a particular direction means the propagation of the wave-front. *The direction in which the wave travels is at right angles to the wave-front and a ray of light represents this direction.* Spherical waves are represented by concentric circles. As the wave travels outwards, its curvature diminishes, and *at a very great distance from the source the surface of the wave becomes a sphere of very large radius, so that a limited portion of the wave-front may be considered to become plane.* This is evidently the case of waves constituting a parallel beam, as in the case of light coming from the sun.

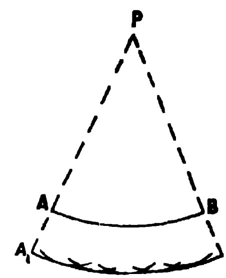


Fig. 127(a)

Huygens' Principle—Suppose *P* [Fig. 127(a)] is a luminous source. The disturbance in ether produced at *P* will spread out uniformly in all directions with the same velocity provided the medium around is isotropic, so that the wave-front at any instant will be on a sphere drawn with *P* as centre and radius equal to the product of time and velocity of propagation. After an interval, let the disturbance lie on the surface of a sphere whose trace is *AB*, on which every point is

in the same state of disturbance, *i.e.* AB is the wave-front. At any later instant of time similarly let the disturbance lie on the trace A_1B_1. Evidently light has travelled from AB to A_1B_1 in the interim period. Huygens considers the mechanism of such propagation as follows. Each ether particle in the wave-front AB becomes a new centre of disturbance from which spherical waves, called **secondary wavelets**, are spread out from each particle as centre with the same velocity as in the case of the wave travelling from P to AB, provided, of course, the medium remains the same. The envelope of these secondary wavelets, *i.e.* the spherical surface, whose trace is A_1B_1, which touches all these spheres, is the new wave-front at the end of the period envisaged. The wave travels outwards from A_1B_1 to succeeding fronts as it did from AB to A_1B_1. Evidently the wave-front at any point moves in the direction of the normal to the wave-front at that point.

116. Reflection and Wave Theory.—According to Huygens' method of construction the laws of reflection are deduced below by taking the case of reflection of a plane wave-front at a plane surface.

Let XY be the trace of a reflecting surface and AF be a plane wave travelling in the direction CA and incident obliquely on XY (Fig. 128). As each point of AF reaches XY in succession, it becomes a centre of disturbance from which spherical waves start. In the time t taken by F to reach E the wave from A will travel through the distance AB in the same medium ($= Vt$, where V is the velocity of propagation of the original wave-front). Now with centre A and radius AB ($= Vt$) describe a sphere. From E draw a plane EB tangential to the sphere and at right angles to the plane of the paper. Then EB is the reflected wave-front travelling in the direction AB.

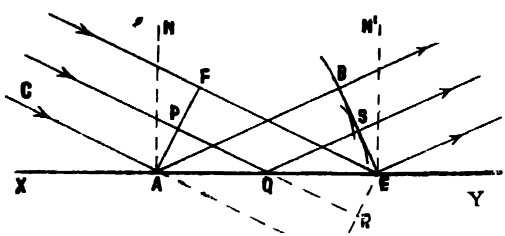


Fig. 128—Reflection on Wave Theory

Now in the triangles AEB and EFA, $AB = AE = Vt$, AE is common ; and $\angle ABE = \angle EFA = 90^\circ$; $\therefore$ the two triangles are equal in all respects.

Hence $\angle AEB = \angle EAF$ ... ... (1)

Here CA, which is perpendicular to the incident wave-front and which is the direction of the incident wave, represents an *incident*

ray at the point of incidence A, and AB, the direction in which the reflected plane wave travels, represents a *reflected ray.*

To prove that the angle of incidence is equal to the angle of reflection, draw AN perpendicular to XY at A. Because AN is perpendicular to the plane AF and AB is perpendicular to the plane EB, we have,

$\angle AEB = \angle NAB$; and because AN and CA are perpendicular to AE and AF respectively, $\angle EAF = \angle NAC$.

$\therefore$ From (1), $\angle NAC = \angle AEB = \angle NAB$.

Thus *the angle of incidence is equal to the angle of reflection.*

Again, the incident ray CA and the reflected ray AB lie in the same plane as the normal AN. Thus both the laws of reflection are deduced from the wave theory.

117. Refraction and Wave Theory.—Let XY be the trace of a plane surface separating two media, say, air and an optically denser medium, say, glass (Fig. 129), and AB be a plane wave travelling in the first medium in the direction EA with velocity V. When the wave-front AB meets the refracting surface XY obliquely at A, the ether particle at A is disturbed and, according to Huygens, it becomes a new centre of disturbance from which the generated wavelets spread out into the second medium with a velocity V', say.

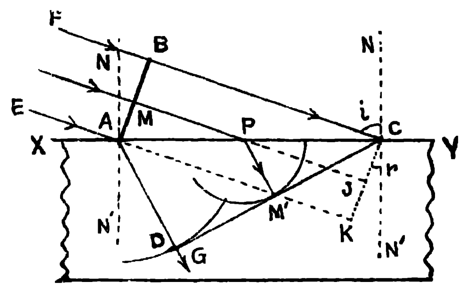


Fig. 129—Refraction on Wave Theory

In the time t, say, taken by B to reach C, let AD be the distance the wavelet from A travels in the second medium. Now with centre A and radius AD ($= V't$) describe a sphere. From C draw a plane tangential to the sphere and at right angles to the plane of the paper. Then CD represents the refracted wave-front and it travels in the direction AG. Thus AG, which is perpendicular to the wave-front CD, represents *the refracted ray*, and similarly EA, which is the direction of the wave in air, represents *the incident* ray.

Now because EA is perpendicular to the plane AB and NA is perpendicular to AC. $\therefore$ $\angle EAN = \angle BAC$; and similarly $\angle DAN' = \angle ACD$.

($\because$ AN' and AD are perpendicular to AC and DC respectively)

If the $\angle$s EAN and DAN' are denoted by i and r respectively, we have,

$$\frac{\sin i}{\sin r} = \frac{\sin EAN}{\sin DAN'} = \frac{\sin BAC}{\sin ACD} = \frac{BC}{AC} \div \frac{AD}{AC}$$

$$= \frac{BC}{AD} = \frac{Vt}{V't} = \frac{\text{Velocity in air}}{\text{Velocity in the second medium}} = \text{a constant.}$$

Thus *the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction*, and this verifies the **second law of refraction**. We know that this constant is called the *refractive index* (μ) of the second medium with respect to the first.

The incident ray EA, the refracted ray AD, and the normals AN and AN' at A all lie in the plane of the paper and so they are in the same plane ; this proves the **first law of refraction**.

Triumph of the Wave Theory over the Corpuscular Theory—By the wave theory of light the refractive index of a medium (b) with respect to another medium (a) is equal to the ratio of the velocity of light in the medium (a) to that in the medium (b).

$$\text{Thus, } {}_a\mu_b = \frac{V_a}{V_b}.$$

Since a ray of light is bent towards the normal when it enters an optically denser medium, *i.e.* ${}_a\mu_b > 1$, it follows from the wave theory of light that the velocity of light is less in a denser medium than that in a rarer one. and this fact has been verified by actual experiment. This is a great truimph of the Wave Theory over the Corpuscular Theory according to which the velocity in the denser medium should be greater.

As an example, ${}_{\text{air}}\mu_{\text{water}} = \dfrac{\text{velocity of light in air}}{\text{velocity of light in water}} = 1\cdot33.$

So, velocity of light in water $= \dfrac{\text{velocity of light in air}}{1\cdot33} = \dfrac{2\cdot99 \times 10^{10}}{1\cdot33}$

$= 2\cdot55 \times 10^{10}$ cms./sec.

Besides giving satisfactory explanation of reflection and refraction of light, the Wave Theory explains also the phenomenon of **interference of light** which takes place as a result of disturbance of any part of a medium by two different sources almost at the same time, and which was unaccountable from the standpoint of the Corpuscular Theory.

Remember :—

(i) Light travels 186,000 miles per second, or
(ii) Light travels 300,000,000 metres per second.
(iii) Light takes 8 minutes to travel from the sun to the earth, and
(iv) Light takes $\frac{1}{7}$ of a second to travel round the earth's equator.

Questions

Arts. 113 & 114.

1. Describe a method of measuring the velocity of light.
(Utkal 1948 ; Del.U. 1940, '43 ; Dac. 1934, '40 ; All. 1922, '24, '45)

2. Describe Romer's method of determining the velocity of light. What is its value ? What is the velocity of light in vaccum ?
(*See Art.* 117) (C. U. 1932, '36, '39, '44 ; Pat. '32)

[**Hints.** $\dfrac{\text{Vel. of light in vacuum}}{\text{,, ,, ,, air}} = {}_{\text{vac.}}\mu_{\text{air}} = 1{\cdot}00029$]

3. Describe an apparatus which can be fitted up in a laboratory for determining the velocity of light. Does the velocity of light depend upon the nature of the medium ? Do you know of any optical property of a medium to which the velocity of light can be related ? (Pat. 1934)

[**Hints.** The refractive index of a medium is related to the velocity of light in that medium (see Art. 117)]

4. Describe Fizeau's method of determining the velocity of light. Explain the principle and give a diagram of the apparatus. (Pat. 1920)

Art. 115.

5. Give reasons for the statement that light travels in straight lines with a finite velocity.

Arts. 116 & 117.

6. Show how the laws of reflection and refraction of a parallel pencil of light at a plane surface may be deduced from the wave theory.
(C. U. 1934, '37 ; Dac. 1941)

7. Explain how the refraction of light is accounted for on the wave theory and point out the physical significance of the refractive index of light.
(C. U. 1945)

8. Prove that at a plane surface of separation between two media $\dfrac{\sin i}{v_1} = \dfrac{\sin r}{v_2}$, where v_1 and v_2 are the velocities of light in the two media and i and r are the angles of incidence and refraction respectively. (Dac. 1942)

9. Refractive index of water is 1·33. The velocity of light in vacuum is 3×10^5 km. per sec. Find the velocity of light in water. (Dac. 1941)

[*Ans.* $2{\cdot}26 \times 10^5$ km. per sec.]

PART V

MAGNETISM

CHAPTER I

Natural and Artificial Magnets
Magnetic Induction

1. Magnets.—

A dark coloured ore, composed of iron and oxygen (Fe_3O_4), called **Magnetite,** first discovered in Magnesia in Asia Minor, was known from ancient times to possess the following two characteristic properties.

(i) **Attractive property**.—When dipped into filings of iron, a lump of magnetite picks up some filings chiefly at the two ends.

(ii) **Directive property**.—When suspended at the end of a fine thread so as to turn freely, it oscillates to-and fro and finally comes to stay with its two ends always directed along the north and the south approximately.

This directive property was used by the ancient sailors to guide the course of the ship on the sea. Hence they called magnetite a **lode-stone** which means a *leading stone.* The word magnet, which means a body possessing attractive and directive properties, owes its name to the magnetite, which is the fore-runner of it.

2. Artificial Magnets.—

Natural magnets like the lode-stone are irregular in shape and have weak attractive or directive properties. This two properties can, however, be very strongly developed in some metals or alloys. Such an act of infusing the properties of a natural magnet in a new body is called *magnetisation.* Magnets so prepared, called artificial magnets, are found in the following types.

(1) **The Bar Magnet.**—This is a rectangular or circular bar of uniform cross-section. Two such bar magnets placed side by side, with opposite polarities towards the same end, having two cross-pieces (called *keepers*) placed on the two ends, is shown in Fig. 1.

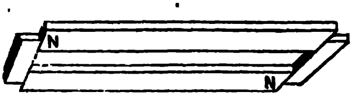


Fig. 1—The Bar Magnet

(2) **The Horse-shoe Magnet.**—This is a bar magnet bent so as to resemble a horse-shoe. The speciality is that the two ends have been brought close together (Fig. 2).

Fig. 2—The Horse-shoe Magnet

(3) **The Magnetic Needle**—This is a short and thin strip of magnetised steel (or suitable alloy) having two pointed ends. This is balanced horizontally in the middle on a point about which it can freely turn (Fig. 3).

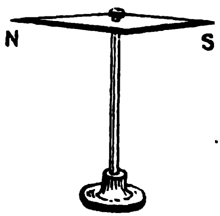


Fig. 3—The Magnetic Needle

(4) **Ball-ended Magnet.**—It is a modified form of a bar magnet of circular cross-section ending in two balls [Fig. 3 (a)]. Its speciality is that the two polarities are localised at the centres of the two balls.

N. B. Besides the above well-known forms, magnets are also found to be of other shapes as well, *e.g.* ring magnets, etc. Sometimes specially shaped *pole-pieces* are fitted on the two ends to produce desired fields in a confined space.

Fig. 3(a)—The Ball-ended Magnet

3. Some Definitions :—

Pole.—If a magnet be dipped into iron filings, the latter is found to adhere chiefly at two regions near the two ends. These regions, where the attraction appears to be strongest, are generally termed its *poles*. The cluster of iron filings picked up by a magnet, however, shows that the "*pole*" property really extends over quite a large region at each end. Each part of this region experiences an attraction or repulsion, as the case may be, when another magnet is brought near ; and the line of action of the resultant of all these individual forces always passes through one point So for all practical purposes we regard each pole as being concentrated *at a point* somewhere near, but not exactly at, the end of the magnet, just as we regard the mass of a body as being concentrated at its centre of gravity. These two points are referred to as the poles of the magnet.

If the magnet be suspended by a thread so as to be free to turn in a horizontal plane, one end of it always points towards the geographical north, and the other towards the south. The pole which points

towards the north is called the **north-seeking pole** or **north pole**, and the pole which points towards the **south** is called the **south-seeking pole** or **south pole**.

Magnetic Axis.—The straight line that joins the two poles of a magnet is called its magnetic axis.

Neutral Region.—On examining the cluster of iron filings picked up by a magnet, it is found that the maximum quantity of filings sticks at the ends and there is a rapid decrease towards the central part where there are practically no filings. This shows that a magnet has no attracting power at the middle. At this part if a belt be imagined round the magnet normal to the magnetic axis, it will have no attracting power and so it is known as the *neutral region*.

Effective Length of Magnet.—It is the distance between the poles of a magnet along the magnetic axis (see Art. 18). It is also called the equivalent length, magnetic length, or simply the length of the magnet. In a good magnet, the effective length is about 85% of the geometric length of the magnet.

Magnetic Meridian.—It is the imaginary vertical plane passing through the magnetic axis of a freely suspended magnetic needle placed at a place, *i.e.* it is the vertical plane containing the direction of the magnetic field at a place.

Geographical Meridian at a place is an imaginary vertical plane passing through the given place and the geographical axis of the earth, *i.e.* it is the vertical plane, imagined at the given place, containing the geographical north and south poles.

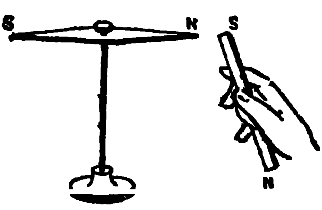


Fig. 3(b)

3(a). Laws of Magnetic Attraction and Repulsion.—If the north pole of a bar magnet be presented before the north pole of a magnetic needle, repulsion takes place, but, if presented before the south pole of the magnetic needle, there will be attraction [Fig. 3 (b)]. The result is stated as :—

Like poles repel and unlike poles attract.

3(b). The Earth as a Magnet.—A magnetic needle, or a bar magnet suspended horizontally at its centre of gravity, invariably sets in a particular direction, with its magnetic axis pointing approximately north and south of the earth. Such behaviour of the needle indicates the existence of a terrestrial magnetic field. From such observations and other considerations, the earth is regarded as a huge magnet having its *south magnetic pole somewhere near the north*

geographical pole and north pole near the south geographical pole. As the result of interaction between the *earth as a magnet* and any other magnet which is free to move, the latter always tends to set itself parallel to the magnetic axis of the earth. For uniformity of nomenclature, the true south magnetic pole of the earth is conventionally called its magnetic N-pole and the true north magnetic pole is called its magnetic S-pole. Thus the geographical N-pole and the magnetic N-pole of the earth, according to this nomenclature, are near each other. Accordingly the magnetic N-pole is near the geographical N-pole. For further studies on terrestrial magnetism refer to chapter IV.

4 Methods of Magnetisation.—(1) Magnetisation by Bar Magnets (*Mechanical method*).—

(i) Method of Single Touch.—The specimen AB to be magnetised is placed on the table and one end of a bar magnet NS is placed on an end A (Fig. 4.) of the piece and then drawn to the other end B, keeping the magnet in the inclined position as shown in the figure. The magnet is then lifted and the process is repeated several times, always beginning from the end A and ending in B. AB will then be magnetised having polarity at B opposite to that of the striking pole.

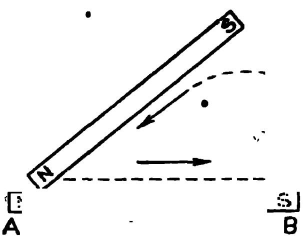


Fig. 4—Method of Single Touch

(ii) Method of Divided Touch.—The specimen to be magnetised is placed on the table (Fig. 5). The *opposite poles* of two bar magnets are placed together at the middle of the specimen and drawn towards the opposite ends keeping the magnets inclined, as in the last experiment. The operation is repeated several times always beginning at the middle. The bar is then turned over, and the other side is also magnetised in the same way. Polarities developed on the ends of the piece are of opposite nature to that of the stroking pole.

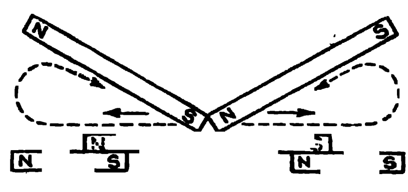


Fig. 5—Method of Divided Touch

The magnet produced becomes stronger if the two ends of the specimen be supported on the two poles of two other magnets, the poles of each being the same as that of the stroking magnet over it.

(iii) Method of Double Touch.—This method is almost the same as the method of divided touch, only a piece of wood or cork is placed between the opposite poles of the rubbing magnet (Fig. 6). The magnets are then moved *together* from the middle to one end of the specimen, then back to the other end and finished at the middle. This is repeated several times.

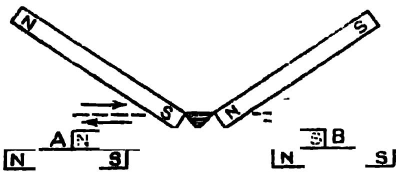


Fig. 6—Method of Double Touch

The polarities developed on the ends of the piece are of opposite nature to that of the nearer stroking pole. For strong magnetisation, the specimen is mounted on two bar magnets as suggested under 'divided touch'.

(2) Magnetisation by Electric Current.—The rod to be magnetised is placed within a thin-walled glass tubing or a card-board cylinder having a spiral of insulated copper wire wrapped round it (Fig. 7). The rod will be magnetised if a strong current is passed through the coil. *The end where the current flows in a counter-clock-wise direction, when looked at from that end, will be a* **north pole**, *and the other end a* **south pole** (Fig. 8). If the rod be of *steel*, then the magnetisation developed in it will be permanent ; if it be of *soft iron*, it will be a strong magnet as long as the current is passing, but will almost completely lose its magnetism as soon as the current is stopped. This is known as the **electro-magnetic method** (see Art. 16, Part VII).

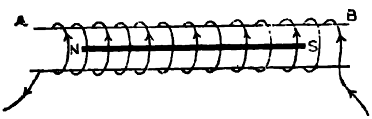


Fig. 7—Magnetisation by Electric Current

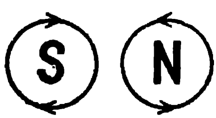


Fig. 8

Strength of the polarities developed will depend on the quality (permeability) of the specimen, the number of turns per unit length of the coil, and the strength of the current (**Ampere-turns**).

(3) Magnetisation by Earth's Induction.—A soft iron rod can be *magnetised by the earth's magnetism*, if it is placed for a few days parallel to the direction of the magnetic meridian, the end pointing towards the north acquiring north polarity. The magnetisation acquired will, however, be very feeble. The method may be hastened by striking the rod with a wooden hammer from time to time.

Again, the bar may be magnetised, if it is kept in a vertical position. In the northern hemisphere it will be found to be magnetised such that the top end will be the south and the bottom end the north pole. This is due to the inductive action of the earth which is regarded as a huge magnet with its south magnetic pole near the north geographical pole and the north magnetic pole near the south geographical pole (see Ch. IV).

4(a). **Consequent Poles.**—Due to faulty or irregular process followed in magnetisation, a magnetic substance may acquire similar polarities at the ends and opposite polarities in the middle, or additional free poles in the middle besides the polarities at the ends. Thus, if similar poles of two magnets are used as the stroking poles in the methods of divided or double touch, two opposite poles will be produced at the two ends and two similar poles will appear in the middle. If a very strong pole is made to touch the middle of a weak magnet, an opposite polarity will be localised there. Such irregular polarities other than those at the two ends of a magnet are referred to as consequent poles. They are, however, short-lived.

5. **Magnetic Saturation.**—The degree of magnetisation developed on a piece of substance depends on the magnetising force and the quality of the substance. But there is a limit to the magnetisation which can be acquired by a magnet, however much the magnetising force may be increased. The substance is called *magnetically saturated* when that limiting value of magnetisation is reached,

6. **Magnetic and Non-magnetic Substances.**—All substances are acted on, more or less, when placed in a *strong magnetic field*. In case of most of the substances. it should be noted that the effects can not be easily detected. A few substances like iron, nickel, cobalt, manganese and some alloys called *ferromagnetics* are, however, *attracted* even by a weak magnet. Those substances are used for preparing permanent magnets. Though magnetism is a universal property of all substances, particularly the ferromagnetics are commonly called *magnetic substances* because of their very pronounced magnetic properties. All other substances besides the ferromagnetics are ordinarily classed as non-magnetics. The subject of magnetic qualities of substances has been dealt with in more details in Art. 17(a).

7. **A Permanent Magnet and a Magnetic Substance.**—A **magnetic substance** is one which is attracted by magnets and is capable of being converted into a magnet. Nickel and cobalt, which are also magnetic substances besides iron, resemble iron in their magnetic behaviour, but to a much less degree.

Many alloys, such as *tungsten-steel*, st alloy, *Cobalt-chrom-steel*, permalloy, *Cobalt-steel*, mumetal, *Alnico*, etc., have been prepared in recent years, which possess the qualities of a magnetic substance to a far greater degree than in case of Iron, Nickel, etc.

A magnetic substance, which has been magnetised and has got a permanent polarity at each end of it, is known as a **permanent magnet.**

Difference between a Magnet and a Magnetic Substance.—

(*i*) A freely suspended magnet always points towards north and south, but a magnetic substance, when similarly treated, point in any direction ; that is, a permanent magnet has definite poles, whereas a magnetic substance has no poles of its own. A magnetic substance can have only induced polarity.

(*ii*) A magnet attracts magnetic substances, but a magnetic substance has got no such property.

(*iii*) If a pole of a magnet be presented before the poles of another suspended magnet there will be attraction in one case and repulsion in another. A magnetic substance, when presented before the suspended magnet, will attract both the poles.

7 (a). How to distinguish between a Magnet, a Magnetic substance, and a Non-magnetic substance.—

(*i*) When an auxiliary magnet is supplied, one end of it is touched to both ends of each of the three specimens successively. The one for which there is attraction at one end and repulsion at the other is the magnet. The other for which there is attraction at both ends is the magnetic substance. The third which is unaffected when the magnet is presented to it is non-magnetic.

(*ii*) When no auxiliary magnet is supplied, but a thread is available by which the specimens can be freely suspended at the centre one after another, it is easy to identify the magnet ; for amongst the three only the one which is the magnet will always set itself in a particular direction (magnetic meridian) when so suspended. The other two will point to any direction in which they are suspended. Knowing the magnet in this way, it is then used to distinguish between the other two, as in(*i*).

(*iii*) When no auxiliary magnet or suspending thread is available, *i.e.* when no external helping agent is to be used, one of the specimens is taken and one end of it is successively presented to the middle point of the other two. The experiment is repeated, taking the other specimens one after another. There will be one case only in which there

will be attraction and in other cases there will be *no attraction*. In the case of attraction, the specimen whose end was presented is the magnet and whose middle part was touched at is the magnetic substance. The third specimen is non-magnetic. This is because the pole of a magnet can attract any region of a magnetic substance only, whereas at the middle part of a magnet which is a neutral region there is no attraction.

8. **Magnetic Induction.**—A rod of soft iron will ordinarily have no action on iron filings placed in contact with it, but if a magnet be made to touch, or brought near one end of the soft iron rod, the filings will cling to the other end of the rod. On removing the magnet the filings will immediately fall off, showing that the soft iron rod was only temporarily magnetised by the influence of the magnet.

Such phenomenon in which temporary magnetism is developed on a magnetic substance by the influence of another magnet, with or without actual contact, is known as **magnetic induction.**

In the above experiment the soft iron rod was magnetised by the inductive influence of the magnet, and the iron filings adhered to the soft-iron rod due to **induced magnetism** acquired by it.

The magnet under whose influence magnetism is developed in the soft iron bar is called *the inducing magnet.*

Nature of Induced Polarities.—A pivoted magnetic needle is taken and one end of a soft-iron bar (Fig. 9) kept on a suitable stand *P* is placed near the north pole of the needle so as to be in the same horizontal plane with it. The pole is attracted to the bar. If now the north pole of a bar magnet (which does not affect the needle directly from that distance) be brought near the other end of the bar, the needle is repelled. So, by induction a N-pole must have been produced at the end of the soft-iron bar near the needle and a S-pole at the other end. This shows that **by induction opposite polarity is created at the end near the inducing pole and a similar polarity at the farther end of a magnetic substance.**

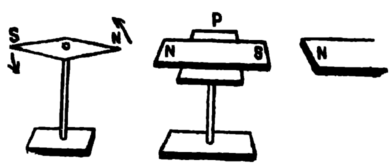


Fig. 9

Thus the soft-iron bar is converted into a *temporary magnet* whose magnetism will last as long as it is before the permanent magnet but the bar will lose its magnetism as soon as the permanent magnet is removed.

Features of Induced Magnetism.—**(a)** If several small soft iron nails are taken and one of them is brought in contact with one of the poles of a bar magnet, it will be supported there. Another nail also can be supported from the free end of the first nail which will become a temporary magnet. In this way a chain can be formed by supporting one nail below another (Fig. 10). The number of nails supported will depend upon the strength of the inducing magnet. On removing the bar magnet from the first nail, the chain is broken and the nails fall down. This experiment demonstrates that *magnetism can be induced in an iron nail by a magnet in contact with it and that the inductive influence can be carried through a number of nails each of which is turned into a temporary magnet.*

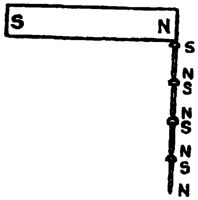


Fig. 10

(b) In the above experiment, suppose inducing pole of the bar magnet is the north pole. If the north pole of a similar second bar magnet be placed over the north pole of the first magnet, the strength is increased and more nails can be supported. If the south pole of the second magnet be placed over the north pole of the first magnet, the nails fall off. When the two inducing poles are dissimilar and are of equal strength, not a single nail can be supported. The above shows that *induced magnetism can be strengthened and weakened by similar or opposite poles.*

9. Magnetisation through Induction.—If a steel rod be placed near the pole of a strong magnet for a considerable period of time, it acquires a *feeble permanent* magnetisation. This is due to the inductive action of the magnet. If long bars of iron or other magnetic substances be kept along the magnetic meridian, or fixed up vertically, (and preferably hammered periodically), it is found after a long time that they get magnetised, though *feebly*. Here the magnetisation is due to the inductive action of the *earth as a magnet.* For the same reasons, the body of a ship acquires some magnetisation during construction.

9(a). Induction Precedes Attraction.—It can now be explained why both the poles of a magnet attract an iron rod. When one end of the rod is brought near the pole of a magnet, the rod no longer remains unmagnetised, but is turned into a temporary magnet by induction, opposite polarity being induced at the near end. These opposite poles attract each other. Thus **induction always precedes attraction between a magnet and an unmagnetised body.**

9 (b). Repulsion is a Surer Test of Magnetisation.—To test whether a given specimen is magnetised or not, it is suspended at its middle from the end of a string, and one pole of a bar magnet is presented to one end of the specimen. If attraction occurs, the specimen may be an unmagnetised magnetic substance or, in the alternative, it may be a magnet having *at the end under test* a pole which is dissimilar to the testing pole. So, without further tests no definite conclusion can be drawn this way or that way. If, however, there is repulsion, the tested end of the specimen must possess a similar polarity, which means that the specimen is magnetised. So repulsion is a surer test of magnetisation. That is why in testing whether a specimen is magnetised or not, a pole of the bar magnet is presented to one end of the specimen and it is looked out whether attraction or repulsion results. If attraction occurs, the other pole of the magnet is to be presented to see if repulsion now takes place or not. If there is repulsion, the specimen is a magnet ; otherwise it is a magnetic substance.

10. Reversal of Polarity.—If one pole of a powerful magnet be gradually approached near the similar pole of a weaker magnet, the two will repel each other, but, when it is quickly brought very near to the weaker pole, repulsion occurs instead of attraction.

This is due to the strong inductive action of the powerful magnet. For example, when the N-pole of a strong magnet is suddenly brought very close to the n-pole of a pivoted needle, which is a weaker magnet, a strong s-pole is induced at that end. As a consequence of superposition of this induced polarity, a s-pole may be built up at that end wholly neutralising the existing n-pole. This inductive development of an opposite polarity at the near end (and a similar polarity accompanying at the remoter end of the needle) explains why attraction should occur where ordinarily repulsion is due.

For a short visit of the inducing pole this pole-reversing effect is temporary, but nevertheless injurious to the needle. That is why a magnet is always to be presented very slowly to a magnetic needle so that the near pole of the needle, if facing a similar pole of the visiting magnet, may have sufficient time to move away. The pole of a strong magnet should not, consistent with the above reasons, be presented to the similar pole of a permanent magnet, for thereby injury is made to the permanent magnetism of the latter through inductive influence.

10 (a). Degree of Induced Magnetism.—The strength of induced magnetism in a substance depends upon (a) the strength of the inducing pole, (b) the distance between the inducing pole and the specimen, (c) the quality of the specimen, and (d) the nature of the intervening medium.

11. Destruction of Magnetism.—The magnetism of a magnet may be weakened or destroyed in the following ways :—

(*i*) *By rough handling.*—A considerable portion of magnetism of a magnet may be lost by hammering or other rough usages.

(*ii*) *By heating.*—A magnet can be made to lose its magnetism completely by heating it beyond a temperature characteristic of the substance.

(*iii*) *By the earth's induction.*—If a magnet be placed parallel to the magnetic meridian with its south pole pointing to the north, its polarity will be weakened by the inductive action of the earth. The north magnetic pole of the earth is similar to the south pole of a magnet, so it induces north polarity in the near end of the magnet facing it, and thus weakens the strength of its south pole.

(*iv*) *By the inductive action of another magnet.*—If a magnet be placed by the side of another magnet with similar poles adjacent to each other, each pole will induce opposite polarity in the other, and thus tend to weaken its strength. *Every magnet has a tendency to demagnetise itself by the inductive action of its two poles upon the magnetised steel which lies between the poles. To avoid this, and also to avoid the demagnetising effect of the two adjacent poles upon one another*, a piece of soft iron is usually placed across the poles of a horse-shoe magnet (see Fig. 2) ; and bar magnets are kept in pairs with opposite poles side by side having a soft iron piece placed across the poles at each end when the magnets are not in use (see Fig. 1). By this method the tendency of one pole acting inductively on the other, or on the steel itself, is prevented, as the effect of each pole of the magnet is neutralised by the opposite polarity induced in the soft iron piece. Such soft-iron pieces are called **keepers**, because they keep the magnetism of the magnets intact by reducing the tendency of the magnets to demagnetise themselves.

Questions

Art. 3.

1. Describe an experiment which will show that a piece of iron attracts a magnet just as truly as the magnet attracts the iron. (C. U. 1920).

[**Hints.**—A rod of soft iron is suspended and one of the poles of a magnet is brought near it. The iron moves towards the magnet, If, on the other hand, the magnet is suspended and the iron is brought near the magnet, the magnet moves towards the iron rod. Hence, both iron and magnet attract each other mutually].

Art. 4.

2. Describe the various ways of magnetising a soft iron.
(Dac. 1932, '43 ; C. U. 1911, '13, '16. '17, '29 ; Pat. 1922, '23)

3. Enumerate the various methods of obtaining a magnetising field for making artificial magnets. (Pat 1932 ; Dac. 1932 : *cf.* C.U. '42 ; cf. Utkal 1947)

Explain how a north pole may be obtained at a definite end of a steel bar.
(C. U. 1942)

4. Describe a good method of magnetising a piece of iron rod. In what respect does an electromagnet differ from an ordinary magnet ? (Pat. 1940)

(See also *Art. 16, Part. VII)*
Explain "Consequent poles'. (Pat. 1946)

Arts. 7 & 8.

5. What is magnetic induction ? How would you distinguish between a permanent magnet and a magnetic substance ?
(C. U. 1913 ; '44 ; Pat. '36 ; Utkal 1947)

6. Distinguish between a permanent and a temporary magnet. (C.U. 1918)

7. Describe suitable experiments illustrating the phenomena of magnetic induction. (C. U. 1920 ; All. 1924 ; Dac. 1928)

7(a). You are given two exactly alike steel bars, and told that one only is a magnet. How would you find out with them which of the two bars is the magnet ? (Cal. 1947)

8. How would you determine whether a given steel rod is a magnet or not ? (C. U. 1915)

Art. 9.

9. 'Repulsion is a surer test of magnetic condition of a body than attraction.' Explain this. (C. U. 1925)

CHAPTER II

Molecular Theory of Magnetism

12. Isolation of a Single Pole is absurd.—When a bar magnet is broken into two parts, each part becomes a complete magnet having a north pole at one end and a south pole at the other. If the pieces are again broken up still further, each piece becomes a complete magnet again (Fig. 11). In practice it is impossible to produce a single pole, *i.e.* a completely isolated pole.

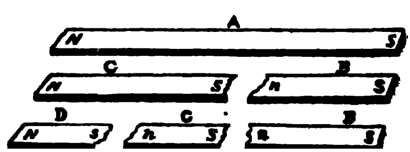


Fig. 11.—Breaking a Magnet

Even in a molecular magnet **two poles will always occur in pair,**

that is, they are *inseparable*, or, in other words, it is *not possible* to have a single or *isolated magnetic pole*.

12 (a). Molecular Theory of Magnetism.—The inseparability of the poles of a magnet led to a theory of magnetism, due to **Weber,** a German scientist, which has been afterwards developed by Ewing. The theory is called the *Molecular Theory of Magnetism.* According to this theory every molecule of a *magnetic substance*, whether magnetised or not, is itself a complete magnet. In an unmagnetised specimen the molecules are arranged either haphazardly [Fig. 12(*a*)], or in closed groups or chains [Fig. 13(*a*)], causing no resultant magnetic effect, the opposite poles neutralising each other throughout the piece. When a magnet is brought near the specimen, the molecules are swung around by the external magnetic force, and, due to the

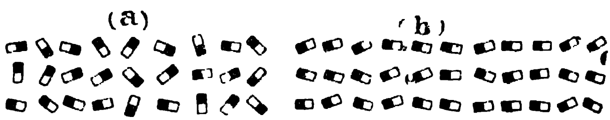


Fig. 12

process of magnetisation, these molecular magnets arrange themselves in definite lines, as shown in Fig. 12(*b*). When all the little magnets are in line, the magnetisation is complete and the substance is then said to be magnetically **saturated.**

On account of the linear alignment of the molecules due to magnetisation, the magnetic effects are neutralised everywhere excepting at the free ends, one of which becomes North pole and the other a South pole; that is, a line magnet is formed. A bar magnet may be looked upon as a number of such line magnets lying side by side having similar poles turned to each end. If that is the picture, the polarities should be on the faces of the ends of the magnet. But it is common knowledge that the attractive action exists also well away from the ends. This is well explained if the line magnets are supposed to be more and more curved towards the ends (Fig. 13). The curvature may be attributed to the mutual repulsion of these line magnets at each end, where there are contributory similar polarities.

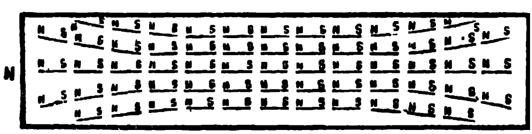


Fig. 13

According to **Ewing**, the molecules of the substance are grouped together in "*closed chains*" [Fig. 13(*a*)]. Such an arrangement of molecular magnets is stable. The process of magnetisation consists in breaking up these chains and arranging them in a linear alignment.

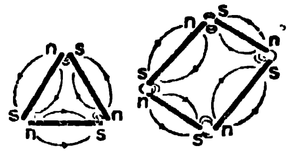


Fig. 13(*a*)—Ewing's Chain Theory

It has been found that soft iron is more readily magnetised than steel but it loses its magnetism more readily than steel. This is explained by the fact that the *binding forces* of closed chains of the molecules are *weaker in soft iron than in steel*, and therefore it is more difficult to rotate the molecular magnets in a definite direction in steel than those in soft iron, but after they are once rotated it is more difficult to rotate them back again, *i.e.* to demagnetise.

12 (b). Support of Molecular Theory of Magnetism—Weber-Ewing Theory of molecular magnetism has found a strong support in the fact that many of the ordinary magnetic phenomena could be successfully explained in the light of this theory.

(i) Induction.—When a particular pole of a magnet is presented to one end of a magnetic substance, *Weber elements* (the molecular magnets) are acted on according to the ordinary laws of attraction and repulsion. They, however, can not leave their places owing to the high rigidity of the substance but can turn about their positions of equilibrium just as pivoted needles can. So the dissimilar poles of the Weber elements will tend to point towards the inducing pole overcoming their mutual actions. Thus the end of the substance facing the inducing pole will contain an excess of free dissimilar poles resulting in a polarity of the opposite kind. Consistently, a stronger inducing pole will rotate a larger number of Weber elements into linear alignments and will cause a stronger magnetism to be induced. When the inducing pole is withdrawn, the Weber elements, if they can easily turn as in the case of soft iron, will turn back into their normal positions under mutual actions.

(ii) Magnetisation by Rubbing.—Rotation of the Weber elements due to an external inducing pole is necessarily very small and so the induced magnetism is feeble. When a magnetic substance is rubbed along its body with a strong magnet, the Weber elements are under the strongest action because of the closest vicinity of the inducing pole, and so the rotation of the elements is more complete than under ordinary induction. So "rubbing" is considered to be a more quick and effective method of magnetic induction.

(iii) Laminated Magnets.—When a specimen is a *thick* bar, magnetisation even by rubbing is also not satisfactory, for the Weber elements well within the body are not appreciably acted on by the rubbing pole. So, in practice, thin bars called lamination are separately magnetised first and are then placed upon each other having similar poles at the same end. The laminated magnets are then rivetted together at the ends. A more uniform magnetisation throughout the body of the magnet is in this way ensured than in the case of a thick single magnet. A thick magnet so made is called a *laminated or compound magnet.*

(iv) Magnetic Saturation.—'Magnetic Saturation' is reached when no further magnetisation can be induced in a specimen. This can be readily explained by the molecular theory ; for, as the inducing action is increased, more and more of the Weber elements will be orientated in the direction of magnetisation. When finally all the elements have turned into linear alignments, the amount of free polarity at either end is maximum and no further magnetism can be produced by increasing the magnetising force after that limiting state is reached.

(v) Equality of Poles of a Magnet.—In the act of magnetisation, according to the molecular theory, the Weber elements set themselves in lines along the direction of magnetisation. On either side of the neutral region there should thus be equal numbers of free poles of the opposite kind. That is, the two poles of a magnet should be of equal strength.

(iv) Demagnetisation at Curie Point.—If a magnet is gradually heated, a temperature is finally reached when its magnetism is wholly lost. This temperature, which is different for different specimens, is known as the *Curie point* or the *temperature of recalescence.* Curie-point for iron is nearly $750^\circ C$.

With increase of temperature the molecular agitation increases and at the curie point the acquired freedom of rotation probably enables the Weber elements to return to their normal configurations forming closed chains again.

(vii) Demagnetisation by Rough Treatment.—A magnet may suffer considerable loss of magnetism due to rough treatments like rough handling, hammering, etc.

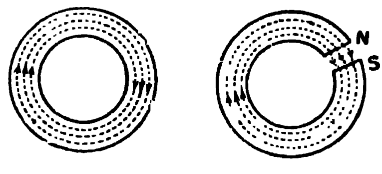
These mechanical processes partially destroy the linear arrangements formed due to magnetisation.

(viii) Heat produced by rapid Magnetisation and Demagnetisation.—When a substance is subjected to a rapid process of alter-

nate magnetisation and demagnetisation, considerable heat is generated in it. This is what it should be, for the Weber elements are thereby alternately brought into lines and flung back into closed chains at a rapid rate. This is equivalent to a vibratory motion of the elements. This vibratory motion is the cause of generation of heat in a body according to the theory of heat.

13. Magnetisation of Rings and Discs.—An iron ring may be magnetised in two ways :—(*a*) By rubbing a bar magnet several times in the same direction along its circumference.

In this way the molecules are arranged in a definite direction forming closed chains having no free poles [Fig. 14(a)]. It can also be magnetised by winding a coil of wire round the whole length of the ring and passing an electric current through the wire. In this case also there is no free polarity. Note that no external magnetic effect is produced. But, on cutting the ring at one place, north pole appears at one end and south pole at the other [Fig. 14(b)].



(*a*) (*b*)

Fig. 14—Magnetisation of Rings

(*b*) By placing the ring within the pole pieces of a strong horse-shoe magnet, or of an ordinary electromagnet, such that a diameter of it is parallel to the axis, two opposite poles are developed at the two ends of that particular diameter and the ring will then behave as an ordinary magnet. Each pole will induce opposite polarity where it touches the ring.

Magnetic Shell.—A thin flat disc magnetised in such a way that its faces possess opposite polarity is known as a **magnetic shell.** For it, the length of the magnet is very small compared to other dimensions.

Questions

Art. 12.

1. What is the reason for the assertion that a magnet cannot be produced which has only one pole ? (C. U. 1933)

2. What do you find when a magent is successively broken into a number of pieces ? What conclusion does it lead to ? (Cf. Pat. 1939)

What do you expect in a bar of magnetic substance if the magnetising field, by which it is magnetised, is gradually increased from a very low value to a high one. (Pat. 1932)

3. Give an outline of the molecular theory of magnetism and show how it accounts for (*i*) magnetic saturation, (*ii*) difference between the behaviour of soft iron and hard steel when under magnetising influence. (Pat. 1931)

3(a). Explain the molecular theory of magnetism and mention any experiment which in your opinion supports the theory. (Del. U. 1940)

Art. 13.

4. Two circular rings of iron are magnetised, the first by being placed between the poles of a strong horse-shoe magnet so that line joining the poles of the magnet is a diameter of the ring, the second by having one pole of a bar magnet drawn round it several times. Describe the magnetic state of each ring. (Punjab. 1931)

CHAPTER III

Magnetic Field and Lines of force.

14. Magnetic Field.—*Magnetic field* is the space surrounding a magnet over which the influence of the magnet is exerted.

Magnetic Lines of Force—If an isolated north pole be supposed to be situated at any point in a magnetic field due to a magnet, it will experience a force of attraction by the south pole and a force of repulsion by the north pole. It will also experience a force due to the earth as a magnet. These forces together produce a resultant force, and the isolated pole, if free, will move in the direction of the resultant force at that point. At every point of the field, the magnitude and direction of the resultant force depend on the position of the pole relative to the magnet and so the direction of motion of the pole will change. By changing the position of the isolated north pole from point to point in the field, the path followed by it will be found to be a curve starting from the north and terminating at the south pole of the magnet. Such a curve which represents the path of motion of an isolated *N-pole* in a magnetic field is called a *line of force.* From each point on the *N*-pole of the magnet one line of force starts and passing through the magnetic field ends on a corresponding point on the *S*-pole; from the *S*-pole it travels *through the body of the magnet* and ultimately returns to the same point on the *N*-pole from which it originated. That is, a line a force is a continuous closed curve. But it must be remembered that these lines of force do not really exist. They are imagined only and are a means of studying the nature

of a magnetic field. So it may be defined as follows :—"*A line of force is a continuous curve drawn in a magnetic field such that the tangent at any point on it shows the direction of the resultant force at that point.*" The *positive direction* of a line of force is the direction in which an isolated free north pole will move.

Properties of Lines of Force.—(i) They are closed curves.

(ii) They always start from a N-pole and end in a S-pole and are continuous through the body of the magnet.

(iii) They never intersect one another ; for, if they did it would mean that at the point of intersection the resultant magnetic force would act in two different directions, which is impossible.

(iv) They are like streched elastic threads and are in a state of longitudinal tension, and mutually repel each other sidewise.

(v) They start from and end on a surface normally.

15. Maps of Magnetic Field.—It has been found that the direction of the magnetic lines of force can be traced by means of an isolated north pole, but as it is impossible to obtain a single pole the above method cannot be practically obtained. The lines of force are therefore traced by means of a small compass needle which, placed at any point, will set itself with its magnetic axis in the direction of the resultant magnetic force at that point.

It follows, therefore, that it is possible to indicate the directions of the lines of force at all points of a field by means of a compass needle instead of a single pole (which is impossible to obtain). Such a diagram is called a **map of the magnetic field**. This idea is due to Michael Faraday, the celebrated physicist of London (1791-1867).

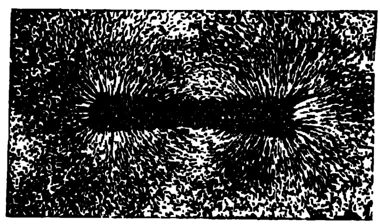


Fig. 15—Map of the Magnetic Field due to a single Bar-magnet

(a) Methods of Plotting Magnetic Maps.—Two methods are adopted for mapping magnetic fields.

(i) *Iron filings method ;* (ii) *Compass needle method.*

(i) **Iron Filings Method.**—This method is suitable only for strong magnetic fields, and not for weak fields like the earth's horizontal field. Iron filings are scattered on a glass plate which is

placed over a magnet. On gently tapping the glass plate, the iron filings will be arranged along certain lines due to the magnetic action of the magnet which takes place across the glass plate (Fig. 15). The setting of the iron-filings gives the nature of the resulting magnetic field and is called the *map of the magnetic-field* produced. From one end of the magnet to the other, the filings set themselves along definite lines which give the lines of force ; the tangent at any point of such a line represents the direction of the resultant magnetic force at the point.

(ii) **Compass-Needle Method.**—When a compass needle (Fig. 16) is placed near a bar magnet, the direction in which the needle rests is the direction of the resultant field due to the two sources, one due to the magnet, and the other due to the earth.

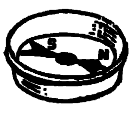


Fig. 16

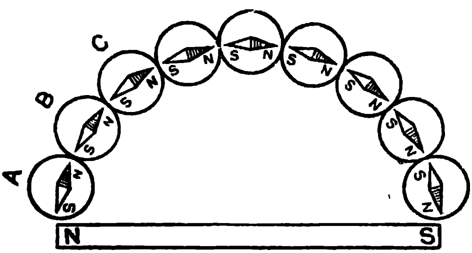


Fig. 17—Compass-needle Method

A bar magnet $N S$ is placed on a sheet of paper (Fig. 17) fixed on a drawing board in such a way that it lies in the magnetic meridian. Its outline is then drawn by a pencil. A compass needle is placed in position A near one end N of the magnet, and two pencil marks are put on the paper exactly at the two ends S and N of the needle. The needle is then moved to the position B so that its first pole is placed on the second mark N, and another mark is put at the other end. In this way the needle is shifted from one position to the next till the other pole of the magnet is reached, as shown in Fig. 17. [The axis of the compass needle will really set along the tangent to a line of force and so the smaller the needle the more accurately will the line, drawn on the paper, coincide with the true path of a line of force]. This process is continued until the whole field is mapped out. It will be noticed that near the magnet the forces due to the earth are negligible in comparison with those due to the magnet, and so the lines are, in reality, due to the magnet only ; but, at more remote points, the earth's field predominates, and we get the lines due to the earth, which are, however, slightly modified by the presence of the magnet.

(b) **Neutral Points.**—Fig. 18 represents, for one side of the magnet, the resultant field due to the earth and a bar-magnet placed in the magnetic meridian with its N-pole pointing north. It will be seen that

the lines of force due to the magnet (passing from N to S) and those due to the earth are in the same direction at all points along the axis of the magnet produced ; so on this line the resultant field is stronger, whereas these two fields are in opposite directions at all points on a line drawn through the centre of the magnet at right angles to its axis, where these two fields, therefore, weaken each other. The strength of the earth's field is uniform and that of the magnet varies from point to point being stronger than the earth's field near the magnet and gradually becoming weaker with the increase of distance from the magnet. Hence at some point, say at A (Fig. 18), the two fields will be equal and opposite, the resultant being zero. Such a point is called a *neutral point*. Notice that **at the neutral point A, the resultant field F due to the magnet exactly balances an equal and opposite force H due to the earth's horizontal field.** Another similar neutral point X like that of A must be on the other side of the magnet, as shown in Fig. 19.

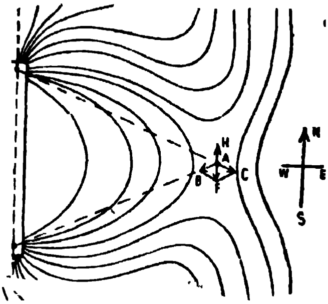


Fig. 18—Method of determining the positions of Neutral Points due to a Bar-magnet and the earth.

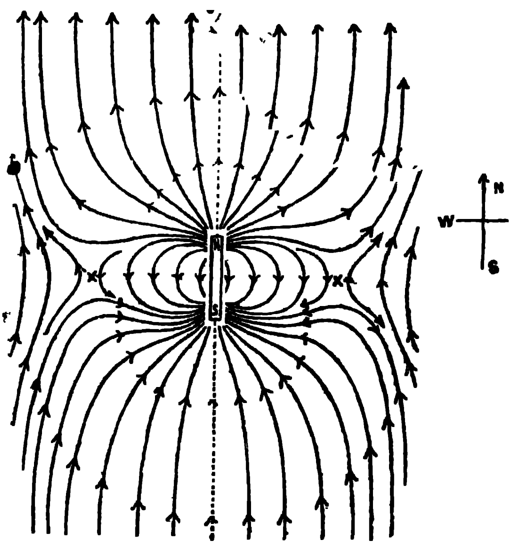


Fig. 19—Resultant Field (N-pole pointing North)

If, however, the magnet is placed N-pole pointing south, the direction of its lines of force will be reversed ; here the two fields will agree in direction on the line drawn through the centre of the magnet at right angles to the axis and will oppose each other along the axis. Thus the neutral points will now be on the axis, as shown in Fig. 20.

A **neutral point** may, therefore, be defined as a point in a magnetic field where the resultant intensity is zero ; so a small needle placed at the point will be at rest in any direction in which it is kept. There are two such points in the map of the magnetic field of a bar magnet placed in the earth's field. At each of these points the field due to the bar magnet is equal and opposite to the horizontal field due to the earth. In figures 17 to 22, maps of the magnetic field of a bar magnet placed at different positions relative to the earth have been given. The points marked X show the positions of the neutral points.

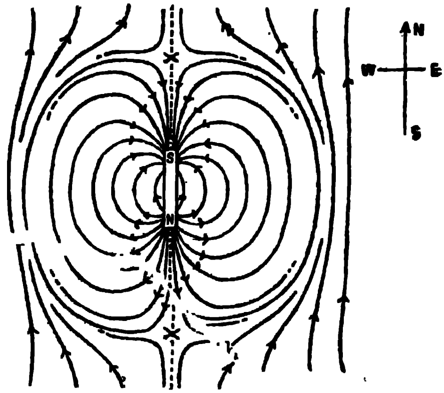


Fig. 20—
Resultant Field (N-pole pointing South)

(c) **Some Important Maps :**—(1) Fig. 19 represents the complete resultant field due to a magnet with its N-pole pointing north.

(2) Fig. 20 represents a complete field where the N-pole of the bar-magnet placed in the magnetic meridian is pointing south.

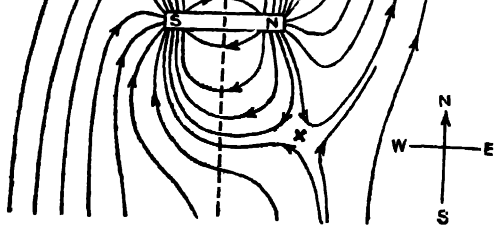


Fig. 21—
Resultant Field (N-pole pointing East)

(3) Fig. 21 represents the resultant field due to the earth and a magnet placed in **east-west direction.**

(4) **Bar Magnet in any position.**—Fig. 22 shows the positions of neutral points (marked X and A) in a field due to a bar-magnet, when placed in any position with respect to the earth's field. In this particular case the axis of the magnet makes an angle of about 70° with the direction of the earth's field.

(d) Some More Cases of Lines of Force.—A few important cases of lines of force are indicated diagrammatically in Figs. 23—25. Fig. 23 shows the map of the lines of force due to a horse-shoe magnet ; Fig. 24 that due to two bar-magnets with their like poles together ; and Fig. 25 that due to two bar-magnets with unlike poles together.

N. B. From the above experiments it might appear that *the lines of force* around a magnet all lie in one plane, but in fact they *extend throughout all the space around the magnet*.

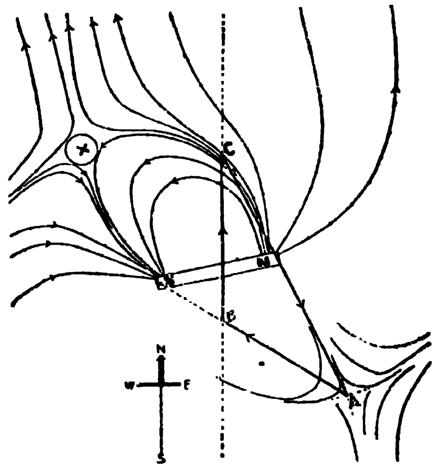


Fig. 22—Resultant Field (Bar-magnet in any position)

16. Lines of Force, Lines of Magnetisation, and Lines of Induction.—When a long rod of a magnetic substance is placed in a magnetic field parallel to the direction of the field, it is magnetised by induction. The number of lines of force per sq. cm. of the cross-section of the field in the space occupied by the specimen is increased due to a number of lines of force added by the induced magnetism of the rod. Thus, within the magnetic substance, we may make a distinction as follows.—

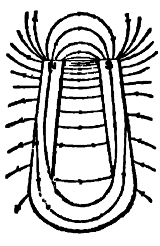


Fig. 23

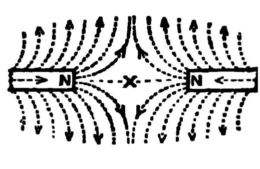


Fig. 24

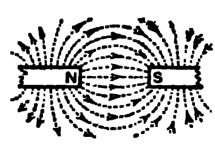


Fig. 25

(*a*) There are *lines in air* which are due to the magnetising field and which would exist if the magnetic medium were not there. They are called the **lines of force**.

(*b*) There are *additional lines within the magnetic substance* which are due to the induced magnetism of the rod. These lines are referred to as the **lines of magnetisation**.

(*c*) The total lines comprising the lines of force and the lines of magnetisation are referred to as the **lines of induction**. The strength of the magnetic field within a magnetic material is due to the lines of induction, i. e. to the joint effect of the lines of force and the lines of magnetisation.

16(a). Intensity of Magnetisation.—In the case of a magnet, which is uniformly magnetised, that is, on which the amounts of free magnetism on the opposite sides of a cross-section taken perpendicularly to the axis at any point are exactly equal but of opposite sign, the *intensity of magnetisation at any point is known as the magnetic moment* (vide Art. 23) *per unit volume taken about that point.* Thus, if M be the magnetic moment, m the pole strength, $2l$ the length in centimetres, a the cross-section in sq. cm., V the volume in cubic centimetres, we have, the intensity of magnetisation

$$I = \frac{M}{V} = \frac{m \times 2l}{a \times 2l} = \frac{m}{a}.$$

Hence *the intensity of magnetisation is also defined as the pole strength per unit area.*

16 (b). Permeability and Susceptibility.—Magnetic lines of force can pass more easily through magnetic substances than through non-magnetic substances like air, etc. Fig. 26 shows that soft iron is more *permeable* to magnetic lines of force than air or it is said to be of greater *permeability*. If a piece of soft iron is placed in a magnetic field, it causes a shrinkage of the lines of force from air into the soft iron piece, which has a very great permeability. The permeability of a substance is always compared relative to air. Thus **Permeability** can be defined as *the conducting power of a medium for magnetic lines of force as compared with that for air, and is measured by the ratio of the number of lines of force passing normally through a unit area placed within the substance (i e. the lines of induction) to the number of lines passing through the same area placed in the same position in air when the substance is removed.*

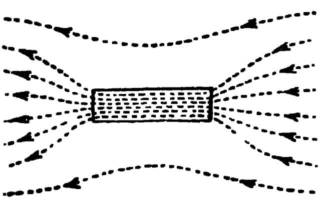


Fig. 26

If B denotes the number of lines of induction per unit area of the substance when placed in a magnetic field (air) H, then B measures the

magnetic induction and H the magnetising force. The ratio of B to H measures the permeability of the substance. Thus,

Permeability $\mu = \frac{B}{H}$. It is greater for Iron than for Nickel or Cobalt.

The *susceptibility* of a magnetic substance is measured by the ratio of the intensity of magnetisation I of the substance to H, the magnetising force. Thus,

Susceptibility $K = \frac{I}{H}$. It is greater for soft iron than for steel.

It can be proved that for a magnetic substance placed in a magnetic field, $B = H + 4\pi I$.

Dividing both sides by H, $B/H = 1 + 4\pi \frac{I}{H}$, or, $\mu = 1 + 4\pi K$.

17. Ferro-magnetic, Para-magnetic, and Dia-magnetic Substances—From a study of the behaviour of substances in a magnetic field it has been found that they may be classified under two distinct classes—para-magnetic and dia-magnetic.

Para-magnetic bodies such as Pt., K, Al. O_2, etc. when freely suspended in a magnetic field ultimately set themselves parallel to the direction of the field, *i.e.* they move from the weaker to the stronger parts of the field. Magnetism is induced in them in such a way that the acquired magnetisation increases the field within them. That is, for them the induction B is greater than the magnetising field H. So for them, permeability μ is greater than unity (*cf.* $\mu = B/H$) and susceptibility K is positive (*cf.* $\mu = 1 + 4\pi K$). So when such a body is introduced in a uniform field, the lines of force become more closely packed within the body than in the space outside, as in Fig. 26. Iron, Nickel and Cobalt should be considered belonging to this general class judging them by the above characteristics. But, in their case, the induced magnetism, when placed in a magnetic field, and hence susceptibility and permeability, are so great in comparison with others that it has been thought fit to group them into a separate class. This is why they have been given a special name—**ferro-magnetics**. Curie's experiments show that for para-magnetics, magnetisation in a magnetic field varies inversely as the absolute temperature, and above the *curie-point*, ferro-magnetics pass into the para-magnetic state. It must not, however, be understood that para-magnetism and ferro-magnetism are only two aspects of the same phenomenon differing from each other only in degree. The difference is really a difference in kind. Ferro-magnetism is not shown by substances which have no definite crystalline structure. Since liquids and gases have no definite structure, they can never be ferro-magnetic.

Dia-magnetic bodies such as *Bi, Sb, Hg, Ag, Zn, Cu, Pb*, water, quartz, etc., when freely suspended in a magnetising field, tend to set themselves at right angles to the lines of force, though the effect is generally only too feeble. In a magnetising field they acquire such magnetisation that they move away from the stronger to the weaker parts of the field, *i.e.* the polarity created in a dia-magnetic body is the reverse of that created in the para-magnetic body. The lines set up within the body due to acquired polarity oppose the magnetising lines of force, and hence the induction B is less than the magnetising field H, *i.e.* the permeability is less than unity and susceptibility negative. So, when such a body is introduced in a uniform magnetic field, the density of the lines of force within the body is less than that in the surrounding space. Unlike para-magnetism, dia-magnetism is independent of temperature.

17(a). Iron and Steel.—Soft iron is readily magnetised, that is, the **susceptibility** of soft iron, *i.e.* the capacity for its being magnetised by a magnetic influence, is greater than that of steel, but the **retentivity**, *i.e.* the power of retaining the magnetisation, after the magnetising force is removed, is smaller.

The retentivity of steel is greater, though it is not readily magnetised like soft iron.

The power of retaining magnetism, inspite of rough handling or any other subsequent demagnetising treatment, is known as **coercivity**. The coercivity of steel is greater than that of soft iron.

It should be noted that steel, containing about 13 per cent. of manganese is non-magnetic.

Any kind of steel which is capable of being hardened can be used as a permanent magnet. In recent years tungsten-steel (3·6 per cent. of tungsten and 0·6 per cent. of carbon), cobalt-chrom-steel (cobalt 15·0, chromium 9·0, molybdenum 1·5, carbon 1·0, and iron 73·5 per cent.), cobalt-steel (carbon, tungsten and cobalt) and "Alnico" (10% Al., 18% Ni., 12% Co., 6% Cu. and 54% iron) are being used for good permanent magnets.

17 (b). Magnetic Screening.—By placing sheet of soft iron D between a point P and a bar-magnet as shewn in Fig 27(a), the point P is shielded from the magnetic effect of the magnet. For the lines of force after entering the iron do not pass out of it in the initial direction for they will traverse the iron which is more *permeable*

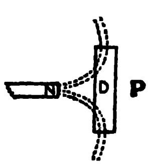
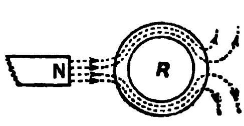


Fig. 27 (a)



Eig. 27 (b)

to them than air, *i.e.* a path of much lower resistance for their passage. The lines of force being thus deviated along the iron-path, the point P is made free from the magnetic field. Similarly, by placing a hollow spherical shell of soft iron in a magnetic field it will be seen that there are no lines of force and, therefore, no magnetic force inside the shell. So the space inside is entirely free from the magnetic effects of the field. If a magnetic needle is placed inside a soft-iron ring, as at R in Fig. 27(b), near a magnet, the needle will be screened from the action of the magnet. In the above cases, the magnetic material, through which the lines of force crowd, leaving any space free from them, acts as a magnetic screen. Such a screen should, therefore, be a substance of high permeability and suitably designed.

18. Localising the Poles of a Bar-magnet.—A compass needle is placed on a sheet of paper fixed on a drawing board. Two large pins (P,P_1) are fixed vertically on the table (outside the drawing board) so that the piece of thread joining the heads of the pins is parallel to the needle in its position of rest. The thread, thus set up, represents the magnetic meridian at the place of the experiment. Now, the compass needle is removed to some distance, the bar-magnet AB is placed on the paper and its outline is drawn by means of a pencil (Fig. 28).

The needle is then placed near one end of the magnet. In this position, neglecting the effect of the other pole, which is at some distance, the needle is acted upon by **two couples**,—one due to the earth's field, and the other due to the adjacent pole of the magnet. The board is then turned until the needle becomes parallel to the thread, *i.e.* until it is in the magnetic meridian. In this position the couple due to the earth's field vanishes, and the needle is acted on only by the adjacent pole of the magnet. The positions of the two ends of the needle are then marked on the paper, and the experiment is repeated by placing the needle in different positions 1, 2, 3, 4, etc., near the same end of the magnet. The magnet and the needle are then removed, and straight lines are drawn through each pair of points. These lines, when produced inside the outline of the magnet, will meet almost at a point near the end. This is the position of a pole.

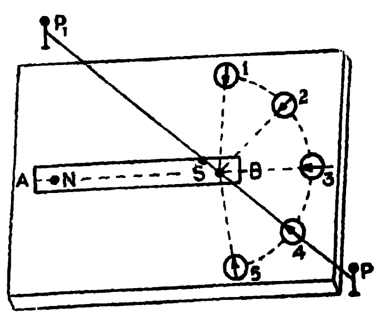


Fig. 28

Similarly, placing the magnet again on the outline and repeating the experiment, with the needle placed near the other end, the position of the other pole can be located. The distance between the poles N and S is called the **magnetic length** of the magnet. The magnetic length is usually about 85% of the length AB of the magnet.

19. Laws of Magnetic Force.—When two magnetic poles are placed near enough there is a force between them, attractive or repulsive, whose value depends on their distance apart. The famous French scientist C. A. Coulomb (1736-1806) first determined the magnitude of such a force. The following are the laws for the action between two poles :

1. Like poles repel and unlike poles attract.

2. (*a*) *The force between two magnetic poles varies directly as the product of their pole strengths, when the distance between them is constant.*

Thus, if m, m' denote the strengths of the two poles, d the distance between them and F the force between the two poles, then,

$\mathbf{F \propto mm'}$, when d is constant.

It is evident that if the pole strengths m, m' are increased, the force will be greater.

(*b*) *The force between two magnetic poles varies inversely as the square of their distance apart ;*

or $F \propto 1/d^2$, when m and m' are constants.

Thus, if the distance is doubled the force between the poles becomes $\frac{1}{4}$ of its former value, or when the distance is trebled the force is reduced to $\frac{1}{9}$ of its initial value, and so on.

This law, viz., $\mathbf{F \propto \frac{1}{d^2}}$, is known as **the Law of Inverse Squares.**

Combining the two laws, we have, $F \propto \frac{mm'}{d^2} = k\frac{mm'}{d^2}$,

where k is a constant depending on the medium and on the units adopted in the measurements.

Unit Pole.—*The unit pole (in the C. G. S. system) is defined as the pole which, when placed at a distance of one centimetre in air or vacuum from another pole of the same strength, is repelled with a force of one dyne.* Thus if $m = m' = 1$, and $d = 1$ cm. of air, F becomes equal to one *dyne* according to this system of measurement. Putting these values in the above equation, k reduces to unity. That is, in the *C. G. S.* system, the force of action between two poles takes the form,

$$F = \frac{mm'}{d^2} \text{ dynes.}$$

20. Strength (or Intensity) of a Magnetic Field.—The strength or intensity of a magnetic field at a point is defined as the force exerted on a unit north pole placed at that point.

A magnetic field has unit intensity when it exerts a unit force on a unit north pole placed in it.

In the C. G. S. system the unit of intensity is called a **Gauss**, after the celebrated German Mathematical physicist, Carl Fiedrich Gauss (1777-1855). *The strength of field at a point is one Gauss, if a unit north pole is acted on with a force of 1 dyne at that point.* Thus, if a unit pole, placed at a point in a magnetic field, is acted on by a force of 10 dynes, the intensity of the magnetic field at the point is 10 gauss.

Remember that if a field has a strength of H gauss, then the force exerted on

a pole of 1 unit strength $= H$ dynes,
a pole of 2 units strength $= 2\,H$ dynes,
a pole of m units strength $= m\,H$ dynes.

Thus, if F be the force in dynes on a magnetic pole of strength m units at a point in a magnetic field of strength H gauss, then $\mathbf{F = mH}$ *dynes.*

[**Note.**—The field strength (H) is a quantity which possesses direction as well as magnitude, *i.e.* it is *a vector quantity*, the direction at any point being determined by the direction in which a N-pole is urged. When it is necessary to indicate the direction as well as the magnitude of this quantity, the term **magnetic force** is used in place of *field strength.*]

Examples.—*1. Two N-seeking point-poles, one of unit strength and the other of strength 2 units, are placed at A and C of a triangle ABC having sides AB, BC, and CA equal to 3, 4 and 5 respectively. Determine the resultant force on unit pole at B.*

Since $5^2 = 3^2 + 4^2$, the angle at B is a right angle.

Repulsive force at B due to $A = \dfrac{1 \times 1}{3^2} = \frac{1}{9}$ dyne ; and that due to $C = \dfrac{2 \times 1}{4^2}$ $= \frac{1}{8}$ dyne. Produce AB and CB and draw the parallelogram of forces at B. The parallelogram will be a rectangle, and hence the resultant force is given by $\sqrt{(\frac{1}{9})^2 + (\frac{1}{8})^2} = \sqrt{\frac{1}{81} + \frac{1}{64}} = 0{\cdot}166$ dyne.

2. Two exactly similar poles are placed at a distance of 8 cms. apart and the force between them is 9 dynes. Calculate the force in gram-weight when they are 4 cms. apart.

Let the strength of each pole be m units. Then by Coulomb's Law, the force $F = \dfrac{mm'}{d^2}$ dynes. Hence $F = \dfrac{m^2}{d^2}$ ($\because m = m'$) ; $\therefore 9 = \dfrac{m^2}{8^2}$. $\therefore m = \pm 24$ units.

The $\pm$ sign shows that both poles may be north poles, or both may be south poles. The force F', when they are 4 cm. apart $= \frac{24 \times 24}{4^2} = 36$ dynes.

Now, 981 dynes = 1 gm. wt.; $\therefore$ 1 dyne $= \frac{1}{981}$ gm. wt.; 36 dynes $= \frac{36}{981}$ $= \frac{4}{109}$ gm. wt.

20 (a). Magnetic Potential.—A magnetic pole placed at a point in the field of another magnet possesses potential energy; for, a single N-pole, if free to move, would move along the line of force, the work being done *by* the system; again work will be done *against* the system if the pole is brought to the point against the magnetic intensity. *The potential energy of a unit N-pole placed at a point in a magnetic field, i.e. the work done in bringing up a unit N-pole from infinity (a point of zero magnetic intensity) to a point against the magnetic intensity is called the* **magnetic potential** *at the point.* The unit of potential (in the C. G. S. system) is the magnetic potential at a point such that 1 erg. of work is needed to bring a unit N-pole to the point from infinity.

Questions

Arts. 14 & 15.

1. Describe what is meant by a line of force due to a magnet. Two bar-magnets are placed end to end with their north poles towards one another separated by a few inches. Draw the lines of force in the plane of the paper, neglecting the effect of the earth's field. (C. U. 1924, '42)

[*See Art. 16*].

2. Define : Magnetic field, Line of force and Neutral point. (Pat. 1948)

3. Trace the lines of force surrounding a bar-magnet when the magnet is placed along the magnetic meridian with the N-pole pointing north. (C. U. 1933 ; Dac. 1932)

4. A bar-magnet is placed in the magnetic meridian with its S-pole pointing North. Explain why neutral points are produced ? (C. U. 1946)

5. Draw roughly the lines of force due to a bar-magnet placed with its north pole towards East, and indicate the position of the Neutral points. (All. 1944)

6. How would you trace the lines of force in the neighbourhood of a bar magnet ? Indicate how the shape of the lines you get depends on the earth's magnetism. (C. U. 1911, '14)

Arts. 16 & 16 (a).

7. Explain why carpenter' tools are sometimes magnetised. How will you protect a watch from magnetic disturbance, if you have to work for long period near powerful magnets ? (Pat. 1924)

[**Hint.**—Carpenters' tools are sometimes magnetised due to the inductive action of the earth's magnetism.]

8. What would be the effect on the magnet's field on placing a small ring of soft iron (with its plane parallel to the plane of the paper) in the space between two north poles ? (C. U. 1909, '21, '24 ; cf. '38)

Art. 18.

9. Describe how you would proceed to determine the position of the poles of a bar-magnet. (C.U. 1922, '26, '30, '31)

Art. 19.

10. What is the force exerted between two magnetic poles of strength 32 and 36 units at a distance of 12 cms. from one another. (C. U. 1928)

[*Ans :* 8 dynes.]

11. Explain what is meant by saying that the pole-strength of a magnet is 50 unit. (C. U. 1949)

12. State the laws of action between magnetic poles. Two north poles repel one another with a force of 2·4 dynes, when their distance apart is 2 cms. What will be their distance apart, when the force is 3·6 dynes ? Find also the repulsive force when their distance apart is 3 cms. (C. U. 1916, '25 ; Dac. 1935)

[*Ans :* $d = 1·68$ cm. ; $F = 1·06$ dynes]

13. Magnetic N-poles of strength 50 and 90 units are placed at the corners B and C of an equilateral triangle ABC of side 10 cms. If a S-pole of strength 80 be placed at A, find the resultant force on A. (Pat. 1948)

[*Ans :* 98·3 dynes.]

CHAPTER III

Magnetic Measurements

21. Uniform Magnetic Field.—A magnetic field is said to be uniform when the strength of the field is everywhere the same, both in magnitude and direction. Such a field is represented by a system of lines of force which are parallel. In such a field a compass needle will vibrate at the same rate at all parts. *The forces exerted on a magnet in a uniform field form a couple.*

Expt.—Suspend a bar of steel in a suitable carrier by a long unspun silk thread. Allow the thread to pass through a hole (without

touching any side) in a horizontal plate fixed to the suspension frame. Now take away the bar, magnetise it, and carefully replace it in the carrier in exactly the same position as before, when the thread will be found to hang in the same way (*i e* without touching the plate). It will, however, turn to-and-fro about the thread and finally set itself along the magnetic meridian at the place. This shows that no resultant horizontal force acts on the magnet as a whole, but the forces form a couple. They tend only to rotate the magnet and not to move it bodily in any direction. Hence **the action of the uniform field is not translatory but only directive.**

22. Equality of the two Poles of a Magnet.—The two poles of a magnet may be proved to be of equal strength. Let m and m' be the pole strengths of the north and south ends of the magnet, then the forces acting on the poles, when placed in a uniform field of strength H (*e.g.* the horizontal component of the earth's field), are mH and $m'H$. Now, if a magnet is freely suspended at its centre of gravity, it will be found to set itself in the magnetic meridian and *there will be no tendency of the magnet to move as a whole along the direction of the field.* This fact can also be shown experimentally by placing a magnetic needle on a cork on the surface of a large bowl of water [Fig. 28 (a)]. *The cork will be found to rotate until the needle lies in the magnetic meridian,* *i.e.* points north and south, or, in other words, there will be *a motion of rotation* of the needle and *not of translation* (see also Ch. IV). In the position of equilibrium the forces, acting on the needle due to the earth's magnetic field, neutralise one another, *i.e.* they are equal in magnitude and opposite in direction,

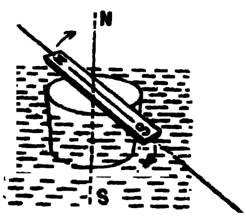


Fig. 28(a)

Thus, $\quad mH = -m'H\,; \quad$ or $\quad m = -m'\,;$

i.e. **the poles of a magnet are equal in strength but opposite in kind. This experiment also proves that the earth's field at a place is only directive.**

23. Non-Uniform Field.—In a non-uniform field the forces acting on a magnet are equivalent to (*i*) a *couple*, and (*ii*) a *force* acting at its centre of gravity, the result being a motion of translation and not merely a directive one.

In the above experiment it is seen that the floating magnetic needle simply rotates into the meridian as the field in this case is uniform. But if another magnet is brought near, the needle will ***rotate***

and also *move* towards the magnet, the latter movement being due to the force acting on the needle as a whole.

24. (a) Magnetic Moment.—The magnetic moment of a magnet is defined as the product of the strength of one of its poles and the distance between them.

If m be the pole strength of a magnet of length $2l$, the magnetic moment $M = 2ml$.

The effect of a magnetic field in causing a magnet to rotate about an axis depends upon the product $2ml$. *i e.* the **magnetic moment** of a magnet.

Couple on a Magnet placed in a Uniform Field.—Suppose a magnet NS of pole strength m and length $2l$ oscillates in the earth's uniform horizontal field (Fig. 29).

Let the axis of the magnet make an angle θ with the magnetic meridian at any instant of time, and let H gauss be the intensity of the uniform field. Then the force acting on the north pole of the magnet is mH dynes in the direction TN, and on the south pole there is a force mH dynes in the direction PS. These two forces are equal and parallel and constitute a couple.

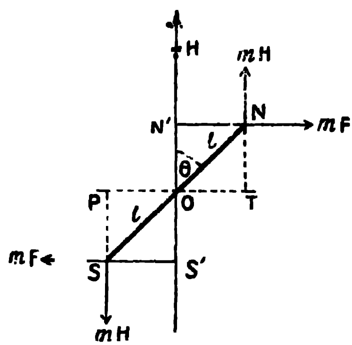


Fig. 29

The moment of the couple on NS = one of the forces × perpendicular distance between them $= mH \times PT$

$= mH \times 2l \sin\theta = 2mlH \sin\theta$

$= MH \sin\theta$, $(2ml = M =$ moment of the magnet).

This expression is equal to M, when $\theta = 90^\circ$ and $H = 1$.

Thus, **the moment of a magnet** *is numerically equal to the moment of a mechanical couple exerted on the magnet to keep the magnet at right angles to a uniform field of unit strength.*

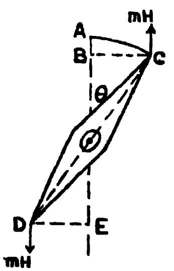


Fig. 30

(b) Work done in Deflecting a Magnet.—When a magnet CD placed in a field H is deflected through an angle θ (Fig. 30) the pole C moves through an arc AC, which is equivalent to the distance AB along the direction of the field ; the work done in moving the pole C against the magnetic force = force × distance $= mH \times AB$.

Hence the total work done on the two poles

$= 2mH \times AB = 2mH\ (AO - BO)$
$= 2mH\ (l - l \cos\theta) = 2mlH\ (1 - \cos\theta)$
$= MH\ (1 - \cos\theta)$, where $2l$ = length of the magnet,

So, when $\theta = 90°$, work done $= MH$, and when $\theta = 180°$, work done $= 2MH$.

25. Two Magnetic Fields at Right Angles to Each Other : Tangent Law :—Suppose a magnet NS (Fig. 29) is hung up in two cross-magnetic fields, one being the earth's magnetic intensity H along the meridian and the other of strength F acting at right angles to the meridian. The magnet will take up a position of equilibrium making an angle θ with the direction of the earth's magnetic field under the action of two couples (one consisting of two parallel forces mF, acting at each end of the magnet due to the force F, and the other of two parallel forces mH, due to the earth's field H). The moments of the two couples are equal and opposite when the needle is in equilibrium.

The moment of the couple due to the forces mF

$$= mF \times 2l \cos\theta = MF \cos\theta.$$

The moment of the couple due to the forces $mH = MH \sin\theta$.

Since the couples balances each other, $MF \cos\theta = MH \sin\theta$ [Art. 23(a)].

At the position of equilibrium, $MF \cos\theta = MH \sin\theta$.

Hence, $F = H \tan\theta$,

i. e. in this case *the tangent of the angle of deflection is equal to the ratio of F to H.*

Again, if H is a constant field (it is constant if it is taken to be the earth's field), we have, $\mathbf{F \propto \tan\theta}$.

This is the **Tangent Law.**

Note.—(*a*) The law is applied in many instruments where a compass-needle is deflected away from the magnetic meridian by a uniform magnetic field acting at right angles to it (see Art. 23, Part VII). In these cases each pole of the needle is acted on by two forces,—

(*i*) **a controlling force** *parallel to the meridian. The force is generally the horizontal component H of the earth's field.*

(*ii*) **a deflecting force** *F acting at right angles to the meridian.*

(*b*) It should be noted that in deducing the Tangent Law, the moment, and hence the pole strength m, of the needle has cancelled out, and so, in applying this law to magnetic experiments, **the actual strength of the needle used is a matter of no importance.**

Examples.—*1. A magnet 8 cms. in length lies in a field of intensity $H=0{\cdot}18$, and the strength of each of its poles is 5. Find the moment of the couple required to deflect it at right angles to the magnetic meridian.* **(C. U. 1932)**

The moment of the couple is given by $MH \sin \theta$. In this case $M=8\times5$; $H=0{\cdot}18$; $\sin\theta=\sin 90^\circ=1$. Thus the moment of the couple $=8\times5\times0{\cdot}18\times1=7{\cdot}2$ C.G.S. units.

2. A magnetic needle of moment 900 and pole strength 50 C.G.S. units is pivoted so that it is free to move in a horizontal plane where the earth's magnetic field is 0·36 gauss in this plane. It is in equilibrium at an angle of 30° from the meridians where it is pulled by a string attached to its north pole in the easterly direction. What is the tension of this string? **(Pat. 1932)**

If H be the horizontal field and T the tension of the string, the moment of the couple due to the pair of forces $mH=MH\sin\theta$, where θ is the angle of deflection and M the moment of the magnet (See Fig. 29). This couple is balanced by the force due to the tension T, the moment of which about the centre of the magnet $=T\times l\cos\theta$, where $2l$ is the length of the magnet. Hence, for equilibrium, $Tl\cos\theta=MH\sin\theta$; $\therefore$ Here $M=900$; $H=0{\cdot}36$; $\theta=30^\circ$; and $l=\dfrac{M}{2m}=\dfrac{900}{2\times50}=9$.

$$\therefore\quad T\times9\times\frac{\sqrt3}{2}=900\times0{\cdot}36\times\tfrac12\text{; whence } T=20{\cdot}8 \text{ dynes.}$$

3(a). A magnet is suspended horizontally in the magnetic meridian by a vertical wire which is untwisted. In order to deflect the magnet through 45° from the meridian the upper end of the wire has to be turned half round. Show how much the upper end has to be twisted to deflect the magnet 60° from the meridian.

For equilibrium two couples act on the magnet when it is deflected through any angle θ,—(*i*) a deflecting couple due to the torsion of the wire tending to turn it out of the meridian, the moment of this couple being proportional, within limits, to the angle of torsion, (*ii*) a controlling couple the moment of which is $MH\sin\theta$ tending to bring the magnet back into the meridian. So, we have,

(*i*) Angle of torsion $=(180^\circ-45^\circ)$, because the upper end of the wire is turned through 180°, while the deflection is 45°.

$$\therefore\quad (180^\circ-45^\circ)\propto MH\sin45^\circ \qquad \ldots \qquad \ldots \qquad \ldots \quad (1)$$

(*ii*) Again, if α be the required angle of torsion,

$$(\alpha^\circ-60^\circ)\propto MH\sin60^\circ \qquad \ldots \qquad \ldots \qquad \ldots \quad (2)$$

$$\therefore\ \text{From (1) \& (2)},\ \frac{(\alpha^\circ-60)}{135^\circ}=\frac{\sin60^\circ}{\sin45^\circ}=\frac{\sqrt3}{2}\div\frac{1}{\sqrt2}=\frac{\sqrt3}{\sqrt2}=1{\cdot}2.$$

$$\therefore\quad \alpha^\circ=(135\times1{\cdot}2)+60=222^\circ.$$

(b) Another magnet B is similarly suspended by the same wire as the magnet A in the above example and in order to deflect B through the same angle the upper end of the wire has to be turned once round. Compare the moments of A and B.

For the magnet A, we have $(180° - 45°) \propto M_1 H \sin 45°$, and for B, $(360° - 45°) \propto M_2 H \sin 45°$; $\therefore \frac{M_1}{M_2} = \frac{180° - 45°}{360° - 45°} = \frac{135}{315} = \frac{3}{7}$.

26. Magnetic Field due to a Bar-Magnet in Two Standard Positions.—

(i) **End-on Position.**—At this position the point P at which the intensity is to be measured is on the magnetic axis of the magnet (Fig. 31).

The intensity of the field at the point P is measured by the force exerted on an imaginary unit N-pole placed at P (Fig. 31).

Now a unit N-pole placed at P will experience (*i*) a force F_n due to the N-pole of the magnet repelling it from the magnet; and (*ii*) a force F_s due to S-pole drawing it towards the magnet.

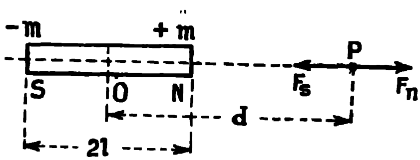


Fig. 31

Because F_n and F_s act in the same straight line, but in opposite directions, and the N-pole is nearer to P than the S-pole, F_n is greater than F_s. The resultant force F will act in the direction SN, and will be equal to $F = F_n - F_s$.

We have, $F_n = \frac{m \times 1}{NP_2}$; and $F_s = \frac{m \times 1}{SP^2}$.

Then $F = \frac{m}{NP^2} - \frac{m}{SP^2}$ (where m is the pole strength).

Let d denote the distance of P from the middle point O of NS, and $2l$ the length of the magnet, then, we have,

$$\mathbf{F} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = m\left\{\frac{(d+l)^2 - (d-l)^2}{(d-l)^2 (d+l)^2}\right\}$$

$$= \frac{4mld}{(d^2 - l^2)^2} = \frac{2\mathbf{M}\mathbf{d}}{(\mathbf{d}^2 - \mathbf{l}^2)^2} \ldots\ldots\ldots (1),$$ since $2ml = M$, the moment of the magnet. If l is very small compared with d, l^2 may be neglected compared with d^2, and the force becomes,

$$F = \frac{2Md}{(d^2)^2} = \frac{2M}{d^3} \text{ units of intensity per unit pole} \quad \ldots \quad \ldots \quad (2)$$

Thus the intensity of the magnetic field at a distant point P due to a short magnet in the End-on position is given by,

$$\mathbf{F} = \frac{2\mathbf{M}}{\mathbf{d}^3}.$$

(ii) Broad-side-on Position.—In this case the point P is on PO, the perpendicular on the axis of the magnet at its middle point. The magnetic force F_n at P due to N-pole $= m/NP^2$ in the direction NP, and F_s due to S-pole $= m/PS^2$ in the direction PS (Fig. 32).

Then the resultant force F acting on an imaginary unit N-pole situated at P is to be calculated to find the field strength there.

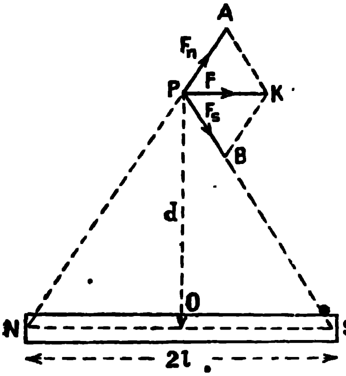


Fig. 32

Denoting the two forces F_n and F_s by the lengths PA and PB respectively and completing the parallelogram, the diagonal PK represents in magnitude and direction the resultant force F at P. Since the two triangles NPO and SPO are equal, we have $NP = SP$. Therefore, the forces F_n and F_s are equal and so the lengths PA and PB of the parallelogram $PAKB$ must be equal. Since $PA = PB$, the diagonal PK, which is the resultant, must bisect the angle APB and will be parallel to NS.

$\therefore$ If $\angle APB = \theta$, the resultant $PK^2 = PA^2 + PB^2 + 2PA.\ PB. \cos\theta$. (vide Art. 32, Part 1). That is,

$$PK^2 = 2PA^2 + 2PA^2 \cos\theta = 2PA^2(1 + \cos\theta).$$

$$= 2PA^2\left(1 + 2\cos^2\frac{\theta}{2} - 1\right)$$

$$= 4PA^2.\ \cos^2\frac{\theta}{2} = 4\left(\frac{m}{NP^2}\right)^2.\left(\cos APK\right)^2$$

$$= 4\left(\frac{m}{NP^2}\right)^2.\left(\cos PNO\right)^2 = 4\left(\frac{m}{NP^2}\right)^2.\left(\frac{NO}{NP}\right)^2 = \frac{4m^2l^2}{(NP^3)^2}$$

$$PK = F = \frac{2ml}{NP^3}.$$

or $$F = \frac{M}{NP^3} = \frac{M}{NP(d^2 + l^2)};\quad (NS = 2l \text{ and } PO = d)$$

$$= \frac{M}{\sqrt{(d^2 + l^2)}\,(d^2 + l^2)} = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}.$$

If l is very small compared with d, this becomes, $F = \dfrac{M}{(d^2)^{\frac{3}{2}}} = \dfrac{M}{d^3}$ units of intensity per unit pole.

$\therefore$ The intensity of the magnetic field at a distant-point P due to a short magnet in the Broad-side-on position is given by, $\mathbf{F} = \frac{\mathbf{M}}{\mathbf{d^3}}$.

Note.—*(i) The force at a point on the bisector of the axis is only half as great as the force at an equally distant point on the prolongation of the axis.*

(ii) The force depends on M. So the force remains unaffected by replacing the magnet with another of different length but equal moment, the axis being placed in the same direction.

27. Determination of Pole Strength and Magnetic Moment of a Magnet (i) By means of Neutral Points :—

When a bar-magnet is placed in certain positions relative to the magnetic meridian (Figs. 19—21), neutral points are obtained due to the neutralisation of the earth's horizontal magnetic force H at the points by the forces due to the poles of the magnet.

(*a*) In Fig. 20, the field F at the neutral point X due to the magnet $= H$.

But $F = \frac{2Md}{(d^2 - l^2)^2}$ [Art. 26 (i)]. $\therefore$ $H = \frac{2Md}{(d^2 - l^2)^2}$; and

$M = \frac{H(d^2 - l^2)^2}{2d}$; where d is the distance of X from the centre of the magnet. But $M =$ pole strength $m \times$ length of the magnet $(2l)$

$$\therefore \quad 2ml = \frac{H(d^2 - l^2)^2}{2d} ; \quad \text{or} \quad m = \frac{H(d^2 - l^2)^2}{4ld}.$$

Knowing the value of H, the value of M or m can be found.

(*b*) In Fig. 19, $F = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$ [Art. 26 (ii)]. $\therefore$ At the neutral point,

$$H = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} \quad \text{or} \quad M = H(d^2 + l^2)^{\frac{3}{2}} : \text{and} \quad m = \frac{H(d^2 + l^2)^{\frac{3}{2}}}{2l}.$$

(ii) Another Method.—In the end-on position, when the magnet is long, the force F at the neutral point may be regarded as entirely due to the north or south pole which is near to it and the force due to the other pole may be neglected.

At the neutral point, $F = H$. But $F = \frac{m \times 1}{d^2}$. $\therefore$ $\frac{m}{d^2} = H$.

or $m = d^2 H$, where d is the distance of the neutral point from the pole. Thus, knowing H, m can be found.

(iii) By Applying the Principle of Triangle of Forces.—

The following two examples will explain this.—

(1) *A bar-magnet with its pole 10 cms. apart lies along the magnetic meridian with its north pole pointing north. A neutral point is obtained 15 cms. from each pole. Find the pole strength of the magnet. The intensity of the earth's field is 0·18 gauss.*

See Fig. 18, where A is the neutral point due to the magnet NS.

By Art. 27 (ii), the resultant force $F = \dfrac{M}{(d^2+l^2)^{\frac{3}{2}}} = H$, at the neutral point. Here $(d^2+l^2) = (AS)^2$ or $(AN)^2 = 15^2$. $\therefore$ $(d^2+l^2)^{\frac{3}{2}} = 15^3$, and $M = m \times 10$, where m = pole-strength. $\therefore$ $\dfrac{10m}{15^3} = 0{\cdot}18$;

or, $m = \dfrac{0{\cdot}18 \times 15^3}{10} = 60{\cdot}75$ units.

Otherwise thus :—This can also be done by the application of the principle of "the triangle of forces". If a unit north pole be placed at the neutral point A (Fig. 18), it will be in equilibrium by the action of three forces, two due to the two poles, which are in this case equal, and the third due to the earth. These three forces can be represented in magnitude and direction by the sides SN, NA and AS of the triangle ASN ; of which SN drawn parallel to the horizontal field H in the direction S to N represents H, and similarly NA and AS drawn parallel to the directions of the other two forces represent the forces $m/15^2$ and $m/15^2$ respectively. According to the principle of the triangle of forces, the forces will be proportional to the sides to which they are parallel, *i e.*

$$\frac{m/15^2}{H} = \frac{AS}{SN}\,;\ \text{or}\ \frac{m/15^2}{0{\cdot}18} = \frac{15}{10}\,;\ \therefore\ m = \frac{15 \times 15^2 \times 0{\cdot}18}{10} = 60{\cdot}75 \text{ units.}$$

(2) In Fig. 22, where the magnet is placed in any position with respect to the earth's field, the pole strength can be calculated thus :— As in the last example, draw the sides CA, AB, BC, of the triangle CAB, parallel respectively to the directions of the forces m/d_1^2, m/d_2^2 and H, the earth's field, where $d_1 = AN$, $d_2 = AS$. So, we have, $\dfrac{m/d_1^2}{H} = \dfrac{AB}{BC}$; and $\dfrac{m/d_2^2}{H} = \dfrac{AB}{BC}$, from any of which m can be calculated after actually measuring the sides of the triangle and also distances of A from the poles of the magnet.

(iv) By Oscillation Magnetometer.—[*Vide* Art. 34 (ii)]

28. Action of a Magnetic Needle in two Magnetic Fields at Right Angles—The combined effect of the earth's horizontal field H and the field due to a distant bar-magnet lying east and west on a small pivoted or suspended magnetic needle is considered, and of this two important cases arise.

(i) Tangent A Position of Gauss.—A bar-magnet NS is placed so that its axis is perpendicular to the earth's field, and is so placed that the axis is in line with the centre of a small suspended magnetic needle $N'S'$ (Fig. 33), *i.e.* the centre O of a magnetic needle is at an **end-on** position with the magnet NS.

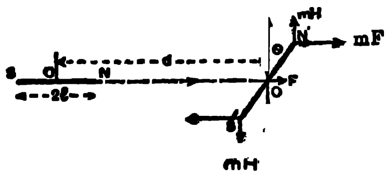


Fig. 33

If m be the pole strength of the magnetic needle $N'S'$, the needle will experience two couples,—one consisting of two parallel forces mH dynes due to the earth's field H acting on N' and S' parallel to the meridian, and the other, of two parallel forces $m \times F$ dynes, due to the field F of the bar-magnet acting on N' and S' perpendicular to the meridian. $N'S'$ being small, the field at N' and S' due to NS may be taken as equal to that at O The latter couple tends to set $N'S'$ at right angles to the meridian.

When the needle is in equilibrium making an angle θ with the direction of the earth's magnetic field, the moments of the couple are equal and opposite (see Art. 24).

The moment of the couple due to the forces mH, called **the controlling or restoring couple** $= M'P \sin\theta$, where M' is the magnetic moment of $N'S'$, and that due to the forces $m \times F$, called **the deflecting couple** $= M'F \cos\theta$.

Since the couples balance each other, $M'F \cos\theta = M'H \sin\theta$;

$$\text{or} \quad F = H \tan\theta. \quad \text{But } F = \frac{2Md}{(d^2 - l^2)^2} \quad \text{[Art. 26 (i)]}$$

(where M is the magnetic moment of NS).

$$\therefore \frac{2Md}{(d^2 - l^2)^2} = H \tan\theta; \quad \text{or} \quad \frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan\theta, \qquad \text{.............. (5)}$$

If l be very small compared with d, l^2 may be neglected, and this becomes $\frac{M}{H} = \frac{d^3}{2} \tan\theta$ ... ... (6)

(ii) Tangent B Position of Gauss.—In this case NS is placed at right angles to the earth's field and the centre of $N'S'$ is in line with the bisector of the axis of NS (Fig. 34), *i.e.* the centre of the needle $N'S'$ is at a **broad-side-on** position with the magnet NS.

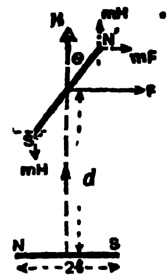


Fig. 34

As before, the moment of the couple due to the forces mH, *i.e.* **the restoring couple** $= M'H \sin\theta$, and that due to the forces $m \times F$, *i.e.* **the deflecting couple** $= M'F \cos\theta$.

Since the couples balance each other, $M'F \cos\theta = M'H \sin\theta$; *or* $F = H \tan\theta$.

But in this case, $F = \dfrac{M}{(d^2 + l^2)^{\frac{3}{2}}}$ [Art. 26 (ii)]

(where M is the magnetic moment of NS).

$$\therefore \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} = H \tan\theta; \quad or \quad \frac{\mathbf{M}}{\mathbf{H}} = (\mathbf{d}^2 + \mathbf{l}^2)^{\frac{3}{2}} \tan\theta. \qquad .(7)$$

If l be very small compared with d, this becomes,

$$\frac{\mathbf{M}}{\mathbf{H}} = \mathbf{d}^3 \tan\theta \quad \ldots \quad \ldots \quad .. \quad (8)$$

Note.—The above two cases are known as Tangent positions of Gauss as in each case the forces are at right angles, and, therefore, the **Tangent Law** (Art. 24) applies.

Problem.—*Two magnets of the same type, but of moments M and 2M, are mounted on a frame so as to form a cross. If the combination is suspended at the centre with a vertical fibre, find the direction in which it will set in the earth's magnetic field.*

Calculate also the intensity of the field at a distance d from the centre of the cross on the prolongation of one of the arms. **(Pat. 1923)**

Let NS be the magnetic meridian, and let α be the angle which the axis of the magnet N_1S_1 makes with NS, and β the angle which the axis of the other magnet N_2S_2 makes with NS (Fig. 35).

Let the couple on the magnet N_1S_1 (having moment $2M$) be $2MH$ *sin* α tending to rotate the system counter-clockwise, and that on the other (having moment M) be MH *sin* β tending to rotate the system clockwise into the magnetic meridian (see Art. 23). For equilibrium,

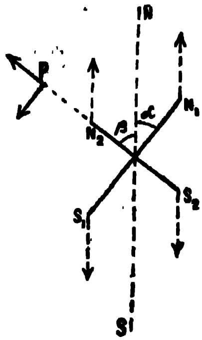


Fig. 35

$$2MH \sin \alpha = MH \sin \beta \text{.......................(1)}$$

$\therefore$ $2 \sin \alpha = \sin \beta$. But, because the angle between the two magnets is 90°, we have,

$\sin \beta = \cos \alpha$;

$\therefore$ $2 \sin \alpha = \cos \alpha$.........from (1) ; *or* $\tan \alpha = \frac{1}{2}$.

Hence the system sets with the magnet of moment $2M$ at an angle with the meridian whose tangent is $\frac{1}{2}$.

(*a*) Intensity at the point P at a distance d from the centre of the cross on the prolongation of the axis of S_2N_2 is $2M/d^3$ in the direction of N_2P produced, and that due to N_1S_1 is $2M/d^3$ in a direction *parallel* to the axis of N_1S_1 (see Art. 26, i and ii).

$$\text{The resultant intensity } R = \sqrt{\left(\frac{2M}{d^3}\right)^2 + \left(\frac{2M}{d^3}\right)^2} = \frac{2\sqrt{2}M}{d^3}.$$

(*b*) When the point P is placed at a distance d on the prolongation of the axis of N_1S_1, the intensity at P due to N_1S_1 is $4M/d^3$ in the direction of N_1P produced, and that due to N_2S_2 is M/d^3 in a direction parallel to the axis of S_2N_2.

$$\therefore \text{ The resultant intensity } R = \sqrt{\left(\frac{4M}{d^3}\right)^2 + \left(\frac{M}{d^3}\right)^2} = \frac{\sqrt{17}\,M}{d^3}.$$

Examples.—*1. A compass needle is placed 30 cms to the east of a small magnet and the needle is deflected through 45°. Calculate the moment of the magnet and the pole strength, if the length is 6 cms. The value of H may be taken as 0·352 gauss.*

We have, $\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \theta$. In this case, $H = 0{\cdot}352$; $d = 30 + 3 = 33$ cms. ; $l = 3$ cms. $\frac{M}{0{\cdot}352} = \frac{(33^2 - 3^2)^2 \times 1}{2 \times 33}$; or $M = 6220{\cdot}8$ units. Pole strength $\times$ length $=$ moment of the magnet ; $\therefore$ Pole strength $= \frac{6220{\cdot}8}{6} = 1036{\cdot}8$ units.

2. There is to be found a neutral point on the prolongation of the axis of a bar-magnet at the distance of 10 cms. from the nearest pole. If the length of the bar be 10 cms., and H = 0·36 C. G. S. unit, find the pole strength of the magnet. (*Pat. 1931*)

The resultant force $F = \frac{m}{10^2} - \frac{m}{20^2} = m\left(\frac{1}{100} - \frac{1}{400}\right) = \frac{3m}{400}$ dynes.

But this force F is exactly neutralised by the earth's horizontal field H, *i.e.* by a force of 0·36 dynes.

Thus, $\frac{3m}{400} = 0·36$; $\therefore$ $m = \frac{400 \times 0·36}{3} = 48$ units ; *or* Pole strength $= 48$ units.

Otherwise :—By Art. 27 (i), $F = \frac{2Md}{(d^2 - l^2)^2} = H$. Here $d = 10 + 5 = 15$ cms. ; $l = 5$ cms. $\therefore$ $M = \frac{0·36(15^2 - 5^2)^2}{2 \times 15} = 480$; $\therefore$ $m = \frac{M}{2l} = \frac{480}{10} = 48$ units.

3. *An unmagnetised steel needle is pivoted at its centre of gravity and rests horizontally. It is then magnetised and it is found that it no longer rests horizontally when pivoted at the same point ; when a load of 0·05 gm. is placed on the needle at a distance of 5 cms. from its centre of gravity, it becomes horizontal again. Calculate the magnetic moment of the needle.* [$H = 0·25$ *gauss* ; $g = 980$ *cm./sec*^{2} ; *Angle of dip* $= 30°$] *(Pat. 1943)*

Let m be the pole strength and $2l$ the length of the magnet, and V the vertical component of the earth's field.

Taking moments of the two forces about the C. G. of the needle, after it is magnetised, we have, $0·05g \times 5 = Vml$; or $0·05 \times 980 \times 5 = H \tan 30° \times ml = \frac{0·25}{\sqrt{3}} \times ml$.

$\therefore$ $ml = 980\sqrt{3}$. Hence $M = 2ml = 1960\sqrt{3}$ C. G. S. units.

29. Magnetometer.— It consists of a small compass needle pivoted, or suspended, at the centre of a graduated circle, divided into four quadrants each reading from 0° to 90°. The needle carries at its centre a long aluminium pointer at right angles to it. The needle is enclosed in a wooden box provided a glass-top through which the movement of the needle can be watched. The cover glass prevents air currents from disturbing the needle. The compass box is mounted on a wooden base provided with two long arms on either side, each being fitted with a metre scale so that the zero marks of both the scales begin from the centre of the needle. Usually there is a reflecting mirror at the bottom of the circular scale. The position of any end of the pointer is read against the scale ; when looked at vertically downwards it coincides with the image produced in the mirror. This type is known as a **Deflection Magnetometer** (Fig. 36).

A magnetometer can be used for *(a) comparing magnetic moments ; (b) comparing magnetic field strengths ; (c) verifying the law of inverse squares.*

30. Comparison of Magnetic Moments of two Magnets by Deflection Magnetometer.—This can be done by using a deflection magnetometer in the Tangent A position, or Tangent B position, of Gauss.

(a) Tangent A Position.—*(i) The magnetometer is placed so that its arms (i e. the metre scales) are at right angles to the magnetic meridian shown by the dotted line NS, and the ends of the pointer read zero-zero* (Fig. 36). This experiment requires the determination of the ratio M/H at the same place for each magnet, from which the ratio of the moments is obtained. Suppose two magnets, preferably of the same dimension, having moments M_1 and M_2, are taken for comparison. Place one of the magnets, east and west, on one of the arms of the magnetometer (as shown in Fig. 36), the centre of the magnet being at a certain distance d from the centre of the needle, and read the deflections for both ends of the pointer. Turn the magnet so that its north and south poles change their places and again read both ends of the pointer.

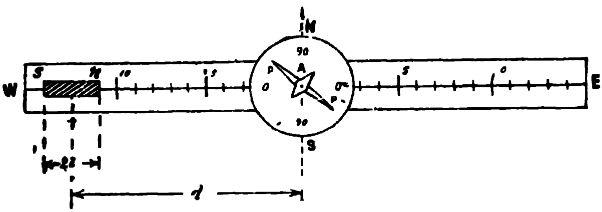


Fig. 36—The Tangent A Position of Gauss

Now place the magnet on the other arm of the magnetometer so that its centre is at the same distance (d) from the centre of the needle and repeat the above four observations. These eight observations may be repeated by placing the magnet upside-down in the two above places once again. Note the mean deflection θ_1 of these sixteen readings. The second magnet M_2 is placed so that its centre is kept exactly at the same distance with respect to the needle and the above observations are repeated. Let θ_2 be the mean of the sixteen readings. Then from formula (6), Art 28, we have,

$$\frac{M_1}{M_2} = \frac{d^3 \tan \theta_1}{d^3 \tan \theta_2} = \frac{\tan \theta_1}{\tan \theta_2},$$ the distance d being the same in both the cases. So, knowing $\tan \theta_1$ and $\tan \theta_2$, the moments can be compared.

Note.—*(i) M is directly proportional to $\tan \theta$.*

(ii) If l is not very small compared with d, then use formula (5) instead of (6), Art. 28.

(ii) Null Deflection Method.—In the arrangement of the previous experiment place one of the magnets on one of the arms of the magne-

tometer with its centre at a distance d_1 from the centre of the needle. Place the other magnet on the other arm of the magnetometer in such a way that the needle is brought back to its zero position, *i e.* until the deflection is made again zero, the distance being (say) d_2 cm.

Because the value of θ is the same, we have from formula (6), Art. 28,

$$\frac{M_1}{M_2} = \frac{d_1^3 \tan \theta}{d_2^3 \tan \theta} = \frac{d_1^3}{d_2^3}.$$

From formula (5), Art. 28, the relation becomes,

$$\frac{M_1}{M_2} = \frac{(d_1^2 - l_1^2)^2}{(d_2^2 - l_2^2)^2} \times \frac{d_2}{d_1}.$$

(b) Tangent B Position—(*i*) The moments can be compared also by using the tangent *B* position of Gauss. *The magnetometer is so arranged that the arms are in the magnetic meridian and the ends of the pointer read zero-zero* (Fig. 37). The magnet is placed *across* one of the arms, the centre of the magnet being placed at a particular distance *d* from the the centre of the needle (as shown in Fig. 37). The deflection θ_1 is noted, and the other magnet is also placed at the same place in the same way and deflection θ_2 is noted. Then from formula (8), Art. 28, we have, $\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}$

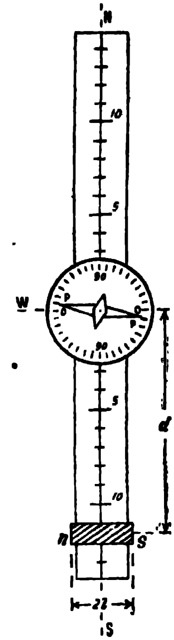


Fig. 37—
The Tangent B
Position of Gauss

(*ii*) *Null Deflection Method.*—Using the null deflection method as in case [Art. 30 a (*ii*)], we have, $\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$

Note.—(i) If long bar-magnets are used, then full formulæ (*i.e.* 5 and 7, Art. 28) must be used in the above two cases, (a) and (b).

(ii) In experiments with the deflection magnetometer, the tangent (A) position of Gauss is preferable to the (B) position, as the adjustments in (A) position can be carried out more accurately and also the deflection obtained with any given magnet for a given distance is greater.

(iii) In both the positions [*tan* (*A*) and *tan* (*B*)] the bar-magnet should always be placed east and west on the arm of the magnetometer.

31(a). Oscillation Magnetometer.—This instrument (Fig. 38) consists of a short magnet *M* enclosed in a wooden box having glass windows. The glass-case prevents air currents from disturbing the

oscillation of the magnet. The magnet is held in a double loop held by silk support and suspended horizontally from a torsion-head at the top of the tube.

Before beginning the experiment see that there is no torsion in the thread. The magnet is placed in the magnetic meridian and is made to oscillate by bringing another magnet at a suitable distance. The magnet now oscillates in the earth's field of intensity, say, H. Take the time t for a complete oscillation by means of a stop-watch. It can be shown theoretically that the period of oscillation t is given by the formula,

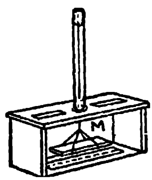


Fig. 38—Oscillation Magnetometer

$$\mathrm{t}=2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{MH}}}, \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where I = moment of inertia of the magnet ; M = magnetic moment of the magnet ; H = horizontal component of the earth's field. We have,

$$t^2 = 4\pi^2 \frac{I}{MH}; \quad \therefore \; t^2 \propto \frac{I}{H} \quad \ldots \quad (2)$$

If n be the number of oscillations of the magnet in a second and t the time for one oscillation, then,

$$n = \frac{1}{t} \quad \text{or} \quad n^2 = \frac{1}{t^2}; \quad \text{so, } t^2 = \frac{1}{n^2}. \quad \therefore \; \mathbf{H} \propto \mathbf{n^2} \quad \ldots \quad (2)$$

Note.—The constant I in formula (1) is called the **moment of inertia,** which can be determined thus. For a rectangular bar-magnet of length a, and breadth b, oscillating about an axis passing through its centre of gravity and perpendicular to its length, $I = m\left(\frac{a^2+b^2}{12}\right)$, where m is the mass of magnet. For a cyclindrical bar-magnet of length l and radius r, $I = m\left(\frac{l^2}{12}+\frac{r^2}{4}\right)$.

(b) Searle's Magnetometer.—This form of the magnetometer (Fig. 39), consists of a small cylindrical magnet (about 1·5 cms. long) fixed at the lower end of a massive brass cylinder which is suspended by a final thread of unspun silk so that the effect of torsion may be negligible. A long aluminium pointer (about 10 cms. long) fixed below the magnet enables the oscillations to be observed more easily. The heavy brass cylinder serves to increase the moment of inertia of the system so that the period of vibration becomes large enough to be accurately measured.

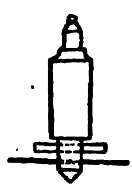


Fig. 39—Searle's Magnetometer

32. Comparison of Magnetic Moments of Two Magnets by the Oscillation Magnetometer.—

Method 1.—We have from equation (2), Art. 31,

$$M_1 H = \frac{4\pi^2 I_1}{t_1^2}, \text{ and } M_2 H = \frac{4\pi^2 I_2}{t_2^2}. \text{ Hence, } \frac{M_1}{M_2} = \frac{I_1}{I_2} \times \frac{t_2^2}{t_1^2}.$$

So the moments of two magnets can be compared by calculating the moments of inertia and finding the time for a complete oscillation of each magnet at the same place.

Method 2.—The disadvantage of the above method is that it necessitates the determination of I_1 and I_2 which may, however, be avoided by causing them to oscillate together as one system by placing them in two slots cut in a small wooden block, one above the other in the same vertical plane, first with their like poles, and then with their unlike poles, pointing in the same direction. In this way the moment of inertia of the whole oscillating system remains the same in both the cases. We have,

$$t_1 = 2\pi\sqrt{I/(M_1+M_2)H}\ ; \text{ and } t_2 = 2\pi\sqrt{I/(M_1-M_2)H}$$

$$\text{or } \frac{t_1}{t_2} = \sqrt{\frac{M_1-M_2}{M_1+M_2}}\ ; \quad \text{or } \frac{M_1-M_2}{M_1+M_2} = \frac{t_1^2}{t_2^2}\ ;$$

$$\text{or } \frac{M_1+M_2+M_1-M_2}{M_1+M_2-M_1+M_2} = \frac{t_2^2+t_1^2}{t_2^2-t_1^2}\ ; \quad \text{or } \frac{M_1}{M_2} = \frac{t_2^2+t_1^2}{t_2^2-t_1^2} \quad \ldots \quad (3)$$

But $n = \frac{1}{t}$, so the above relation can also be written as

$$\frac{M_1}{M_2} = \frac{n_1^2+n_2^2}{n_1^2-n_2^2}.$$

*__Examples.__—(1) *A small magnet oscillates in the earth's field (0·36 C.G.S. unit). A bar magnet placed end-on to it and east of it deflects it through 60°; what will be the strength of the resultant field? If the rate of oscillation in the earth's field was 10 per minute, what will be the new rate of oscillation?*

(*Pat. 1926*)

The two fields, one H due to the earth and the other F due the bar-magnet, are acting at right angles to each other (see Fig. 29). Then the resultant field R is given by $R^2 = F^2 + H^2$.

Here $F = H \tan\theta = 0{\cdot}36 \times \tan 60° = 0{\cdot}36 \times \sqrt{3}$; $\therefore R^2 = (0{\cdot}36 \times \sqrt{3})^2 + (0{\cdot}36)^2 = (0{\cdot}36 \times 2)^2$; $\therefore R = 0{\cdot}36 \times 2 = 0{\cdot}72$ C. G. S. unit.

In this field H, the number of oscillations was 10 per minute. $\therefore H \propto 10^2$.

If n be the number of oscillations in the new field R, $R \propto n^2$.

Hence $\frac{R}{H} = \frac{n^2}{10^2}$; $\therefore \frac{0{\cdot}72}{0{\cdot}36} = \frac{n^2}{100}$, whence $n = 10\sqrt{2}$.

(2) *An iron bar 100 cms. in length and 1 mm. × 1 mm. in section, is uniformly magnetised and its period of vibration is found to be 5 seconds. It is then broken into two equal halves. What will be the period of vibration of each half?* (Pat. 1940)

We have (Art. 31), $t_1 = 2\pi\sqrt{\dfrac{I_1}{M_1 H}}$; and $t_2 = 2\pi\sqrt{\dfrac{I_2}{M_2 H}}$,

where t_1 is the period of the iron bar and t_2 is the period of each half.

$$\therefore \frac{t_2}{t_1} = \frac{\sqrt{I_2 \times M_1}}{\sqrt{I_1 \times M_2}}; \quad I_1 = \left(\frac{100^2 + (0{\cdot}1)^2}{12}\right)W$$

$I_2 = \left(\dfrac{(50)^2 + (0{\cdot}1)^2}{12}\right) \times \dfrac{W}{2}$; $M_1 = 100m$; $M_2 = 50\ m$ (where m is the pole strength). $\therefore \dfrac{t_2}{5} = \sqrt{\dfrac{2500{\cdot}01}{1000{\cdot}01}}$; whence $t_2 = 2{\cdot}5$ secs.

(3) *Two bar-magnets, the moment of one of which is double that of the other, but otherwise similar, are arranged parallel one above the other, first with their like poles in contact and then with their unlike poles in contact. Find the ratio of the periods of vibration of the combination in the same magnetic field.* (Pat. 1941)

Proceed as in Art. 32 (*b*). Here $\dfrac{t_1}{t_2} = \sqrt{\dfrac{2M - M}{2M + M}} = \sqrt{\dfrac{1}{3}}$;

$$\therefore t_1 : t_2 :: 1 : \sqrt{3}.$$

33. Verification of the Law of Inverse Squares —

(1) By Method of Oscillation ; Vibration Magnetometer.

Expt.—A magnetic needle, say, a searle's magnetometer (Fig. 39), is made to oscillate at a point under the action of the earth's field H alone. Let n be the number of oscillations per minute. Then, we have,

$$H \propto n^2 \quad \ldots \quad \ldots \quad (1)$$

Let a long magnet, preferably ball-ended, be placed vertically to the north of the needle with its south pole turned towards the north pole of the needle. The magnet being long, the north pole of the magnet will have very little influence on the deflection on the needle and the field at the point may be regarded as due to the south pole only. The south pole of the magnet is to be placed in the same horizontal plane as the needle and at a distance (say) d_1 from the needle.

So the needle is oscillated under the joint influence of the field due to the magnet and due to the earth's horizontal field H. Here the total field at the point is the sum of the two fields. The magnet being long, the effect of the other pole on the needle is neglected.

Let the number of oscillations under the action of the combined field be n_1 and let F_1 be the field due to the pole of the magnet, then

$$(F_1 + H) \propto n_1^2 \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The magnet is then moved to a distance d_2 from the needle, and let n_2 be the corresponding number of oscillations per minute. If F_2 be the field due to the magnet in this position, $(F_2+H) \propto n_2^2$...(3)

From (1) and (2), $\frac{F_1+H}{H}=\frac{n_1^2}{n^2}$; or $\frac{F_1}{H}=\frac{n_1^2-n^2}{n^2}$ ... (4)

Again, from (1) and (3), $\frac{F_2+H}{H}=\frac{n_2^2}{n^2}$; or, $\frac{F_2}{H}=\frac{n_2^2-n^2}{n^2}$ ... (5)

$\therefore$ From (4) and (5), $\frac{F_1}{F_2}=\frac{n_1^2-n^2}{n_2^2-n^2}$.

But by actual experiment it will be found that,

$$\frac{n_1^2-n^2}{n_2^2-n^2}=\frac{d_2^2}{d_1^2}; \quad \therefore \frac{F_1}{F_2}=\frac{d_2^2}{d_1^2};$$

or, $F_1 : F_2 :: \frac{1}{d_1^2} : \frac{1}{d_2^2}$, which proves the Law of Inverse Sqs.

[**Note** :—If the magnet had been placed north of the needle with its *N*-pole turned towards *N*-pole of the needle, the field at the place would have opposed the earth's field, *i.e.* the total field would have been F_1-H, instead of F_1+H, assuming F_1 to be greater].

(2) **By Coulomb's Torsion Balance.**—It consists of a cylindrical glass case *G* (Fig. 40), graduated in degrees along its middle, having a co-axial glass-tube attached at the middle of its top. The tube is provided with a screw-head *P* by which any rotation can be given to the magnet *AB*, which is suspended from it by means of a fine silver thread. The rotation of the magnet is read from the scale on the glass case, and that of the screw-head, from a scale provided at the top of the tube. Through a slot on the top of the case another magnet *C* is introduced vertically so that its lower end is in level with the suspended magnet.

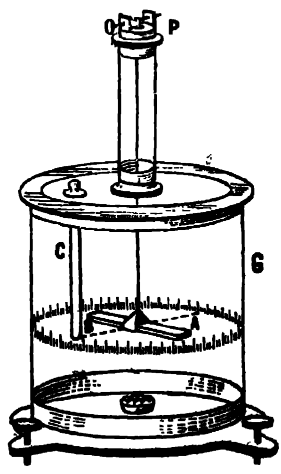


Fig. 40

On removing *C*, the top-screw *P* is rotated until the magnet is brought to the magnetic meridian. From this position, the top-screw is slowly turned until the magnet is deflected through one degree. The controlling force of the earth as a magnet is thus determined in terms of the

torsional couple in the wire. Suppose θ° rotation of the screw-head corresponds to 1° rotation of the magnet against the earth's field.

Bring the suspended magnet back into the meridian by rotating the screw-head in the opposite way. Next introduce the similar pole of the magnet C such that it is just in level with the magnet AB. Suppose, thereby, that the magnet AB deflects through α° which equals $(\alpha\theta + \alpha)^\circ$ in terms of torsional couple. Now rotate the screw-head in the opposite direction until the deflection becomes one half, *i.e.* $\alpha/2$. If the necessary rotation of the screw-head be β, the torsional equivalent of it will be $\left(\beta + \frac{\alpha}{2} + \frac{\alpha\theta}{2}\right)^\circ$. Now it may be supposed that for small rotations, α is proportional to the distance between the two poles. From the experiment it will be found that $\dfrac{\beta + \frac{\alpha}{2} + \frac{\alpha\theta}{2}}{\alpha\theta + \alpha} = \dfrac{4}{1}$, when the angular separation, *i.e.* the distance between the poles, is made half. This proves the inverse sq. law. The effect of the distant poles being small is neglected in this experiment.

(3) **Graphical Method.**—A bar magnet NS is placed on a sheet of paper fixed on a drawing board, and its outline is drawn by a pencil. Let N and S be the positions of north and south poles respectively of the bar-magnet (Fig. 41). A compass needle is now placed with its centre at a point P in the field due to the magnet. In this position the needle is acted on by two opposite couples—one, due to the earth's field acting parallel to the magnetic meridian, and the other due to the bar magnet. The board is now turned until the needle becomes parallel to the magnetic meridian (the direction of which should be determined in the beginning by stretching a thread between two long vertical pins fixed outside the board). In this position the couple due to the earth's field vanishes and the direction of the needle, say, PK indicates the direction of the resultant of the two magnetic forces due to the two poles. Join NP and SP, and produce NP to A.

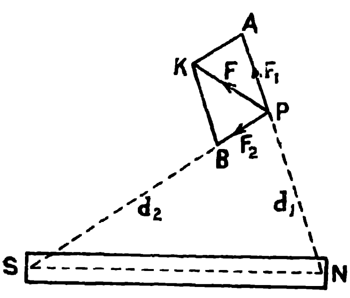


Fig. 41—Graphical Method

We know that if a unit north pole be placed at P, and if m be the pole strength of the magnet, then, by the law of inverse squares, the

force of repulsion $F_1 = m/NP^2$. This force acts in the direction NP. Again, the force of attraction $F_2 = m/SP^2$. This force acts along PS.

$$\therefore \quad \frac{F_1}{F_2} = \frac{m}{NP^2} \div \frac{m}{SP^2} = \frac{SP^2}{NP^2}. \qquad \ldots (1)$$

Take any point K on the resultant, and draw KA and KB parallel to SP and NA respectively. Then, $F_1 \propto PA$, $F_2 \propto PB$;

$$\text{or,} \quad \frac{F_1}{F_2} = \frac{PA}{PB}. \quad \therefore \text{ From (1),} \quad \frac{F_1}{F_2} = \frac{PA}{PB} = \frac{SP^2}{NP^2} \qquad \ldots (2)$$

This can be verified by actually measuring the distance NP, SP, and PA, PB. So it is proved that $\frac{F_1}{F_2} = \frac{SP^2}{NP^2} = \frac{d_2^2}{d_1^2}$ (if $NP = d_1$ and $SP = d_2$). Thus $F_1 : F_2 = \frac{1}{d_1^2} : \frac{1}{d_2^2}$. Prove this for other points also in the field of the magnet.

This proves the Law of Inverse Squares.

(4) Deflection Magnetometer Method.—It has been proved in Art. 26 that the field F_1 due to a small magnet in the "end-on" position is twice the field F_2 due to the same magnet in the "broad side-on" position at the same distance, and this result was obtained by assuming the truth of the law of Inverse Squares for a magnetic pole.

Now place a magnet in the "end-on" position in a magnetometer experiment and let θ_1 be the mean of the sixteen readings as taken in Art. 30. Place the same magnet in the "broad side-on" position, and let θ_2 be the mean of the sixteen readings now.

We have, $F_1 = \frac{2M}{d^3}$; $F_2 = \frac{M}{d^3}$, when d is large compared to the length of the magnet; $\therefore \frac{F_1}{F_2} = \frac{2}{1}$.

$$\text{Again, } F_1 = H \tan\theta_1 \text{ ; } F_2 = H \tan\theta_2 \text{ ;} \qquad \therefore \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{F_1}{F_2} = \frac{2}{1}.$$

So, if it can be proved experimentally that $\tan\theta_1$ and $\tan\theta_2$ are in the ratio of 2 to 1, then the Law of Inverse Squares is proved.

34. Comparison of the Earth's Fields at Two Places.—

(i) By the Deflection Magnetometer.—Use a deflection magnetometer, say, at the Tangent A position and note the deflection θ_1 with the centre of a bar-magnet placed at a distance d. Again, let θ_2 be the

deflection with the same magnet placed in the same way in another place. If H_1 and H_2 denote the field strengths of the earth at the two places, we have from eq. (6), Art. 28,

$$H_1 = \frac{2M}{d^3} \times \frac{1}{\tan\theta_1} \; ; \; H_2 = \frac{2M}{d^3} \times \frac{1}{\tan\theta_2}.$$

Since the same magnet is used in the two places in the same way, M and d remain the same. Hence, $\dfrac{H_1}{H_2} = \dfrac{\tan\theta_2}{\tan\theta_1}$.

[**Note.**—H is inversely proportional to $\tan\theta$].

(ii) By the Oscillation Magnetometer.—By noting the number of oscillations n_1 of the needle in a given time at a place where the earth's field is H_1, and also noting the corresponding values n_2 and H_2 at the second place, we have from Art. 31,

$$H_1 \propto n_1^2, \text{ and } H_2 \propto n_2^2 \; ; \quad \therefore \quad \frac{H_1}{H_2} = \frac{n_1^2}{n_2^2}.$$

Again, if t_1 be the time of one complete oscillation in a field of intensity H_1, and t_2 that in H_2, we have, $\dfrac{H_1}{H_2} = \dfrac{t_2^2}{t_1^2}$.

35. (a) Comparison of Pole Strengths of Two long Magnets by the Oscillation Magnetometer.—From eq. (4), Art. 33, we get,

$$F_1 = H\frac{n_1^2 - n^2}{n^2}.$$

If m_1 be the pole strength of a long magnet, d the distance of the long magnet from the oscillating needle, $F_1 = \dfrac{m_1}{d^2}$. Hence $m_1 = d^2 H \dfrac{n_1^2 - n^2}{n^2}$. The pole strength m_1 may, therefore, be known if H be known. Again, repeating the above experiment for another long magnet of pole strength m_2, keeping it in the same position and at equal distance d, we have, $m_2 = d^2 H \dfrac{n_2^2 - n^2}{n^2}$, if n_2 be the number of oscillations per minute now. Therefore, $\dfrac{m_1}{m_2} = \dfrac{n_1^2 - n^2}{n_2^2 - n^2}$.

(b) Determination of Pole Strength of a Magnet.—By using any method to determine M/H, M will be known when H is known, from which the pole strength m of a magnet can be calculated by the relation $M = 2ml$, where $2l$ is the length between the two poles.

Examples.—(1) *Two magnets A and B are caused to oscillate in the same magnetic field. A performs 15 vibrations per minute and B 10 vibrations per minute. The magnet A is then caused to oscillate in one magnetic field and B in another : A now performs 5 vibrations per minute and B 20 vibrations per minute. Compare the intensities of the fields in which A and B now oscillate, and compare also the magnetic moments of the magnets.* (*Pat. 1931*)

Let the intensity of the first field in which both A and B oscillate be H_1, the intensity of the second field in which only A oscillates be H_2, and that in which only B oscillates be H_3 ; then for A, $H_1 \propto 15^2$ and $H_2 \propto 5^2$.

$\therefore \dfrac{H_2}{H_1} = \dfrac{5^2}{15^2} = \frac{1}{9}$ (see Art. 31). Similarly for B, $H_1 \propto 10^2$; and $H_3 \propto 20^2$,

$\therefore \dfrac{H_3}{H_1} = \dfrac{20^2}{10^2} = \frac{4}{1}$. $\therefore \dfrac{H_2}{H_3} = \dfrac{H_2}{H_1} \div \dfrac{H_3}{H_1} = \frac{1}{9} \div \frac{4}{1} = \frac{1}{36}$; *or,* $H_2 : H_3 :: 1 : 36$.

That is, the intensity of the second field where B oscillates is 36 times as great as that where A oscillates a second time.

The formula for the vibrating magnet A is given by $t_1 = 2\pi\sqrt{\dfrac{I}{M_1 H}}$ ...(1),

where t_1 is the time for one complete vibration, M_1 the magnetic moment and H the field in which A and B oscillate.

Similarly, for magnet B, $t_2 = 2\pi\sqrt{\dfrac{I}{M_2 H}}$ ... ... (2)

assuming the magnets to have the same moment of inertia.

From (1) and (2), $\dfrac{M_1}{M_2} = \dfrac{t_2^2}{t_1^2} = \dfrac{n_1^2}{n_2^2}$, where n_1 and n_2 are the number of vibrations performed in one minute by A and B respectively. $\therefore \dfrac{M_1}{M_2} = \dfrac{15^2}{10^2} = \dfrac{9}{4}$.

2. *A small magnet vibrating horizontally in the earth's field makes 4 vibrations in 16 seconds, and when another magnet is brought near it, it makes 5 vibrations in 16 seconds. Compare the intensity of the field due to the magnet with the earth's horizontal field, (i) when the two fields are in the same direction, and (ii) when they are in opposite directions.*

Let H_1 be the earth's field, and H_2 the resultant field.

Here the number of vibrations per second n_1 in the field $H_1 = \dfrac{4}{16} = \dfrac{1}{4}$;

and the number of vibrations per second n_2 in $H_2 = 5/16$.

We have $H_1 \propto n_1^2 = \frac{1}{16}$; and $H_2 \propto n_2^2 = \dfrac{25}{16 \times 16}$; $\therefore \dfrac{H_2}{H_1} = \dfrac{25 \times 16}{16 \times 16} = \dfrac{25}{16}$.

(*i*) When both the fields are in the same direction, the field due to the magnet only $= H_2 - H_1$. We have $\dfrac{H_2}{H_1} = \dfrac{25}{16}$; $\therefore \dfrac{H_2 - H_1}{H_1} = \dfrac{25 - 16}{16} = \dfrac{9}{16}$.

(*ii*) When the fields are in opposite directions, the field due to the magnet $= H_2 + H_1$. $\therefore \dfrac{H_2 + H_1}{H_1} = \dfrac{25+16}{16} = \dfrac{41}{16}$.

3. *A compass needle makes 10 vibrations in 9 seconds in the earth's field alone. It makes 10 vibrations in 12 seconds when a bar-magnet is placed near it and finally comes to rest in the same direction as before. What will be the period of oscillation of the needle when the bar-magnet is reversed ?*

Let H be the field due to the earth and F that due to the magnet.

We know from Art. 31 that the intensity of the field in which the needle vibrates is proportional to the square of the number of oscillations per second. So the intensity of the second field which is the resultant of the two fields is less than the first as the number of oscillations per second diminishes, and because the needle comes to rest in the same direction as before $H > F$. So, the resultant field, $(H-F) \propto (\frac{10}{12})^2$; and $H \propto (\frac{10}{9})^2$;

$$\therefore \frac{H-F}{H} = \frac{(\frac{10}{12})^2}{(\frac{10}{9})^2} = \frac{9}{16}\ ; \quad \text{or } F = \frac{7}{16}H.$$

In the second case, the intensity of the resultant field $= (H+F)$.

$$\therefore \frac{H+F}{H} = \frac{n^2}{(\frac{10}{9})^2} = \left(\frac{9}{10}\right)^2 n^2\ ; \text{ or, } \frac{H+\frac{7}{16}H}{H} = \frac{23}{16} = \left(\frac{9}{10}\right)^2 n^2\ ; \text{ or } n = 1{\cdot}33.$$

If t be the period of oscillation, we have $t = \dfrac{1}{n} = \dfrac{1}{1{\cdot}33} = 0{\cdot}752$ second.

36. Determination of the Earth's Magnetic Field (H) and of the Magnetic Magnet (M) of a Magnet in Absolute Measure.—The measurement of the moment of a magnet, or the strength of any field, in terms of the units of these quantities, is called an *absolute* measurement, and is quite different from measurements of the *comparative* kind as done in the above articles, where it has been determined *how many times* magnetic moment of one magnet, or strength of one field, was greater than another.

(i) Using a deflection magnetometer, an equation for M/H is obtained by formula (6), or (8), of Art. 28. Then $M/H = a$ (say).

Again, by determining the moment of inerita I, and the period of vibration t of the same magnet at the same place by using the oscillation method, a value for MH is obtained from Art. 31.

Let $MH = b$. Now to find H, divide b by a, *i e.* $MH \div M/H = H^2 = b/a$.

$$\therefore \mathbf{H} = \sqrt{\frac{\mathbf{b}}{\mathbf{a}}}.$$

(ii) To find M, multiply a and b : *i.e.* $M/H \times MH = M^2 = ab$

$$\therefore \mathbf{M} = \sqrt{\mathbf{ab}}.$$

[M can also be determined by means of neutral points (See Art. 27)]

Note.—The *pole strength* m of the magnet may be found by determining M as above, and by knowing the length $2l$ of the magnet, $m = M/2l$.

Questions

Art. 21.

1. Describe the behaviour of a freely suspended magnet in a uniform magnetic field after it is disturbed from its position of rest. (Pat. 1932)

2. What is meant by uniform magnetic field ? Explain why a magnetic needle does not tend to move bodily along the lines of force in a uniform magnetic field. (C. U. 1923 ; cf. All. '30)

Art. 22.

3. What is meant by the statement that the strength of a magnetic pole is 'm' units ? How would you show experimentally that the two poles of a bar-magnet are of equal strength ?

[*See* also Art. 20]

Art. 23.

4. Define the term : Moment of a magnet. (Utkal 1948 ; Pat. 1930 ; All. '46)

How would you calculate the moment and the intensity of magnetisation in a horse-shoe magnet ? (Pat. 1930)

5. Define the magnetic moment of a magnet. Find an expression for the moment of the couple acting on a magnet placed in the earth's horizontal magnetic field when it is deflected through an angle θ from the magnetic meridian. (C. U. 1932)

6. A magnet placed at an angle of $30°$ with a uniform field of intensity 0·32 experiences a couple whose moment is 8 ; calculate the magnetic moment of the magnet and, the length of the magnet being 5 cms., calculate also its pole-strength.

[*Ans* : Moment of magnet = 50. Pole-strength = 10].

Art. 24

7. State and prove the law of tangents. How would you verify them experimentally ? (Pat. 1936)

(For verification see Art. 33(4)].

Art. 25.

8. What do you understand by intensity of magnetisation ? (Pat. 1943 ; All. '44)

Arts. 26 & 27.

9. Prove that the magnetic force due to a short magnet at great distances compared with its length varies inversely as the cube of the distance for points

along the direction of its axis, or in the plane through the centre of the magnet perpendicular to its axis. How would you verify these results experimentally?
(Pat. 1921, '23)

[**Hints.**—For verification place a magnet at two different distances d_1 and d_2 from the centre of a magnetometer needle. Let F_1 and F_2 be the corresponding forces, then $F_1 = \frac{2M}{d_1^3}$; $F_2 = \frac{2M}{d_2^3}$. $\therefore \frac{F_1}{F_2} = \frac{d_2^3}{d_1^3}$.

But $\frac{F_1}{F_2} = \frac{H \tan \theta_1}{H \tan \theta_2} = \frac{\tan \theta_1}{\tan \theta_2}$; where θ_1 and θ_2 are deflections corresponding to F_1 and F_2.

It will be actually found that $\frac{\tan \theta_1}{\tan \theta_2} = \frac{d_2^3}{d_1^3}$. This verifies the law.]

10. Find the strength of the field due to a bar-magnet at a point on the line bisecting the magnet at right angles.
(U. P. B. 1943 ; P. U. 1918 : Bom. U. 1928 : Pat. 1944)

Describe and explain the behaviour of a small compass needle placed at that point, if the bar-magnet is placed with its north pole pointing due east.
(Pat. 1911)

[For the first part see Art. 26 (ii). For the second part see Art. 28 (ii).]

11. A short bar-magnet is placed in the magnetic meridian with its north pole pointing south. The neutral point is 24 cms. north of the south pole of the magnet, and upon the prolongation of its axis. Find the intensity of the field at a point on this axis 20 cms. from the south pole and north of it. ($H = 0{\cdot}18$) (Pat. 1929)

[**Hints.**—At the neutral point, $F = H$. But $F = 2M/24^3$ [Art. 26 (i)].

$\therefore \frac{2M}{24^3} = H = 0{\cdot}18$: or, $M = \frac{0{\cdot}18 \times 24^3}{2}$. $\therefore$ The intensity of the field (due to the magnet only) at a distance of 20 cms. from the south pole of the magnet of moment $M = \frac{2M}{20^3} = \frac{2 \times 0{\cdot}18 \times 24^2}{2 \times 20^3} = 0{\cdot}31$ gauss. So, the resultant intensity (due to the earth and the magnet) $= 0{\cdot}31 - 0{\cdot}18 = 0{\cdot}13$ C.G.S. units (Here the earth's field is opposite to that of the magnet).]

12. AB is a thin magnet 20 cms. long, the strength of each of its poles being 12 units. Upon AB as base an equilateral triangle is constructed. Find the magnitude and direction of the force that a unit pole would experience, if it were placed at C; also the force upon the magnet caused by the unit pole at C. (Pat. 1931)

[*Ans* : 0·03 gauss]

13. A thin magnet of 20 cms. length with its north pole pointing south just balances the earth's field (magnitude 0·2 C. G. S. unit) in its plane at a

distance of 20 cms. from its pole. Find its magnetic moment and pole strength. (Pat. 1934)

[*Ans* : $M = 2133{\cdot}3$ units ; $m = 106{\cdot}6$ units.]

14. A magnet whose poles are 12 cms. apart is placed in the magnetic meridian. The field due to this magnet counterbalances the earth's horizontal field (0·35 C. G. S. unit) at a point 10 cms. from each pole. Find the pole strength of the magnet. (Pat. 1944)

[*Ans* : 29·17 units]

15. A bar-magnet, having poles 10 cms. apart, is placed in the magnetic meridian with the north pole pointing south. The neutral point is at a distance of 20 cms. from the nearer pole. Find the intensity of the resultant field at a point on the perpendicular bisector of the axis of the magnet and at a distance of 10 cms. from the centre of the magnet. [$H = 0{\cdot}4$ gauss.]

[*Ans* : 2·46 gauss. (Note that H should be added)] (Pat.1945)

Art. 28.

16. What is meant by the tangent positions of Gauss ? Explain how, in one of these positions, the relation between the movement of a bar-magnet and the horizontal component of the earth's magnetic field is obtained. (Pat. 1938)

Art. 30.

17. Define Magnetic moment.

In an experiment with a magnetometer a small magnet A produces a deflection of 30° when it is placed at a distance of 40 cms. from the centre of the magnetometer needle in the tangent A position of Gauss ; another small magnet B produces a deflection of 20° when it is in the tangent B position of Gauss, and at a distance of 30 cms. Compare the moments of the two magnets. (Pat. 1927)

[*Ans* : 1·17 nearly]

18. Define the magnetic moment of a magnet. Describe how you would compare the magnetic moments of two magnets. (C. U. 1944 ; Dac. 1943 ; Pat. '43, 45 ; All. '44)

18(a). Describe some form of magnetometer, and explain how you could use it to compare the magnetic moments of two bar magnets. (Pat. '47)

Art. 31.

19. Describe an oscillation magnetometer, and explain its uses. (All. 1937, '32 ; Pat. 1940, '45)

*20. A compass needle makes 10 oscillations per minute in the earth's field alone, and 20 oscillations per minute when the north pole of a long bar-magnet is placed to the south of the needle in the same level with it and at a distance of 4 inches from it. The distance of the magnet is then reduced to 3 inches. How many oscillations per minute will the needle now make ?

[*Ans* : 25·18]

21. A compass needle makes 30 oscillations per minute in the earth's field. When a bar-magnet is placed near it so as not to alter the direction of the needle, it makes 40 oscillations in one minute. How many times will the needle oscillate per minute if the magnet is reversed ? (Pat. 1944)

[*Ans :* $10\sqrt{2}$]

22. A small magnetic needle, suspended by means of a silk-fibre, makes 20 oscillations per minute in the earth's field. When a bar-magnet is placed in the magnetic meridian near the needle, the number of oscillations is increased to 25 per minute. If the bar-magnet be reversed and replaced in its former position, how many oscillations will the needle make per minute ?

[*Ans :* $5\sqrt{7}$] (Pat. 1947)

Art. 33.

23. How would you prove the law of inverse squares for magnetic forces ; given (*a*) a magnetised rod of steel about a metre long, (*b*) a small suspended magnetic needle, (*c*) a measuring rod, and (*d*) a stop watch.

[C. U. 1910 ; Cf. Pat. 1918, '44)

24. How would you prove experimentally that the force of attraction or repulsion between two magnetic poles varies inversely as the square of the distance between them ? (C. U. 1934, '37 ; Pat. '48)

Art. 34.

25. Explain the method of comparing the intensities of magnetic fields by the observations of the times of oscillation of a magnetic needle.

(All. 1927, '32 ; Pat. 1929)

26. Describe some form of deflection magnetometer and explain how you would use it to compare the earth's horizontal field at two places. (Pat. 1942)

Art. 36.

27. Describe a method of measuring the moment of a magnet. (All. 1932)

28. How is the horizontal component of the earth's magnetic field at any place determined in absolute measure ? (Pat. 1941)

29. How would you determine the horizontal intensity in your laboratory with the help of a deflection magnetometer and a suspension arrangement. Give a complete theory. (Pat. 1944)

30. A bar-magnet has a pole-strength of 50 units, and the distance between its poles is 10 cms. Mention and describe any method you would use to verify the value of the pole-strength. (C. U. 1949)

[**Hints.**—Use either the method of *neutral points* (Art. 27(i)] or the method described in Art. 36 to determine the moment of the magnet and see if it comes out to be equal to 50×50 *i.e.* 500 units or not.]

CHAPTER IV

Terrestrial Magnetism

37. The Magnetic Field of the Earth.—A compass needle, or a magnet suspended horizontally at its centre of gravity, invariably sets in a particular direction with its magnetic axis approximately north and south. This indicates the existence of a magnetic field on the surface of the earth.

If a compass needle, which is suspended *perfectly freely* at its centre of gravity, is taken from one pole of the earth towards the other, it will be found that the angle between the horizontal and the magnetic axis of this needle, *i.e.* the angle of inclination, will change from place to place

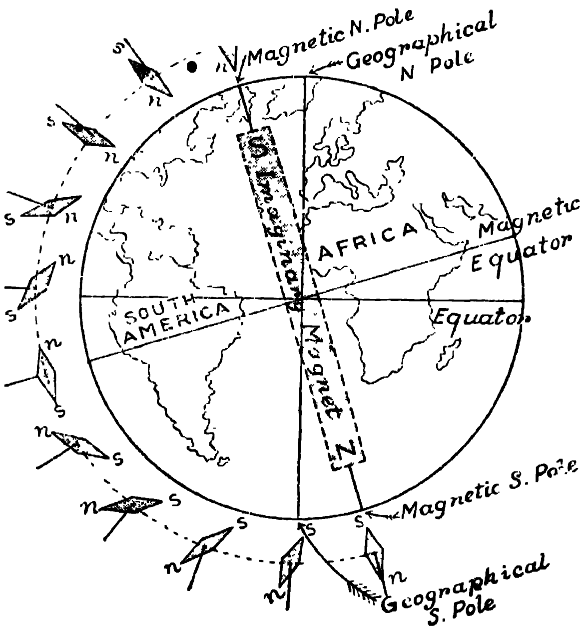


Fig. 42—Earth as a Magnet

on the surface of the earth showing that the direction of the resultant magnetic intensity is different at different places on the earth's surface. This angle of inclination, which is also known as the "**angle of dip**",

is zero near the equator and 90° at two places, one of which is near Boothia Felix (in Canada about 1000 miles from north geographical pole) in longitude 96° W. and latitude 70°75 N., and the other is in the southern hemisphere near South Victoria Land in longitude 155°E. and latitude 73°S. These two points on the earth's surface are, therefore, called **the magnetic poles** of the earth. They are not the same as the geographical poles. The angle between the two lines, one joining the two magnetic poles and the other the two geographical poles, *i.e.* the angle between the magnetic axis and the geographical axis, is about 17°. It will be seen in Fig. 42 that the needle is parallel to the earth's surface near the equator, where the *angle of inclination is zero*, and is vertical at the two magnetic poles, where the *angle of inclination is 90°*. The line joining the points where the angle of inclination is zero is called **the magnetic equator.**

The earth behaves like a magnet, but upto the present no satisfactory explanation has been put forward as to the cause of the earth's magnetism. The magnetic behaviour of the earth, however, can be explained to a certain extent by considering an imaginary short magnet placed at the centre of the earth *along its magnetic axis*, though we do not believe that there is any actual magnet there, and also the temperature of the centre of the earth is too high for this to be the actual fact. By the law of attraction and repulsion it is supposed that **the northern regions of the earth possess south polarity and the southern regions north polarity.** The actual cause of the earth's magnetism may be due to electric currents in the earth or in the upper layers of the atmosphere, or it may be connected in some way with the sun. But these may again be only mere conjectures.

38. Magnetic Elements.—In order to specify completely the magnetic field of the earth at any place the following three qualities are usually chosen : (1) **the Declination** or **Variation** ; (2) **the Inclination** or **Dip** ; and (3) **the Horizontal Intensity.** They are chosen, for they lend themselves most readily to experimental determination. They are called the **magnetic elements** of the earth at a place.

(1) Declination (or **Variation**).—*It is the angle which the magnetic meridian at a given place makes with the geographical meridian.*

The geographical meridian is taken, by convention, as the plane of reference. The angle which the magnetic meridian makes with respect to this plane is a measure of the declination of the place. If θ is the declination at a place, it is expressed as θ°E or θ°W, depending on whether the magnetic meridian, *on the north of the place*, is to the

east or to the west of the geographical meridian. Declination at Delhi 2°E means that the N-pole of a horizontal compass needle will point 2°E of the geographical north-and-south direction.

(2) **Dip (or Inclination).**—*The dip or inclination at a place is the angle which the earth's resultant magnetic intensity there makes with the horizontal direction.*

In Fig. 43, the resultant intensity I at the place B makes an angle δ with the horizontal direction BD there. So it is the dip at the place It will be $\delta°$ north or south according as the place B is in the Northern or the Southern hemisphere. For, a perfectly freely suspended needle at B would not in general set in the horizontal direction BD but along the direction BR of the resultant magnetic intensity I, where the plane BDC represents the magnetic meridian at the place B. Its N-pole will dip downwards if the place B is in the Northern hemisphere and its S-pole will dip downwards if the place is in the Southern hemisphere. The dip at Delhi is 40°N means that a dip-needle will be inclined to be horizontal at 40° with its N-pole dipping downwards at Delhi.

(3) **Horizontal Intensity.**—It is the resolved part of the earth's resultant intensity at a place in the horizontal direction in the magnetic meridian.

Vertical Intensity.—It is the resolved part of the earth's resultant intensity at a place in the vertical direction.

38 (a). Resultant Intensity and its Components.—In Fig. 43, the plane ABC represents the geographical meridian and DBC the magnetic meridian. Let BR represent, in direction and magnitude, the total magnetic intensity I at a place B, and BE and BF represent the horizontal and vertical components of I in magnitude and direction. The horizontal component $H=I\cos\delta$ and the vertical component $V=I\sin\delta$, where I is the resultant force and δ the angle of dip. Thus, we have, $H^2=I^2\cos^2\delta$, and $V^2=I^2\sin^2\delta$.

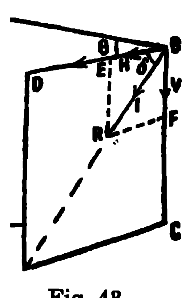


Fig. 43

$$H^2+V^2=I^2(\cos^2\delta+\sin^2\delta)=I^2 \qquad \text{or } I=\sqrt{H^2+V^2} \tag{1}$$

And, $$\frac{V}{H}=\frac{I\sin\delta}{I\cos\delta}=\tan\delta. \tag{2}$$

39. The Directive Couple of the Earth.—The action of the earth's field on other magnets is simply directive. This can be shown experimentally, as in Art. 22, by fixing a magnet on a cork and float-

ing it on water contained in a large vessel. It will be noticed that the cork turns round until the needle is in the magnetic meridian. There is no tendency of the magnet to move bodily towards the side of the vessel. Therefore the earth's horizontal force which is acting on the magnet is *directive* only.

This action of the earth is unlike the action exhibited by other magnets. This may be explained by the fact that the magnetic poles of the earth being at very great distances from the poles of the magnet (as the size of the earth is very large in comparison with that of the magnet), the distances of both poles of the magnet from either pole of the earth may be considered to be equal. In other words, the resultant forces acting on the two poles of the magnet are equal and opposite, and hence form a couple directing the magnet to the magnetic meridian.

40. Determination of the Magnetic Meridian.—(1) By a Bar-Magnet.—A bar-magnet is suspended horizontally over a table by a silk loop and unspun silk fibres (Fig. 44). A short piece of fine wire is attached by wax at the middle of each end of the magnet so that the wires are quite vertical in this suspended position of the magnet. When the magnet is at rest, two *brass* pins A, A' are fixed vertically on the table against the two ends so that the pins and the two fine wires appear to be in the same straight line. The magnet is then turned upside down in the same loop, and two more pins B, B' are fixed in the same way. In both the cases, the magnetic axis of the magnet lies in the magnetic meridian. In all probability the line joining the fine wires at the ends of the magnet does not lie on the magnetic axis. In that case, in both positions of the magnet, this line must be equally inclined to the magnetic axis. Therefore the straight line bisecting the angle between the lines AA' and BB' represents the **magnetic meridian.**

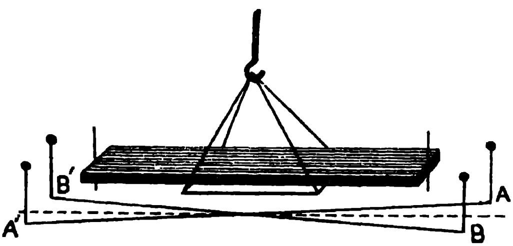


Fig. 44—Determination of Magnetic Meridian

It should be remembered that, strictly speaking, the magnetic meridian at a given point is not *a line* of definite direction, but an imaginary vertical plane passing through the magnetic axis of a freely suspended magnet placed at the point of observation, *i.e.* it is the ***imaginary vertical plane*** through the above bisecting line which gives the magnetic meridian at the place.

Magnetic Axis of the Magnet.—To determine the magnetic axis of the magnet, it is again suspended in the same loop and allowed to come to rest. A line is then drawn on the face of the magnet in the same vertical plane as the straight line representing the magnetic meridian. This line on the face of the magnet indicates the direction of the **magnetic axis.**

(2) **By a Magnetised Steel Disc.**—The above method applies both to symmetrical and unsymmetrical magnets. If a magnet has consequent poles, or is otherwise irregularly magnetised, the resultant poles of the magnet may be far out of the centre line of the bar, as assumed in the case of symmetrical magnets.

The same method of finding the direction of the magnetic axis is also applied in the case of a magnetised steel disc. Mark one of the diameters of the disc as a reference line and suspend it in a suitable carrier. Draw a line on a sheet of paper attached to the table just below the disc exactly under the reference line when the disc comes to rest. Now invert the disc and draw a second line as before. The line bisecting the angle between the two lines is the **magnetic meridian** line, and a line drawn on the disc exactly over the meridian line is the **magnetic axis** of the disc.

N. B. It should be noted that the same method can be applied for the determination of magnetic meridian or, magnetic axis, even if the bar-magnet, or the magnetised disc, be enclosed in a wooden box.

41. Determination of the Declination of a Place.—To determine the declination at any place it is necessary to determine both the geographical and magnetic meridians at the place. The determination of the magnetic meridian has been explained in Art. 40. The determination of the *geographical meridian* involves careful astronomical observations, but a simple method of doing this is as follows :—

Expt.—Fix a straight rod about one foot long vertically on a level ground where the sun can shine. Observe the length of the shadow about an hour or two before noon. Describe a circle round the rod with the rod as centre and the length of the shadow as radius. Mark the direction of the shadow again in the afternoon when the shadow is of the same length, and touches the circle. The line bisecting the angle between these two marked directions of the shadows is the true north and south line, and the angle between this line and the magnetic meridian line gives the **declination** of the place.

It should be noted that for all places the magnetic and geographical meridians are nearly coincident.

42. The Dip at a Place.—If a magnetic needle be freely suspended at its centre of gravity by a silk fibre, the angle which the magnetic axis of the needle makes with the horizontal line passing

through its point of support measures the **inclination** or **dip** at a given place. The direction taken up by the magnetic axis of the needle represents the direction of the total magnetic intensity of the earth at the place.

A magnetic needle which is perfectly freely suspended at its centre of gravity will no doubt show the dip at a place, but the practical difficulty with such a device is with regard to the mechanical support which must influence the angle at which the needle will set. If, however, the needle is mounted on an axle (passing through the C. G. of the needle) resting on horizontal knife-edges, it will rotate in the vertical plane, and if this plane is made to coincide with the magnetic meridian at the place, the inclination of the needle to the horizontal will give the dip. Such a needle is called a **dip-needle**.

Measurement of Dip.—The dip at any place is determined by means of an instrument called the **Dip-circle** (Fig. 45). The dip-circle consists of a magnetic needle AB mounted on an axle which rests horizontally on two agate knife edges and can rotate in the plane of the vertical circle S, which is graduated in degrees, $0° - 0°$ being on the horizontal line and $90° - 90°$ on the vertical line. The needle and the scale are enclosed in a glass case, which can be rotated about a vertical axis, the angle of rotation being indicated by a pointer E moving on a horizontal graduated circle P. The instrument is supported by three levelling screws, and the levelling can be done by the help of a spirit level fixed on the base (not shown in the figure).

In determining the dip, the instrument is first levelled and the case is then turned until the needle is vertical, *i.e.* points to 90° on the vertical scale. In this position the needle is acted on only by the vertical component of the earth's field, and so the plane of the needle must be at right angles to the magnetic meridian. The position of the pointer E on the *horizontal scale* is noted, and the case is rotated through 90° from this position. The plane of the vertical circle is now in the magnetic meridian and the needle points along the direction of the earth's resultant magnetic field. *The angle (read on the vertical scale) which the axis of the needle makes with the horizontal line (zero degree line) gives the value of dip at the place.*

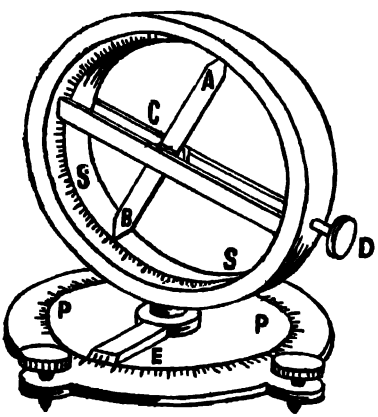


Fig. 45—The Dip-circle

Errors in a Dip-circle.—The readings obtained as explained above may not be accurate, as allowances must be made for the following errors :—

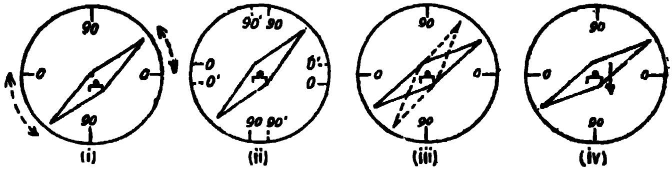


Fig. 45 (a)

(1) **Error of Eccentricity.**—The axis of rotation of the needle may not pass through the centre of the vertical scale. To correct for this error, the readings of both ends of the needle are to be taken [Fig. 45 a, (i)].

(2) **Error of Zero-Zero Line.**—The Zero-Zero line of the vertical scale may not be truly horizontal. To eliminate this error, turn the case through 180° and again read both ends of the needle [Fig. 45 a, (ii)].

(3) **Error of Magnetic Axis.**—The magnetic axis of the needle may not coincide with its geometrical axis. To correct for this error, reverse the needle on its bearings and repeat observations (1) and (2) [Fig. 45 a, (iii)].

(4) **Error of Centre of Gravity.**—The axis of rotation may not pass through the centre of gravity of the needle ; so a small couple due to gravity may cause the needle to turn from its position of true dip. To correct for this error, remagnetise the needle in the opposite direction to the same strength, so that the end which dipped previously now turns upwards, and repeat the above eight observations.

The mean of the above sixteen readings gives the true dip. In a properly constructed instrument, the individual readings should not differ by more than a degree from the mean.

The following are the magnetic *declinations*, *inclinations* and *horizontal intensities* at different places.

Place	Declination	Inclination or Dip	Horizontal Intensity H (C. G. S.)
Bombay	0°41′*E*	24°21′*N*	0·3648
Calcutta	0°38′*E*	30°59′*N*	0·3725
Madras	0°10′*W*	34°37′*N*	0·3690
Delhi	2°02′*E*	40°56′*N*	0·3400
Paris	13°31′*W*	64°40′*N*	0·1972
Greenwich	14°18′*W*	66°54′*N*	0·1850
New York	10°14′*W*	72°13′*N*	0·1822

42(a). Changes in the Values of the Magnetic Elements.—The magnetic field of the earth at any place is not constant but is subject to changes which may be classified as follows :

(1) *Secular change.*—The magnetic elements undergo a gradual cycle of changes which extend over a long interval after which they return to their original values. These changes are relatively large and take place steadily.

(2) *Annual change.*—Such changes are periodic and the value of an element varies gradually between a maximum value and a minimum value in course of a year. As an example, suppose the declination at a place attains the maximum value in February and the minimum value in August every year.

(3) *Daily change.*—A periodic change extending over 24 hours in the value of an element is also noticed. An element reaches the maximum value at some hour of the day and the minimum value at some other hour, characteristic of the element.

(4) **Magnetic Storms.**—It has been found that during volcanic eruptions, display of Aurora Borealis, appearance of sun-spots, etc., sudden and violent changes occur in the indications of recording instruments measuring the magnetic elements. These are said to be due to magnetic storms. They are obviously non-periodic.

Example.—*The value of the angle of dip at a place (A) is 45° and the total force of the earth's magnetism is 0·536 gauss. At another place (B), the dip is 60°, and the total force is 0·62 gauss. At which place will a compass needle oscillate more rapidly ?*

The horizontal component of the earth's field $H = I \cos \delta$.

At A, $I = 0{\cdot}563$; $\delta = 45°$; $H = H_a$; $\therefore$ $H_a = 0{\cdot}536 \times \cos 45°$ $= 0{\cdot}536 \times \frac{1}{\sqrt{2}} = 0{\cdot}379$ gauss. At B, $I = 0{\cdot}62$; $\delta = 60°$; $H = H_b$;

$\therefore$ $H_b = 0{\cdot}62 \times \cos 60° = 0{\cdot}62 \times \frac{1}{2} = 0{\cdot}31$ gauss.

Now, $H \propto n^2$, *i.e.* in a stronger field the number of oscillations of the needle per unit time will be greater, and hence the time for one oscillation (t) will be smaller, or, in other words, the needle will vibrate more rapidly.

Here H_a is stronger than H_b, so the needle will vibrate more rapidly at A.

43. Determination of the Angle of Dip without bringing the needle into the Magnetic Meridian.—The angle of dip can be determined by observing the apparent angles of dip taken in any two vertical planes ***at right angles to each other.***

Let δ_1 be the apparent angle of dip, *i.e.* the angle of inclination of the magnetic axis with the horizontal, in a plane OA making an angle θ_1 with the magnetic meridian OK, and δ_2 be the apparent angle of dip in another plane OB at right angles to the plane OA (Fig. 46). Let V and H be the vertical and horizontal components of the earth's field. The components of H along OA is $H \cos \theta_1$ and along OB is $H \cos \theta_2$. The vertical component V is the same for both the planes. So, we have [see Art. 38(*a*)],

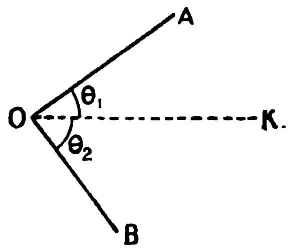


Fig. 46

$$\tan \delta_1 = \frac{V}{H \cos \theta_1}, \text{ and } \tan \delta_2 = \frac{V}{H \cos \theta_2}. \text{ But } \cos \theta_2 = \cos (90 - \theta_1)$$

$$= \sin \theta_1 \qquad \therefore \quad \cot \delta_1 = \frac{H \cos \theta_1}{V} \; ; \; \cot \delta_2 = \frac{H \sin \theta_1}{V}.$$

$$\text{Or,} \quad \cot^2\delta_1 + \cot^2\delta_2 = \frac{H^2 \cos^2\theta_1}{V^2} + \frac{H^2 \sin^2\theta_1}{V^2} = \frac{H^2}{V^2}.$$

But, if δ be the true dip in the plane of the magnetic meridian, *i.e.* in the plane OK, $\cot \delta = \frac{H}{V}$. $\therefore$ $\cot^2\delta = \frac{H^2}{V^2} = \cot^2\delta_1 + \cot^2\delta_2$.

Thus, knowing δ_1 and δ_2 in any two planes at right angles to each other, the true dip δ can be calculated.

Example.—*In an experiment for finding the value of dip at a place, it is observed that apparent dip in one plane is 30° and that in a plane at right angles to the first plane is 20°. Calculate the true dip at the place.* (*Pat. 1927*).

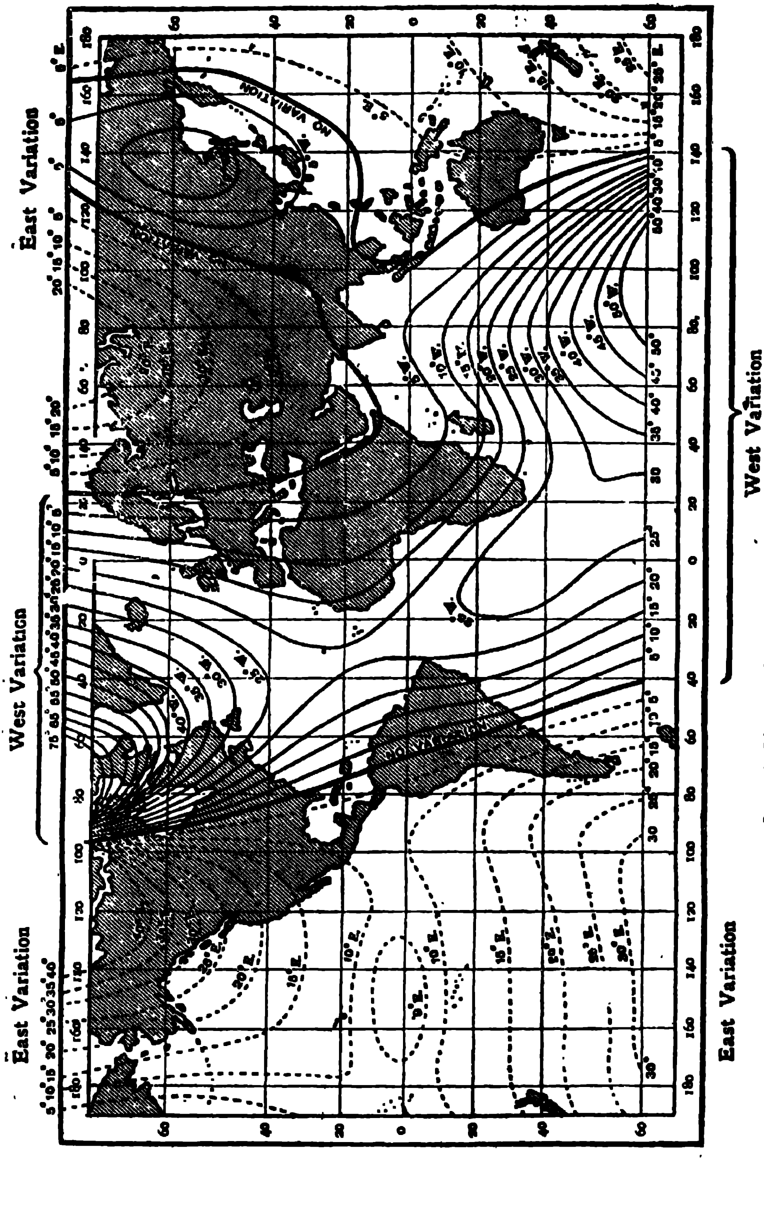
If δ be the true dip, $\cot^2\delta = \cot^2\theta_1 + \cot^2\delta_2$

$= \cot^2 20^\circ + \cot^2 30^\circ = (2{\cdot}74)^2 + (1{\cdot}73)^2 = 7{\cdot}5076 + 2{\cdot}9929 = 10{\cdot}5005$

$\therefore \quad \cot \delta = \sqrt{10{\cdot}5005} = 3{\cdot}24.$

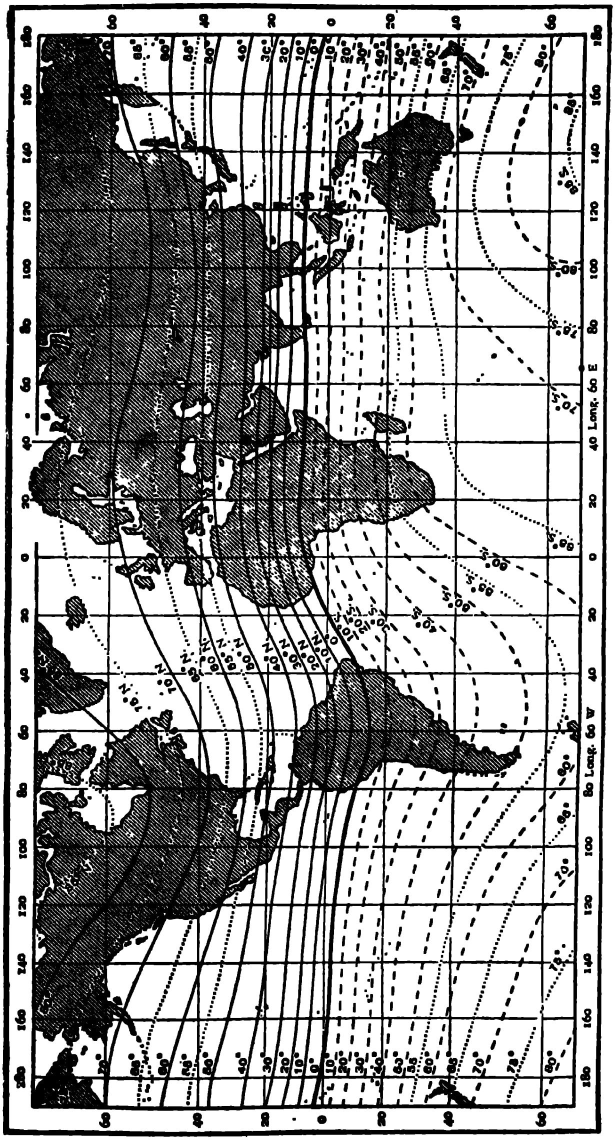
Reference to a table of natural cotangents will show that the angle whose contangent is 3·24 is 17°12′ nearly. $\therefore$ True dip = 17°12′ nearly.

44. Magnetic Maps.—The values of magnetic elements of different places are different, and magnetic maps have been drawn by joining those places on the geographical maps in which a magnetic element has equal values. In magnetic maps we have the following lines :—



Isogonic Lines, or Lines of Equal Declination. (For the Epoch 1932.)

Fig. 47



Isoclinic Lines, or Lines of Equal Dip. (For the Epoch 1922.)

Fig. 48

(i) **Isogonic and Agonic Lines.**—*Isogonic lines* (Fig. 47) are lines joining places on the map of the earth where the declination is the same. *Agonic* lines are those which pass through places having zero declination.

(ii) **Isoclinic and Aclinic Lines** —*Isoclinic lines* (Fig. 48) are lines joining places on the map of the earth where the magnetic dip is the same. The line passing through places having no dip is called the *aclinic line.*

The dip of the places situated on the magnetic equator (not the geographical equator) *is zero.* So **the magnetic equator is the line of no dip**. At the two magnetic poles the dip needle points in a vertical direction, so the dip is 90°. Starting from the magnetic equator, the dip will go on increasing both towards the north and south poles ; but towards the north, the north pole of the dip-needle dips downwards, and towards the south, the south pole dips downwards.

(iii) **Isodynamic lines.**—These lines join up places on the map of the earth where the value of the horizontal intensity is the same.

45. The Ship's (or Mariner's) Compass —This is an invaluable instrument (Fig. 49) to the mariners for determining the directions in which to guide the course of their ships.

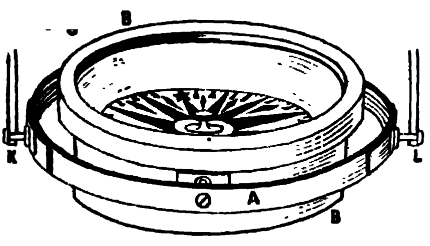


Fig. 49—Mariner's Compass

It consists of a magnetised needle attached underneath a circular card which turns with it and the circumference of which is divided into thirty-two directions, called the **points of the compass.**

The *N-S* direction on the card is marked along the axis of the needle. To distinguish the *N*-pole it is *Crown-marked.*

The needle with the direction-giving card on it rests horizontally on a vertical pivot fixed to the base of a hemispherical bowl *BB*. It is essential to make a contrivance so as to keep the needle always horizontal irrespective of any movement of the supporting base caused by the rolling of the sea. The contrivance is called a **Gimballs arrangement.** What is done is that the bowl is suspended inside an outer ring *A* at two diametrically opposite points about which it can freely turn, while the ring, on its part, is suspended at two diametrically opposite points *K* and *L* about which it has every freedom to rotate, the two axes of rotation being mutually perpendicular to each other.

To find the geographical north at a place on the sea, the declination of the place is first determined by referring to the Nautical Almanac. If it is $\theta^\circ W$, it means that the crown-mark lies θ° to the west of the true North.

For reliability of the indications of such an instrument, permanence of magnetisation of the magnetic system, steadiness of the system under violent vibration, alignment of the N-S direction marked on the card with the magnetic axis of the needle, and quick damping of any oscillations imparted to the needle must be secured. In order that these conditions may be fulfilled, the magnetic system should be *of short length, large magnetic moment, should have a large time period* and *should be quickly damped.* In the **Kelvin's Compass**, the first three conditions have been sought to be fulfilled by the use of a set of eight small parallel needles suspended from an aluminium ring which acts as the carrier of the card. In the liquid compass, the oscillations of the needle are quickly damped because the needle is immersed in a fluid which is a mixture of water and alcohol.

The indications of the needle are liable to be affected by the magnetisation (both temporary and permanent) of the iron of the ship. Compensation is accomplished by the suitable placing of small magnets attached to the shaft carrying the needle. The positions of these magnets are carefully adjusted until all external fields except the field of the earth have been neutralised. Sometimes hollow spheres of soft iron are also suitably placed near the needle for exact compensation.

Questions

Art. 37.

1. How would you prove that the earth is a magnet ? (C.U. '45 ; All. '24).

2. Give a short account of the earth's magnetic field. (Dac. 1933)

Art. 38.

3. What are the magnetic elements of a place ? Briefly describe a method of finding each of them. (Pat. 1930 ; Cf. '32, '36 ; Utkal 1948 ; All. '46)

(See Arts. 36, 41 and 42)

Describe how the intensity of the earth's field can be determined at a point. [See Art. 36] (All. 1946)

4. Show how from a knowledge of the horizontal component of the earth's field and the dip, the total intensity of the earth's magnetic field at any place is calculated ? (Pat. 1941)

5. The apparent dip at a place in a plane 60° away from the magnetic meridian is 45° and the total intensity of the earth's field is 0·46 C.G.S. units. Explain clearly what are meant by the terms and figures in the above statement. Calculate also the true dip at the place. (Pat. 1934)

[**Hints.** $V/H \cos 60^\circ = \tan 45^\circ$. $\therefore \tan \delta = \tan 45^\circ \times \cos 60^\circ$; or $\delta = \tan^{-1} \frac{1}{2}$.]

*6. At A the total magnetic intensity is 0·5 and the angle of dip is 68°, while at B the total intensity is 0·55 and the angle of dip 72°. Compare the horizontal intensities at the two places. ($\cos 72^\circ = 0{\cdot}3090$, and $\cos 68^\circ = 0{\cdot}3748$).

[*Ans :* $H_a : H_b = 1{\cdot}1 : 1$] (C. U. 1935)

*7. At a place A, the total magnetic intensity is 0·98 gauss and the dip is 45° ; at another place B, the total intensity is 0·5 gauss and the dip is 60°. The time period of a magnet vibrating horizontally at A is 3 seconds. What is the time period at B ? (Pat. 1945)

[*Ans :* 5 secs..]

8. What is the earth's horizontal intensity ? Explain what observations are necessary for the determination of total intensity of earth's magnetic field at any given place. (C. U. 1947 ; Utkal 1947)

Art. 39.

9. Prove that the earth's action on a magnet is simply a directive one. (C. U. 1928)

Art. 40.

10. A bar-magnet is provided to you such that its poles are not in the axis of symmetry. Show how you will use it to determine the magnetic meridian. (Pat. 1928 ; Cf. Pat. 1930 ; Cf. C.U. '42)

11. Describe an experiment for determining the magnetic axis of a magnet fixed inside a flat rectangular wooden box without opening the box. (Pat. 1943)

Art. 42.

12. Describe a Dip-circle. How will you use it to determine the magnetic inclination at a place ? (Pat. 1927, '37, '39, '40, '41, '48 ; All. 1931)

Mention the errors that may arise and explain how they can be eliminated. (Pat. 1948)

Art. 44.

13. What are meant by the terms : (*a*) magnetic equator, (*b*) isogonic lines ?

A dip-circle is placed so that the needle sets vertical. The circle is then roated through θ about a vertical axis and the dip as measured in this position is found to be ϕ. Find its true value. (C. U. 1945)

Art. 45.

14. What is Mariner's compass and how is it used ?

PART VI

STATICAL ELECTRICITY

CHAPTER I

Fundamental Ideas

1. **Electrification.**—Many substances, such as glass, ebonite, sealing wax, resin, etc., when rubbed with silk, flannel, catskin, or other suitable materials, acquire the property of attracting light bodies like bits of paper, pieces of pith, etc. The bodies in such a state are said to be **electrified**, or to possess electric charges, or they are simply called *charged bodies.*

The electricity so produced by friction is called **Frictional Electricity**. It is also called **Statical Electricity**, as it does not move from one place to another in the body in which it is produced.

2. **Two kinds of Electrification.**—If a pith-ball is suspended by means of a silk thread, and if a dry glass rod is rubbed with silk and then held near the pith-ball, the pith-ball is attracted by the glass rod, touches it, and is then repelled by the rod. If the glass rod now approaches the pith-ball, the pith-ball will move further away. If another pith-ball is similarly suspended, and if a rod of sealing-wax, rubbed with a piece of flannel, is held near it, the pith-ball is attracted by the rod, touches it, and is then repelled. If now the glass rod is held near the second pith-ball, it attracts the pith-ball, and, if the rod of sealing wax is held near the first pith-ball, it also attracts the pith-ball. This experiment clearly indicates that the charge of the first pith-ball, received from the glass rod rubbed with silk, is different in nature from that of the second pith-ball received from the sealing-wax rubbed with flannel. The repulsion of the first pith-ball by the glass rod and the second one by the rod of sealing-wax show that similar electrical charges repel each other ; while the attraction between the charged glass rod and the second pith-ball, which received charge by contact with the rod of sealing-wax rubbed with flannel, show that dissimilar charges attract each other. Thus we have the following **Fundamental Law of Electrostatics :—**

Two bodies with like charges repel each other and with unlike charges attract each other.

The kind of electricity or charge excited on glass by rubbing it with silk was called, by the ancients, **vitreous**, and that excited on sealing-wax when rubbed with flannel was called **resinous**. These names are now obsolete. Now-a-days the *vitreous electricity* is called **positive**, and *the resinous* **negative**.

A list of substances is given below which have been arranged in such a way that any one of them becomes positively charged when rubbed with another coming later in the list.

Fur	Glass	Human body	India-rubber
Flannel	Paper	Wood	Sulphur
Sealing-wax	Silk	Metals	Ebonite.

3. Conductors and Non-Conductors (or Insulators).—A charged pith-ball suspended by silk retains its charge for a time. But, if the pith-ball were suspended by a copper wire, it would lose its charge as soon as it is given. If a glass rod held in the hand is rubbed with silk, it is electrified only in the portion where it is rubbed ; while a brass rod, when similarly treated, shows no sign of electrification. In the first case, the silk thread is a non-conductor or **insulator**, so it does not allow electric charge to pass through it ; while the copper wire is a good conductor and conducts away the electricity quite readily. Similarly, in the second case, the glass being an insulator does not conduct away the electricity developed by friction, and so the charge remains there ; but, in the other case, brass being a good conductor, the electricity developed on the brass rod at once escapes to the earth through the human body which is a conductor. If the brass rod is fitted with a glass or ebonite handle, it will be strongly electrified by friction.

The best **insulators** are ebonite, mica, glass, sulphur, shellac, paraffin, sealing-wax, silk, oils, dry air, etc. The best **conductors** are the earth, silver, copper, etc. There is, however, no sharp line of division between the two classes,—for example, there are some substances like wood, paper, etc., which are semi-conductors or partial conductors. In the universe no material can be said to be a perfect conductor or a perfect insulator.

The earth, and specially the moist portion of it, **is regarded as a huge conductor**. Any body which is to be discharged is connected with the earth, and *the body is said to be* **earthed**.

4. Electroscopes : (Pith-Ball Electroscope and Gold-leaf Electroscope).—*Electroscope* is an instrument for detecting the presence of electricity and also for determining whether the charge on a body is positive or negative.

(a) **Pith-Ball Electroscope.**—The *pith-ball electroscope* is one such instrument which consists of a pith-ball (preferably gilded) suspended from a support by a single silk fibre.

(i) **Detection of Charge.**—An uncharged pith-ball is brought near the body under examination. If the pith-ball is first attracted and then repelled after touching the body, the body is charged. The repulsion is due to the same kind of charge acquired by the pith-ball after contact with the body.

(ii) **Detection of the Nature of Charge.**—Let a positively charged pith-ball be brought near the body. If there be *repulsion*, the charge on the body under examination is positive, but if there be *attraction*, the body may be either uncharged or negatively charged. Whether the body is charged or uncharged that is determined by the first method.

(b) **Gold-leaf Electroscope.**—The *gold-leaf electroscope* is a more sensitive instrument than a pith-ball electroscope used for the detection of a charge and its nature. It is therefore very commonly used. It consists of two gold leaves L, L attached to the two sides of the lower end of a metal rod R (Fig. 1). The rod passes through an insulating stopper S, usually made of amber or sulphur, fitting into the neck of a glass vessel B which protects the leaves from the disturbing effects of air currents. The rod terminates above in a metal disc or knob C. The delicacy of the instrument is increased by pasting two strips t, t of tin-foil on the inside of the glass vessel just opposite to the gold leaves. The tin-foils begin from the level of the gold leaves and pass down to the metal base of the instrument in order to be communicated with the earth. These strips make the instrument more delicate, the utility of which will be fully understood after studying the principles of electrostatic induction and potential. In order to keep the air dry, a small vessel, containing calcium chloride or pumice stone soaked in strong sulphuric acid, is placed inside the instrument, otherwise charges of the leaves may leak through moist air which is conducting.

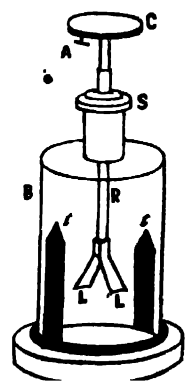


Fig. 1—Gold-leaf Electroscope

Action.—If a glass rod rubbed with silk touches the disc of the electroscope, a positive charge is communicated to the leaves, and the leaves being charged with the same kind of electricity repel each other and diverge. If the divergence be too great, the leaves touch the tin-foil strips, which conduct away their charges to the earth, and they

collapse again. *The divergence of the leaves indicates the presence of an electric charge.* The extent of the divergence depends on the quantity of the charge given to the leaves. This process of charging the electroscope is called **charging by conduction.**

(For a better method of charging the electroscope, see Art. 9 and for the detection of a charge, see Art. 11.)

5. Repulsion is a Surer Test of Electrification—A charged glass-rod is presented to a suspended pith-ball. Suppose the pith-ball is attracted to the glass-rod. From this it can not be concluded that the pith-ball is electrified with opposite charge. For, a charged body can attract an uncharged body and also a body charged with opposite kind of electricity. So, only by attraction it cannot be definitely said whether the second body is charged or not. If, however, the pith-ball is repelled, it can be concluded at once that it carries the same kind of charge as the glass-rod, because repulsion is possible only between two bodies charged with the same kind of electricity. Hence, repulsion is a surer test of electrification.

6. Simultaneous Development of Equal and Opposite Kinds of Electricity.—Whenever a charge of one kind of electricity is produced by friction, an *equal quantity of charge of the opposite kind* is also produced at the same time. Thus, when glass is positively charged by being rubbed with silk, an equal quantity of negative charge is developed on the silk. This is clearly verified by the following experiment.

Expt.—A flannel cap having a long silk thread attached to it is fitted on an ebonite rod (Fig. 2). The rod is then rubbed round by the cap several times. The rod and cap together, when presented before an uncharged gold-leaf electroscope, will produce no divergence. The cap is then separated from the rod by means of the silk thread and presented before a positively charged gold-leaf electroscope. The divergence increases, showing the charge on the cap to be *positive.* Now, on presenting the rod before the electroscope, the divergence diminishes showing the charge of the rod to be *negative.* The two pieces, the rod and the flannel cap, are thus both charged, while the two together behave as neutral. This means that by friction equal charges of the opposite kind are produced on the substances rubbed.

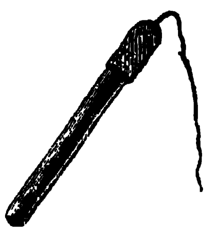


Fig. 2

7. Theories of Electricity.—From time to time various theories have been put forward, e.g. (i) *One-Fluid Theory,* (ii) *Two-Fluid*

Theory, but the latest of them is the (iii) *Electron Theory*, which is now-a-days universally applied.

(i) **The One-Fluid Theory** is due to Benjamin Franklin, who first suggested the terms, *positive* and *negative*, to denote the two kinds of electricity. According to this theory, electricity is a kind of indestructible, subtle and weightless fluid and is possessed by all uncharged bodies in a normal amount A body containing more than the normal amount of this fluid is said to be **positively** charged ; and if it contains less than the normal amount, it is said to be **negatively** charged. By the act of rubbing, the fluid is transferred from one body to another as it were by squeezing. The amount lost by one shows to be in excess in equal quantity on the other. This also explains equality of the charges on the rubber and tho rubbed. This theory was replaced in course of time by a more rational one, called the two-fluid theory.

(ii) **Two-Fluid Theory**.—Robert Symmer envisaged the existence of two kinds of fluids having opposite nature corresponding to the two kinds of electrification, positive and negative. He thought that all bodies, in unelectrified state, contain these two fluids in equal quantities whereby the effects are neutralised. By the act of rubbing the positive fluid of one is transferred to the other. The one that gets it has its positive fluid in excess over the negative and shows positive electrification, while tho other that loses it has its negative fluid in excess over the positive to the same extent and shows equal negative electrification. Thus the equality of charges on the rubber and the rubbed is explained This theory is obviously more complex that the one-fluid theory but, nevertheless, the two theories were prevalent until up to the end of the last century.

(iii) **The Modern Electron Theory.**—Within recent years the existence of particles, far smaller than the atom, has been provéd. It has now been definitely established that atoms contain as constituents tiny particles, one kind of which is associated with a negative charge ; and it is believed that it is nothing but a minute particle of negative electricity. This ultra-atomic particle is called an **electron.** Each electron has the same mass, about $\frac{1}{1840}$ of that of a hydrogen atom, which is the lightest known atom, and each is associated with the the same negative charge, the value of which is $4{\cdot}77 \times 10^{-10}$ electrostatic unit. This is the smallest quantity of negative electricity which has been found existing by itself. Inside the atom there is a **nucleus** of *positive* electricity surrounded by electrons. These electrons are revolving in definite orbits round the positive nucleus, much in the same manner as the Earth, Mars, and other planets revolve in their orbits round the Sun. The positive nucleus is formed of positively

charged particles, called **protons**, and some uncharged particles, called **neutrons**, both of which are massive particles compared to electrons. A proton or a neutron is as heavy as a hydrogen atom ; the former carries a positive charge equal to that of an electron, while the latter is electrically neutral. These three particles, electron, proton and neutron, are regarded as the fundamental bricks with which all matter is composed.

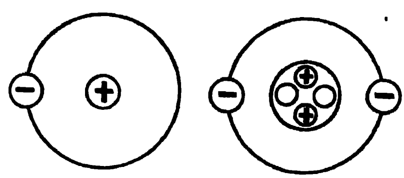
7(a). **The Structure of an Atom.**—An atom consists of a central part, called the **nucleus**, about which a number of electrons, called the **orbital electrons**, revolve in the outer part of the atom, called the shell.

(1) **The Nucleus (or Core).**—It is the central portion of the atom like the sun of the solar system. It mainly consists of units of positive electricity, called **Protons**, which are elementary positively charged particles, together with a certain number of embedded **neutrons**, which are also elementary particles each having nearly the same mass as that of a proton but are electrically uncharged. So the charge of a nucleus is positive. The nucleus is surrounded by the outer part of the atom, called the **shell**, in which electrons, equal in number to the protons in the nucleus, continually revolve. In the case of a normal atom, the electric charges of the protons and electrons are equal in quantity but opposite in kind ; so, their resultant charge is zero. The nucleus in the case of a hydrogen atom has only one proton in it. It is rather special. It is not only called a hydrogen nucleus but also a *Proton* and is without any embedded neutron. A proton is supposed to be even smaller than the electron though, relatively, its mass is many times greater. *The number of positive units of electricity at the nucleus (which is equal to the number of revolving electrons in the shell) is what is called the* **atomic number** *of an element. The atomic number is the number of the place occupied by an element in a table arranged in increasing order of protons in the nucleus*. For hydrogen, this number is 1, as hydrogen has only 1 proton in the nucleus ; for helium 2, as in the nucleus of helium there are 2 protons and 2 neutrons, the positive charge being 2 units (or 2 protons). Similarly for oxygen this number is 8, for copper 29, and finally for uranium it is 92. This table is the modern *periodic table* for the elements.

(2) **The Shell.**—It is the outer part of an atom in which electrons continually revolve round the nucleus in definite orbits.

The number of these revolving electrons in a normal atom gives the

atomic number of the element. Thus, in a **hydrogen** atom there is 1 *electron* and 1 *proton* [Fig 3 (*a*)] ; in the **helium** atom the nucleus consists of 2 protons and 2 neutrons, so there are two resultant positive units of charge in the nucleus [Fig. 3 (*b*), where a white circle and a positive circle respectively denote a neutron and a proton] and in the shell there are *2 external electrons* : in **oxygen 8 protons** together with 8 neutrons in the nucleus and 8 external *electrons*, and so on. These electrons are revolving round the positive nucleus in definite orbits, much in the same manner as planets revolve in their orbits round the sun, with enormous speed.



(*a*) Hydrogen atom (*b*) Helium atom

Fig. 3

The mass of an electron being about 1840 times as small as that of a proton, it is obvious that *the whole mass of an atom is concentrated in the nucleus*, each of which contains a number of protons and neutrons compared to which the weight of the electrons is almost entirely negligible. Thus the **atomic weight** of an element is proportional to the *total number of protons and neutrons* in the nucleus of its atom, while the *chemical nature* of each element is determined by the **atomic number** and **arrangement of the oribital electrons**.

So, a heavier atom contains a greater number of protons and neutrons in its nucleus than a lighter one. The atom is kept as a definite stable system by the electrical forces of attraction and repulsion between the different electrical units. Thus, the atom is not a solid thing, the electrons, neutrons and protons inside it only occupying a small part of the total space of the atom. An English scientist has said that if we imagine the St Paul's Cathedral to be the size of an atom, the particles, of which atoms are formed, would be like specks of dust, which are practically invisible.

Another has said that the size of an electron relatively to that of an atom to which they are attached may be compared as the volume of an airship to that of the earth. On the whole the constituent particles of an atom are exceedingly small.

The modern theory of structure of atom is chiefly due to Prof. J. J. Thomson, Prof. Sir Earnest Rutherford of Cambridge, and it has been developed by Prof. Bohr of Copenhagen, Prof. Milikan of America, and others.

7(b). Positively and Negatively Charged Bodies.—The rotating electrons of an atom may be dislodged from their orbits and pass

from one atom to another. When, at any instant, an atom loses one or more of its electrons, it has an excess of positive electricity (as it was neutral in the beginning), and it is said to be **positively charged;** while an atom, which gets one or more electrons above its normal number, has an excess of negative electricity, and is said to be **negatively charged.**

Thus, **according to the modern view,** when a glass rod is electrified by being rubbed with silk, all that is done is that some of the electrons are removed from the rod and transferred to the silk piece. Thus, glass has got a deficit of electrons and is *positively* electrified, while the silk piece has a surplus and is *negatively* electrified.

Similarly, in the case of the experiment described in Art. 6, the ebonite rod gains electrons from the flannel cap and is negatively charged, but the cap, having lost the same electrons, is positively charged to an equal amount. *This explains the simultaneous development of two kinds of electricity in equal quantity.*

7 (c). **Conduction : Electric Current, etc.**—In **solids**, the positions of the nuclei are more or less fixed and the electrons behave in two different ways. In some, the electrons cling to their own nuclei and it is very difficult to move them, while in others they can roam about freely. The electrons moving in the outer layers of certain atoms, chiefly those of the metals, are supposed to be loosely bound to the atoms and may easily be detached to migrate to neighbouring atoms. So, according to this theory, the materials in which the so-called 'free' electrons may easily migrate from one atom to another are called **conductors**, while substances, in which the electrons are strongly bound to the atoms, and in which free movement of electrons within the interior of the substances is seldom allowed, are termed **insulators**. Metals are good conductors, and the earth, which is a good conductor, is a big reservoir of electrons as the ocean is of water. A negatively charged conductor, *i.e.* a conductor with surplus electrons, when connected with the earth, gives up its surplus electrons to the latter and becomes neutral. Similarly a positively charged conductor, *i.e.* a conductor short of electrons, when connected with the earth, receive electrons from the earth and becomes neutral. The earth receives or loses electrons in this way, but the earth remains unaffected as any gain of water during rains or loss of water by evaporation has no appreciable effect on the level of water in the ocean.

In **liquids**, the electrons are *generally* bound to their nuclei, so liquids can conduct electricity only when their molecules are "ionised" (see Ch. VI, Part VII).

7(d). Electric Current and Electromotive Force.—If there is a force or pressure which can direct the movement of the free electrons in a conductor to a particular direction, there will be a stream of electrons moving in that direction, which is termed an **electric current**, and the force or pressure producing it is called an **electromotive force** (see also Art. 8, Part. VII).

Questions

Art. 2

1. What do you mean by the statement that a body is electrically charged ? When you electrify a glass rod by rubbing it against flannel, what is the source from which electrical energy is obtained ? (Pat. 1924)

Art. 4.

2. Describe the construction, and explain the use of a gold-leaf electroscope. (C. U. '16, '22, '24, '27, '45 ; Dac. 1933)

Art. 6.

3. How would you prove that positive and negative electrifications are produced in equal quantities ? (C. U. '27, '35, '42, '44 ; Cf Pat. 1923, '32.)

Explain clearly, with reasons, the names given to these by the ancients. Why have these names been changed into 'positive' and 'negative' ? (Pat. 1932)

(See also Art. 2 and 7)

Art. 7.

4. What is an electron ? Explain with its help the phenomenon of electrification by friction or by induction. (C. U. 1932)

[See Arts. 7(b) and 8].

5. Describe the structure of atoms. Define electron, proton, nucleus, and in this connection state what is meant by conduction.

CHAPTER II

Electrostatic Induction

8. Proof-plane.—A *Proof-plane* (Fig. 4) consists of a small metal disc P mounted on an insulting handle. It is used for testing the kind of electricity on a body by touching the metallic disc with the body and presenting before a charged electroscope the small amount of charge taken from the body. When a conductor is touched by it, the proof-plane becomes a part of the

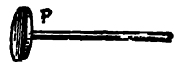


Fig. 4 —Proof-plane

surface of that conductor, and when removed, it carries a part of the charge of the conductor. The amount of the charge removed will depend on the charge at the point touched. Thus, it simply acts as a carrier of small charge taken from a body.

8(a). **Electrification by Induction (or Influence).**—When a charged body A, say, positively charged, is brought near an insulated uncharged body BC (Fig. 5), but not in contact with it, the latter is affected by the electric charge of the former acting through air or some other insulating medium. Due to the presence of the charged body A, the electric equilibrium of BC is disturbed, and the side of it near the electrified body is charged with electricity opposite to that of the charged body, *i.e.* *negatively*, the remote side being charged with electricity of the same nature, *i.e.* *positively*; thus the end B is negatively charged and the end C is positively charged. This may be proved in the following way: Place the disc of a proof-plane against the end B and present it before a negatively charged gold-leaf electroscope. Observe that the divergence of the leaves increases which shows that the charge of the proof-plane, and consequently that at the end B of the conductor, is negative. Now discharge the proof-plane by touching it with the hand; touch the end C of the conductor with it and bring it before a positively charged gold-leaf electroscope, when the leaves will diverge more, showing the presence of positive charge on the proof-plane, and consequently on the end C of the conductor. It can similarly be proved with the help of an uncharged electroscope that there is practically no charge at the middle of BC.

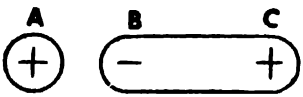


Fig. 5

So it is seen that the electricity of a charged body attracts, as it were, the opposite electricity of an uncharged body towards it and repels an equal quantity of electricity of the same kind to the remote side. Such electrification by influence is known as **electrostatic induction**, and the second body is said to be charged by induction. The charge on the first body is called the **inducing charge**, while the charge on the second body is called the **induced charge**. The opposite charge induced at the near end is called the *bound charge*, bound as it is to the inducing charge by force of attraction; while the similar charge induced at the remote side is called the *free* charge.

Development of Equal but Opposite Charges by Induction.—If a positively charged body A is brought near two insulated

metal spheres B and C placed in contact (or connected by a wire), the nearer sphere B will be negatively charged, and the farther one C positively charged (Fig. 6). If the two spheres B and C are now separated *in the presence* of the positively charged inducing body A, then, on testing with a proof-plane and a charged electroscope, it is found that the nearer one B shows negative charge and the other C positive charge. If the spheres are again placed in contact and the combination is presented before an uncharged gold-leaf electroscope, the leaves remain unaffected showing that there is no charge on the combination. This fact proves that **the induced negative charge on B is equal to the induced positive charge on C** Further proof of this is furnished in Art. 14.

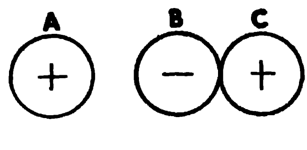


Fig. 6

Explanation of Induction by Electron Theory.—A conductor is supposed to contain a number of free or mobile electrons When a positively charged body is brought near it, its electrons are attracted so that the near end of the conductor gets more of these electrons, and so becomes negatively charged, while the remote end is left at a deficit by the same number of electrons and so becomes equally positively charged. When, however, the conductor is earthed in the presence of the positively charged body, the shortage of electrons of the remote end of the conductor is made up by electrons coming from the earth. If now the positively charged body, *i e.* the attracting force, is removed, and the earth connection cut off, the electrons of the near end are distributed over the whole surface.

Facts about Induction.

(*i*) Two kinds of electricity are always separated by induction.

(*ii*) Opposite kind of charge is induced on the near end and similar charge is induced on the remote end.

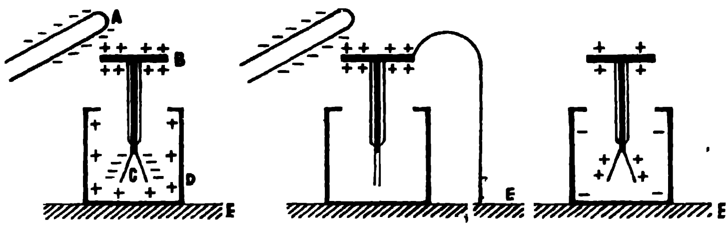
(*iii*) The two induced charges are equal in amount.

(*iv*) The induced opposite charges are temporary ; they neutralise each other when the inducing body is removed.

8 (b). Induction precedes Attraction.—When a charge is brought near an uncharged body, electric separation (separation of positive and negative charges) takes place in the body and the phenomenon is called *induction* ; in the conductor the opposite kind of charge comes nearer to, and the same kind moves farther away from, the inducing charge ; so the attraction between the unlike charges which are nearer will be

greater than the repulsion between the like charges and the net result will be attraction. Hence *induction always precedes the attraction between a charged and an uncharged body.*

9. Charging a Gold-leaf Electroscope by Induction.—Suppose a negatively charged rod is brought near the disc of an uncharged gold-leaf electroscope. Induction takes place ; the disc acquires a positive charge, and the lower part of the metal rod and the leaves acquire negative charge ; the leaves consequently diverge with negative electricity [Fig. 7(*a*)]. If now keeping the charged rod in position, the disc



(*a*) (*b*) (*c*)

Fig. 7

is touched momentarily with the finger, the negative charge of the leaves escapes to the earth and the leaves collapse [Fig. 7(*b*)]. The positive charge on the disc is held in position by the attractive influence of the inducing negative charge. On now removing the charged rod, positive electricity, which is no longer confined to the disc due to the presence of the inducing charge, spreads over the disc, the rod, and the leaves ; and thus the leaves diverge with positive electricity, as in Fig. 7(*c*). So the electroscope is now charged positively by induction, that is, **charging by induction produces the opposite kind of charge.** This is a very convenient way of charging an electroscope, as the charge on the leaves can be regulated by altering the distance between the charged body and the electroscope. The charge on the leaves will be less, if the distance be greater.

Note.—Strongly charged bodies should never be brought close to the electroscope, for in that case repulsion between the leaves may be strong enough to tear away the leaves. For testing such strongly-charged bodies proof-planes are used.

To Charge a Gold-leaf Electroscope Negatively.—Suppose a positively charged glass-rod is brought near the disc of an uncharged electroscope (compare with Fig. 7). Induction takes place, the disc is negatively charged and the leaves positively. The disc is momentarily connected to the earth whereon the positive charge of the leaves

escapes to the earth, and the leaves collapse ; but, according to the **modern view**, it would be more correct to say that due to the influence of the positively charged glass-rod, some of the electrons are brought nearer (on the disc) and thus the leaves of the electroscope are left deficient in electrons, *i.e.* positively charged, and so they diverge. Then, if the disc is touched, the leaves collapse on account of neutralisation due to the electrons rushing *from* the earth *to* the electroscope ; and, if the rod is taken away, *i e.* the influence is removed, the electrons distribute themselves over the leaves and so the leaves again diverge with negative electrification.

10. Three steps in the Process of Charging by Induction.—

(*a*) Bring the charged body near the one to be charged. (*b*) Touch the latter *momentarily*, *i. e.* connect it with the earth. (c) Remove the inducing charge.

11. Detection of the Character of a Charge.—To detect the the character (positive or negative) of an unknown charge, bring the charged body near an electroscope charged, say, *negatively*. If the unknown charge be *negative*, it will act inductively on the electroscope giving the leaves more negative charge ; so the divergence of the leaves will *increase*. If the unknown charge be *positive*, the divergence will *diminish*. Then a greater or less divergence of the leaves for a negatively charged electroscope indicates whether the unknown charge is negative or positive

The result of the experiment can be tabulated as below :—

Brought up near the disc	Electroscope charged	
	Positively	Negatively
Positively charged body	Increased divergence	Collapse or partial collapse
Negatively charged body	Collapse or partial collapse	Increased divergence.

12. Magnetic and Electric Induction Compared.—

Resemblance.—(*i*) As induced magnetism is temporary, so electricity induced in a conductor is also temporary and lasts only as long as the inducing body is present near the induced body.

(*ii*) There is no change in strength either in the inducing electrified body, or in the inducing magnet, when induction takes place.

Difference.—(*i*) In magnetism, induction may also take place between two bodies *in contact*, but in electricity, induction is only possible from *some distance* from each other.

(*ii*) In electric induction, a charged body will induce a charge on any insulated conductor, but magnetic induction is limited to magnetic substances only.

13. Three ways of Producing a Charge.—

(*i*) *By friction* ; (*ii*) *By conduction ;* (*iii*) *By induction.*

(*i*) In this case, the sign of the charge depends upon the material used. (*ii*) In the second case, the charge is of the *same sign* as that which causes it : and (*iii*) in the third case, the charge is of *opposite sign.*

14. Faraday's Ice-pail Experiment.—To perform this experiment a hollow spherical or cylindrical metallic can C open at the top is placed on an insulating stand R, the outer side of the can being connected with a gold-leaf electroscope G by a wire (Fig. 8). The experiment can be divided into three parts :—

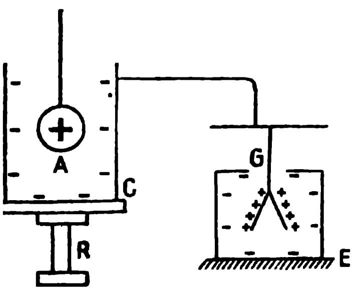


Fig. 8—Faraday's Ice-pail Experiment

(*i*) A positively charged metal ball A held by a silk thread is slowly introduced inside the can, care being taken not to touch any part of the can during the process. It will be found that as soon as the ball is almost surrounded by the can, the divergence of the leaves becomes maximum. If at this state the ball is moved sideways, the divergence does not change but remains constant. The charge on the ball being positive, negative charge is induced on the inside surface of the conductor ; the leaves of the electroscope get positive charge which causes the leaves to diverge. The leaves collapse again when the ball is withdrawn. This proves :—

When a charge is almost completely surrounded by a conductor, the magnitude of the induced charge becomes maximum and remains the same, whether the position of the inducing charge inside the conductor is changed or not, provided it continues to be completely surrounded by the conductor.

(*ii*) The electrified ball is again introduced well inside the conductor. The maximum induction takes place as in the previous case and the leaves diverge with the induced positive charge. The ball is then made to touch the inside surface of the conductor. The divergence of the leaves does not change, and it remains constant even when the ball is withdrawn. Now, on testing the ball, it is found to be completely discharged. The inducing charge being completely neutralised, the induced free charge remaining constant, the neutralisation is due to the induced opposite charge which must be equal in magnitude. Thus :—

When an inducing charge is almost completely surrounded by a conductor, the induced opposite charge is equal in magnitude to the inducing charge.

(*iii*) The positively electrified ball is introduced as before, and the leaves diverge. The outer surface of the can is momentarily touched by the finger, the leaves collapse due to neutralisation of the induced positive charge by the flow of electrons from the earth. The ball is then withdrawn, and the leaves immediately diverge again The amount of divergence is found to be the same as before, but, on examination, the charge is found to be negative As long as the ball is inside the conductor, the induced negative charge is held in position, but, after the withdrawal of the ball, this negative charge spreads and goes to the outside of the conductor, *i e.* to the electroscope, which was previously occupied by the induced positive charge. The equality of the divergence now shows equality of their strength.

If, again, the ball (which retains its original charge) is introduced into the hollow vessel and allowed to touch it, the whole arrangement becomes neutral, and the leaves collapse. This proves :—

The induced positive and negative charges are equal in magnitude.

This experiment was first performed by Faraday who used an ice-pail in place of the cylindrical conductor, hence the experiment is known as **Faraday's Ice-pail experiment.**

Note : *The above experiment (iii) gives a method of transferring the whole of a charge from one conductor to another which is hollow and larger in size.* When the inside of the hollow vessel is touched by the charged conductor, the charge goes on to the outside of the vessel.

15. The Seat of Charge on a Conductor.—If one end of an insulating rod be electrified, the electricity is confined to that end only, but, if an electric charge is given to any part of an insulated conductor, the charge at once distributes itself all over the surface.

(*i*) **Biot's Expt.**—A charged insulated metal sphere C is taken (Fig. 9). Two metal hemispheres A and B provided with insulating handles are put against the charged spheres so as to completely cover it. On removing the covers by the handles, and presenting them before a gold-leaf electroscope, they are found to be charged. The metal sphere C on being similarly tested shows no trace of any charge ; the charge must have passed on to the two hemispheres which formed the outer surface of it.

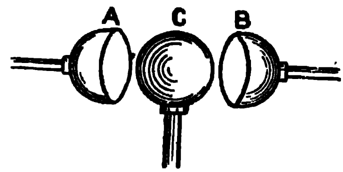


Fig. 9—Biot's Spheres

This shows that **electricity resides on the outer surface of a conductor**. This is known as *Biot's Experiment.*

(*ii*) **Faraday's Butterfly-Net Expt.**—Another experiment, known as *Faraday's Butterfly-Net Experiment,* also confirms the fact that the charge resides on the outer surface of a conductor. A conical muslin net attached to a brass ring mounted on an insulating stand is furnished with two silk threads by which it can be turned inside out (Fig. 10). A charge is given to the net. Test for the charge inside and outside by means of a proof-plane. The charge will be found to be present entirely on the outside. Now turn the net inside out by the silk thread (not by the hand), and test again. The charge will again be found to be only on the outside.

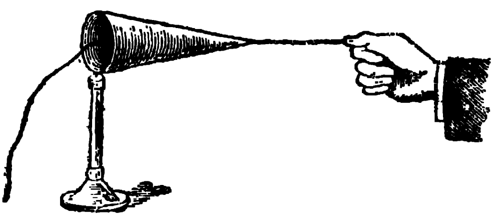


Fig. 10—Butterfly-Net Experiment

(*iii*) **No Charge inside a Hollow Charged Conductor.**—A hollow insulated charged metal sphere with a hole is taken. A proof-plane is inserted to touch different parts of the inside of the sphere and then it is made to touch the disc of a gold-leaf electroscope, when no divergence takes place ; but the leaves diverge if the proof-plane touches the disc after touching any part of the outside surface of the metal sphere. When the proof-plane is placed on the conductor, it becomes for a moment the outer surface of that portion of the conductor and acquires the charge covered by it. It carries that charge when lifted off by the handle. The absence of any divergence in the first case shows that there is no charge inside the conductor (solid or hollow) ; the charge resides solely on the outside surface.

16. Distribution of Charge on Conductors : Surface Density of Charge.—Although the charge on a conductor distributes itself all over the surface, it should not, however, be concluded that the distribution is uniform all over the surface. **The distribution depends upon the shape of the conductor,** *the greater the curvature at any point the greater will be the accumulation of electricity at that point. The distribution is also greatly affected by the neighbourhood of other conductors.*

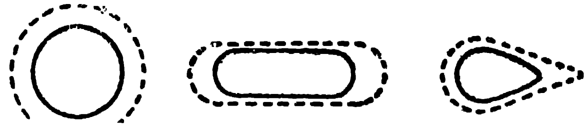


Fig. 11—Distribution of Charge on Conductors

Surface density *is the amount of charge per unit area surrounding a point on the surface of the conductor.* It depends on the (*i*) shape of the conductor, and the (*ii*) neighbourhood of other conductors. The density of the charge is greatest where the curvature of surface is greatest, *i.e.* at sharp bends or pointed portions (see Action of Points in Ch. V). On a sphere the distribution of electricity is uniform (Fig. 11). The distribution of charge on the surface of a conductor at different points can be tested with the help of a proof-plane, as described in Art. 9.

The distribution of charge on conductors of different shapes will be clear from the dotted lines in Fig. 11. The density of the charge in each case is roughly represented by the distance of the dotted line from the conductor.

In the case of a sphere, the curvature at all points being equal, the distribution of charge is uniform. Therefore,

$$\text{Surface density } \rho = \frac{\text{total charge}}{\text{surface area}} = \frac{Q}{4\pi r^2} \text{ units per sq. cm.,}$$

where Q is the total charge, and r the radius of the sphere.

N.B. It should be carefully noted that there is no relation between the shape of a magnet and the distribution of magnetism on it.

Questions

Art. 1.

1. Explain the meaning of the expression "electrification by induction".
(C. U. 1920, '42, '44 ; Pat. 1918, '47)

2. Describe an experiment to show that when an insulated conductor is

electrified by induction, two opposite charges are induced on it, that which is farther from the inducing charge being of the same kind.

Under what circumstances is it possible to transfer the whole of the charge to another insulated conductor ? (Utkal 1947 ; C. U. 1930)

[Hints.—The whole charge can be transferred when the charged conductor touches the inside surface of a hollow conductor. [See also Art. 14)].

3. What experiments will you do to show that electricities generated by electrostatic induction are equal but of opposite kinds. Compare this induction with magnetic induction and explain fully how you can protect an apparatus from the effect of induction in each case.

(Pat. 1928 ; Cf. Utkal 1947 ; Cf. C. U. '42)

(See also Arts. 32 and 33)

4. A bar magnet is divided in the middle and the parts are separated. An insulated conductor (cylindrical with the ends rounded off) is placed in front of the electrified ball with its axis passing through the centre of the ball ; and, while in the presence of the ball, the cylinder is divided in the middle, and the farther half is removed to a great distance. Contrast and explain the state of affairs in the two cases. (C.U. 1912 ; Cf. Pat. 1923).

[Hints.—On dividing the magnet each piece becomes a magnet with a north and a south pole. It is not possible to get an isolated pole. In the next case, when the cylinder is divided in the presence of the inducing charge, the two opposite charges on the two halves are separated, differing in this respect from the first case, where two poles are inseparable.]

Art. 9.

5. If a charged ebonite rod be placed in contact with the knob of an electroscope, the leaves diverge, and on its removal they partially collapse. Explain this. (Pat. 1923 ; C. U. '17)

[Hints.—The ebonite rod being an insulator the leaves diverge : (i) due to the charge received by conduction only from that part of the rod with which the knob is *actually in contact* ; (ii) due to the charge on the remaining portion of the rod received by *induction*. The charges on the leaves, both by conduction and by induction, are of the same kind, so the leaves partially collapse due to the absence of the induced charge when the rod is taken away.]

6. Using an ebonite rod and flannel how would you charge a gold-leaf electroscope (*a*) negatively, (*b*) positively ? (C. U. 1935)

7. Describe a gold-leaf electroscope. Give an uncharged body A on an insulating stand and a body B charged negatively ; how by means of B can you give (*a*) a positive, (*b*) a negative charge ? (C. U. 1924)

8. An electroscope has its leaves charged with positive electricity. An electrified rod is brought close up to the plate of the electroscope, and it is observed that the leaves first collapse and then again diverge. Explain these observations. (All. 1916)

9. How will you obtain a large quantity of negative charge from a small positive one ? Give details of the arrangement ? (Pat. 1932)

[**Hints.**—Take a body A with a small positive charge and two other insulated bodies B and C. Bring B near A, then touch B. B now has got negative charge, induced positive charge being gone to the earth. Now take B near to C and touch C. C gets positive charge. Now take A and C to B, when more negative charge will be induced on B. Repeat this process].

Art. 14.

10. On an inslating stand is placed a metal can the outside of which is connected to a gold-leaf electroscope. A charged metal ball hung by a silk thread is gradually lowered into it till it touches the bottom. Describe and explain the effects produced. (Pat. 1921, '23 ; C. U. 1914, '17)

11. Describe Faraday's Ice-pail experiments, and state clearly the principles that these experiments illustrate. (Dac. 1934 ; Pat. 32, '45 ; Cf. C. U. '40)

Describe an experiment to show that the induced charge equals the inducing charge. (Pat. '44)

Art. 16.

12. What is meant by "the surface density at a point" ? (Pat. 1924, '47)

How does it depend on the shape of the conductor ? (Pat. 1947)

One pole of a battery of many cells is earthed. Two insulated metal balls of 1 cm. and 5 cms. diameter respectively are put one after the other in contact with the other pole of the battery. Compare the surface densities of charge on the two balls. (Pat. 1947)

[*Ans :* 5 : 1]

CHAPTER III

The Electric Field : Potential

17. Electric Field.—The space surrounding an electric charge, or a system of charges, over which the electric force of attraction or repulsion exists, is called its **electric field**. The idea expressed is the same as that of a magnetic field. Theoretically, the field of a charge extends up to infinity but practically it becomes inappreciable after a certain distance.

Electric Lines of Force.—As it is seen in the study of magnetism that a magnetic field can be represented by a series of lines, called magnetic lines of force, so an electric field can be similarly represented, but owing to experimental difficulties it cannot be so easily done in

this case. The idea of lines of force is due to Faraday. A **line of force** *is defined as a curve in an electric field such that the tangent at any point of it shows the direction of the resultant electric intensity of the field at that point. The positive direction of a line of force is that in which a free positive charge tends to move.*

The properties attributed to electric Lines of force according to Faraday are :—

(a) The lines of force originate from a positively charged conductor and terminate on a negative charged conductor. In Figs. 13, 14 and 15, it will be observed that the lines of force start from a positively charged conductor and, as they must end somewhere, they end on the walls of the room when no direct negative charge is near about.

(b) They touch the surface of a conductor at right angles (see Art. 29).

(c) Equal and opposite charges are distributed at the two ends of each line of force.

(d) The positive direction of a line of force is that in which a small positively charged conductor will tend to move, if free to do so.

(e) Lines of force are like stretched elastic threads ; they tend to contract lengthwise and, whilst proceeding in the same direction, they mutually repel each other sideways ; these properties may be used to explain the attraction or repulsion between two charged bodies.

(f) Lines of force never intersect one another ; for if they did, then at the point of intersection there will be two directions for the resultant electric force, which is impossible.

Difference from Magnetic Lines of Force.—(*a*) Electric lines of force always leave the surface of a conductor normally, *i.e.* at right angles to the surface ; but the magnetic lines of force need not leave a magnet or a magnetic substance normally ; electric lines of force are not closed curves like magnetic lines of force.

(*b*) An electric line of force cannot exist inside a conductor while a magnetic line of force is continuous and exists inside a magnetic substance.

Tubes of Force.—To explain electric forces in a medium, Faraday supposed the medium around a charged body to be filled up by a number of imaginary tubes, just as the medium may be supposed to be permeated with lines of force.

Imagine an electric field permeated with lines of force. Suppose, according to Faraday, that the lines are grouped into tubes which touch each other laterally and fill the entire space. If a *definite*

number of tubes are conceived to emanate from a definite charge, they are usually referred to as unit tubes. If *one tube* is conceived to emanate from a unit charge, whatever is the medium, such a tube is called a **unit Faraday tube**.

A tension is supposed to act along the axis of each such tube and a stress normal to the axis. Forces of attraction or repulsion between charged bodies are attributed to the stresses and strains in such tubes.

A tube of force is assumed to start from a positive charge and end on an equal negative charge. Since a tube is subjected to an axial tension, it tends to shorten along its length and thereby bring the opposite charges nearer to each other. The attraction between unlike charges is in this way explained. The stress at right angles to the axis makes the tubes repel each other. In the space between two like charges, the tubes tend to displace each other sideways and thus the like charges are mutually repelled.

18. Law of Force between Electric Charges.—The force of attraction between two unlike charges and of repulsion between two like charges depend on the distance between them. The relationship is expressed by the following law :—

The force of attraction or repulsion between two charged bodies varies directly as the product of their charges and inversely as the square of the distance between them. The latter part of the law is known as the **Law of Inverse Squares**.

If F be the force of attraction or repulsion between two charges q, q', and d the distance between them, $F = c\dfrac{qq'}{d^2}$, where c is a constant depending on the medium and the units chosen.

If the medium be air, and if the unit quantity of electricity be so chosen that this quantity, when placed at a distance of one centimetre from an equal and similar charge, repels it with a force of one dyne, then, for $q = q' = 1$, $d = 1$ cm., and $F = 1$ dyne, we have $c = 1$. Hence, with the unit for charge so defined,

$$\mathbf{F}\ (\text{in air}) = \frac{\mathbf{qq'}}{\mathbf{d^2}}\ \text{dynes.}$$

The force between two electric charges (and also that between two magnetic poles) was first directly measured by Coulomb (1736—1806), a Frenchman, and so this law of force is known also as Coulomb's law of force.

Unit charge.—We have thus got the definition of the **C. G. S. electrostatic unit quantity of electricity** (**E. S. U.**). *It is the quantity of electric charge which exerts a force of repulsion of one dyne on an equal and similar charge placed one centimetre apart in air.* It has no special name.

The **Practical unit** is the **Coulomb** ; 1 coulomb $= 3 \times 10^9$ electrostatic units. (See also Part VII).

19. Influence of the Medium.—The actual magnitude of the force between two charged particles placed at a distance apart depends largely upon the nature of the medium of their separation. This is shown by the following experiment :

Expt.—Suspend from the same point two similarly charged pith-balls. They will repel each other with a certain force and come to rest with a definite distance between them. The medium in this case is air. Now interpose a glass or ebonite plate and notice that the divergence decreases very much. It will be noticed that the distance between the balls will be different for different substances. It is greatest when the medium is air.

This experiment shows that *the electric force depends upon the nature of the medium*, or, in other words, different insulating media have different powers of transmitting electrical influence through them. Faraday gave the name **di-electric** to such a medium. It is an insulator.

We have seen in Art. 18 that, $F = \dfrac{qq'}{d^2}$ dynes.

But, if instead of air, the charged bodies are placed in any other medium, the constant c cannot be equal to unity. The equation then assumes the form,

$\mathbf{F} = \dfrac{1}{k} \cdot \dfrac{\mathbf{qq'}}{\mathbf{d^2}}$, where k is a constant depending on the medium. This constant is called the **di-electric constant** or the **Specific Inductive Capacity** (**S. I. C.**) of the medium (see Art. 39). That is, if the intensity in air is F, the intensity will be F/K when air is replaced by a medium of di-electric constant K.

20. Intensity of an Electric Field.—A somewhat similar procedure to that adopted in defining magnetic field is used for electric field also, unit charge being substituted for unit magnetic pole.

The **intensity** or **strength** of an electric field at a point is measured by the force in dynes exerted on a unit positive charge placed at that point.

The **electric force** exerted on a charged body placed in a field depends on : (*i*) *the intensity or strength of the field* ; (*ii*) *the amount of charge on the charged body.*

An electric field has unit intensity at a point when a unit positive charge placed at the point is acted on by a force of one dyne. Thus,

force on unit charge in a field of unit intensity			= 1	dyne
,, ,, 2 units	,,	,,	= 2	dynes
,, ,, q ,,	,,	,,	$= q$	,,
,, ,, q ,,	,,	f	$= f \times q$	,, .

So, the force $F = f \times q$, where f is the intensity of the field and q the charge on the body.

Electric Intensity in terms of Lines or Tubes of Force.—The intensity of an electric field at any point is also measured by the number of lines of force passing through a unit area surrounding that point, placed normal to the direction of the lines of force. Thus F lines of force will normally pass through a unit area around a point where the intensity is F c. g. s. units. The number of Faraday tubes that will pass through the same point is calculated as follows :—Consider a charge Q e. s. u. placed in a medium of specific inductive capacity K, and imagine a spherical surface of radius r cms. drawn in the field with Q as centre and passing through the point, where, suppose, the intensity is F. The number of Faraday tubes passing per unit area

of the spherical surface $= \dfrac{\text{number of Faraday tubes}}{4\pi r^2} = \dfrac{Q}{4\pi r^2}$

$$= \frac{K}{4\pi} \times \frac{Q}{Kr^2} = \frac{K}{4\pi} \times \text{ intensity of field} = \frac{K}{4\pi} \times F.$$

$$\frac{K}{4\pi} \times \text{number of lines of force.}$$

Number of Lines of Force associated with Unit Charge.—Consider a small body having unit charge. If another body having an equal like charge be placed in air at a distance of one centimetre, the mutual force of repulsion is one dyne. The intensity at this distance is, therefore, unity. If we imagine a sphere to be constructed having a radius of 1 cm. with its centre at one of the bodies, the intensity at all points on the circumference is unity. Since intensity is unity, one line of force, according to definition of intensity in terms of line of force, would pass through each sq. cm. area of the sphere. Therefore, the total number of lines of force cutting the sphere would be 4π. Since all the lines of force that emanate from the unit charge must thread through the surface of the sphere, the total number of lines of force associated with unit charge is 4π.

According to definition of a unit Faraday tube, only one tube originates from unit charge. Therefore a unit Faraday-tube contains 4π lines of force.

Lines of Force associated with Charged Conductor.—In Art. 16, the distribution of density of charge on a conductor has been discussed. Fig. 12 depicts the lines of force associated with different types of charged conductors. In the case of the uniform sphere, the lines are uniformly distributed all around the body. At curved ends or pointed places in other conductors density of charge being greater, more lines are assumed to start from such places.

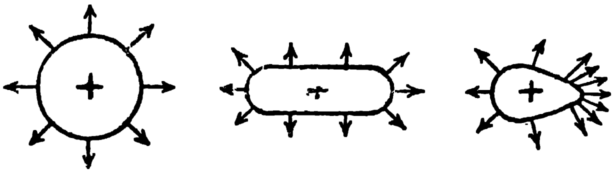


Fig. 12—Distribution of Lines of Force

Examples. *1.—Two small spheres each of mass one decigram are suspended from a point by threads each 50 cms. long. They are equally charged and they repel each other to a distance of 20 cms. If $g = 980$ cms./sec.^{2}, what is the charge on each?*

(All. 1930)

Make a sketch of the arrangement. Let C be the point of suspension. The two threads CA and CB and the line joining the spheres make an isosceles triangle. D is the middle point of AB. The forces acting on one of the spheres A are :—

(*i*) The weight mg dynes acting downwards at A. (*ii*) The repulsive force q^2/d^2 in the direction BA (where q is the charge on each, and d the distance between them). (*iii*) The tension of the thread in the direction AC.

Here the forces mg, q^2/d^2 and that due to the tension are parallel to the sides of the triangle CDA. Hence the forces are proportional to the sides of the triangle to which they are parallel. Therefore, we have, $\dfrac{q^2/d^2}{mg} = \dfrac{DA}{DC}$.

$DA = 10$, $DC = \sqrt{50^2 - 10^2} = 10\sqrt{-24}$, $AB = d = 20$, $m = \frac{1}{10}$ gm.

$$\therefore \frac{q^2}{20^2} = \frac{1}{10} \times 980 \times \frac{10}{10\sqrt{24}} = \frac{98}{\sqrt{24}}\text{ ; whence } q = 89\cdot4 \text{ units.}$$

2. A small charged sphere is made to touch a similar uncharged sphere suspended by a silk thread from the hook of a balance and then held vertically below it. When the difference between the centre of the spheres is 5 cms, the apparent loss of weight is 0·002 gram. What was the initial charge on the sphere?

Let $2Q$ be the initial charge on the sphere, so that after touching and sharing charge, charge on each $= Q$.

The mutual force between them at distance 5 cms. $= \frac{Q^2}{5^2} = \frac{Q^2}{25}$ dynes.

This is equivalent to the apparent reduction in weight, *i.e.* earth's force of attraction, of 0·002 gm. wt. $= 0{\cdot}002 \times 981$ dynes.

$\therefore \quad \frac{Q^2}{25} = 0{\cdot}002 \times 981 = 1{\cdot}962$; whence $Q = 7$ units (approx).

So the initial charge was 14 units.

3. Two small pith-balls of the same size and the same weight (each 0·1 grm.) are suspended from the same point with silk-fibres, each of 5 cms. length. On charging them together they repel each other and their suspensions make an angle of 60°. Calculate the quantity of electricity on each ball (Take 'g' as 980 units.) (Pat. 1939)

(Proceed as in *Ex.* 1). Here $\angle CAB = 60°$ and the two sides equal, so the ΔCAB is equilateral. Now proceeding as above we get,

$\frac{q^2/25}{980 \times 0{\cdot}1} = \frac{DA}{DC} = \frac{\frac{5}{2}}{\frac{5}{2}\sqrt{3}} = \frac{1}{\sqrt{3}}$; whence $q = 37{\cdot}59$ units.

21. Lines of Force in a Few Cases.—There is no convenient way of mapping the electric lines of force practically. In a strong field the direction of the forces can be demonstrated by a small piece of paper held quite loosely on the thin end of a glass rod. The diagrams, shown in Figs. 13 to 15, indicate the distribution of lines of force in several cases. (*a*) Fig. 13 represents the electric field due to a positively charged spherical conductor placed in the centre of a large room. The arrows indicate the positive direction of the lines of force. It is evident that the lines are all radial. They appear to have originated from the centre of the spherical conductor. In the case of a negatively charged conductor the field will be similarly represented except that the direction of the arrows would be reversed. (*b*) Fig. 14 represents the case of two conducting spheres having equal but opposite charges. There is a tendency for the two spheres to draw together as the lines of force tend to contract lengthwise. (*c*) Fig. 15 shows the case of two spheres having equal positive charges. In this case, the lines of force travelling in the same direction repel one another sideways, and hence there is repulsion between two like charges. There will be a **neutral point** in this case (marked X) between the two spheres.

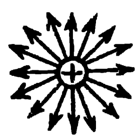


Fig. 13

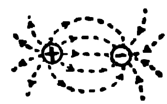


Fig. 14

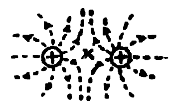


Fig. 15

Note that the *lines of force* do not pass through a conductor but they end on the surface. In the figures, the lines of force are represented in one plane but actually they are going in all directions.

Inspite of these diagrams it should be remembered that the lines of force are only imaginary, they have no actual existence.

22. Potential.—Suppose, we have two insulated conductors charged positively. If they are put into electric communication by means of a pair of discharging tongs (Art. 42), or by a metal wire, electricity will generally flow from one conductor to the other, but the transference of electricity, and the direction in which it will flow, will depend on the **electric condition** of the two conductors ; and, in some cases, it may be possible that there will be no transference from one conductor to the other.

Let us take two analogous cases from other branches of Physics :—

(1) **The Hydrostatic Analogy** :—Consider two cylindrical vessels containing water, connected by a tube provided with a stop-cock. On opening the stop-cock, it will be found that water flows from the vessel in which water-surface has the higher level to the other, even though the quantity of water in the first vessel may be less than that in the other. There will be *no flow if the levels are the same.*

(2) **The Temperature Analogy** :—If a small copper piece be heated red-hot and placed in a bucketful of water at the room temperature, heat will flow from the copper piece to the water though the total quantity of heat in water may be much greater. Heat flows from the copper piece to water only because the former is at a higher temperature. There will be *no flow of heat* from either of them to the other *if their temperatures were the same.*

The **analogy** between temperature and potential may be considered thus :—

Temperature	**Potential**
(*a*) Temperature is the *condition* of a body on which the flow of heat from it to other bodies, or *vice versa*, depends.	(*a*) Potential is the *electric condition* of a charged body due to which electricity tends to pass from it to other charged bodies, or *vice versa*.
(*b*) Heat can be added due to which the temperature of a body rises, and heat may be taken out due to which temperature falls.	(*b*) Potential of a body is raised by successive additions of positive electricity, and it is lowered by the addition of negative charge or withdrawal of positive charge.

(*c*) Two bodies at different temperatures, when in thermal contact, acquire an intermediate temperature.	(*c*) Two conductors at different potentials, when electrically connected, acquire an intermediate potential.
(*d*) Heat, when added to a homogeneous conductor, is *uniformly distributed* throughout the conductor at *one temperature.*	(*d*) When a charge is given to a conductor, the quantity of charge on it may be more at one part than at another, *depending on its shape and neighbourhood of other conductors*, but the conductor will have the *same potential throughout.*
(*e*) In measuring temperature, two standards are taken, (i) melting point temperature of ice, and (ii) boiling point temperature of water.	(*e*) One standard is taken—the potential of the earth, which is taken as zero.

The relation of *potential* to quantity of electricity is analogous to the relation of *pressure* to quantity of water, or *temperature* to quantity of heat. So electric potential is also termed **electric pressure.** Electricity flows from a conductor at a higher potential to that at a lower one until their potentials become equal, when there will be no further flow.

• **So, the potential of a charged conductor** *may be defined as the electric condition of the body which determines the direction of flow of electricity, when put in conducting communication with another body.* If positive electricity flows from it to the other, it is at a higher potential, and if positive electricity flows to it from the other, it is at a lower potential. It is, therefore, a measure of its electric pressure or level.

Explanation of Potential from Modern Theory.—Unattached electrons on a charged body repel each other and also repel other electrons near them with forces acting in the surrounding medium along lines which are termed "**lines of electric force**"

It is considered that the *medium* surrounding a charged body is in a *state of strain.* If the medium be air, then the air, an insulator, is considered to possess the power of withstanding this force, or 'pressure', due to the mutual repulsion among the electrons of the charged body. This pressure is termed **electric potential**. Conductors may be regarded as media which are unable to support this electric pressure, so there is a movement of the electrons in conductors (see Art. 30), which is the reason of electrification by induction (or influence).

23. Potential of the Earth : Positive and Negative Potential.—We know that in measuring the temperature of a body by the Centigrade scale, the melting point of ice is taken as the standard, or the zero, the temperatures above this point are considered as positive ; while those lower than this are considered as negative. Similarly the potential of any body is measured with reference to **the potential of the earth**, which is taken as the standard, and whose value **is always taken to be zero.** The justification for this selection is that the potential of the earth can not be changed by addition or subtraction of charges ; it maintains its potential constant. The potential of a *positive* charge is *above* the potential of the earth and the potential of a *negative* charge is *below* that of the earth.

The electric field around a positively charged body is a region of *positive potential*, the value of which gradually *diminishes* with the increase of distance from the charged body. The electric field around a negatively charged body is a region of *negative potential*, the value of which gradually *increases* with the increase of distance from the charged body.

So, (*i*) *A positively charged body tends to travel from a point of higher potential to that of lower potential*, that is, *down the gradient of potential*, and (*ii*) *a negatively charged body tends to travel from a point of lower potential to that of higher potential*, because the direction of the forces acting on a negatively charged body is *opposite* to that of forces acting on a positively charged body.

An earth-connected conductor has **zero potential**. A positively charged conductor has a positive potential, and a negatively charged conductor has a negative potential. The **potential of a conductor** is considered **positive** when, if earth connected, the positive electricity flows *from* the conductor *to* the earth, (according to modern view the *electrons* would *flow* in the opposite direction, *i.e.* from the earth to the conductor). The **potential of a conductor** is considered **negative** when, if earth connected, the positive electricity flows *from* the earth *to* the conductor (the *electron flow* would be from the conductor to the earth)

When two charged conductors at different potentials are joined together, the flow of electricity from one to the other is entirely a matter of relative potential and does not depend on the actual amount of electricity present. There will be no flow if they are at the same potential, irrespective of the charges present in either.

24. Measurement of Potential.—Suppose we have a body positively charged with q units of electricity. If another body positively charged with q' units is brought near it, there will be a force of

repulsion between them. The mutual force exerted, when the second body is placed at a distance r from the first body, is qq'/r^2. To bring the second body towards the first body against the force of repulsion, work must be done. Suppose the second body has got unit quantity of positive electricity ($q' = 1$) and is at a very great distance where the electrical force due to the first body is negligibly small. As the second body is gradually brought towards the positively charged body, more and more work will be done, and the total amount of work done in bringing the unit charge of the second body up to any point in the field of the first body is taken as a measure of the potential at that point.

Thus, **the potential** *at any point in an electrical field may be defined as the work done by or against electric forces in bringing a unit positive charge from an infinite distance up to that point.* From this it follows that,

The difference of potential *between two points in an electric field is the amount of work done by or against the electric forces in moving a unit positive charge from one point to the other.*

Potential of a conductor may be measured by the work done in bringing a unit positive charge from infinity (point of zero potential) to a point close upon the conductor.

Unit of Potential.—If the work done in bringing a unit positive charge from infinity (*i.e.* a point of zero potential) up to a point is 1 erg, then the potential of the point is 1 electro-static unit (*E. S. U.*). This unit has no special name. The practical unit is the **volt** (see Art. 28, Part. VII) ; 1 volt. = $\frac{1}{300}$ *E. S. U.*

• Thus, *in the c. g. s. system, the difference of potential is measured in ergs per unit charge.*

N. B.—The leaves of an electroscope diverge more than a centimetre for a potential difference of about 1000 volts.

24(a). Potential at a Point due to a Charge.—The potential at P due to a charge at A, according to definition, is measured by the amount of work done in bringing *a unit positive charge* from infinity up to the point P (Fig. 16.)

Suppose there is a small spherical conductor at A having a charge of $+q$ units. Let us find the potential at the point P at a distance r from A. Join A to the point P and produce the straight line to infinity. Let P_n represent a point at infinity, *i.e.* at a very great distance r_n from A

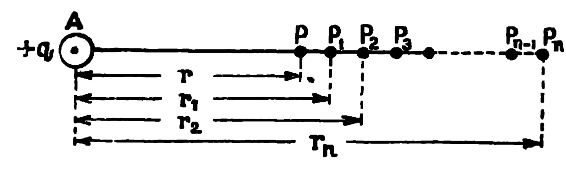


Fig. 16

upon this straight line. Let the distance $(r_n - r)$ be divided into a large number of parts PP_1, P_1P_2,......$P_{n-1}P_n$, each being equal but infinitely small.

The force acting on a unit positive charge at $P = q/r^2$,

and ,, ,, ,, ,, ,, $P_1 = q/r_1^2$.

$\therefore$ The average force between P and P_1 may be represented by the geometric mean and may be put equal to q/rr_1 (the distance PP_1 being very small).

[Alternatively, the average force approximately is, $\dfrac{q/r^2 + q/r_1^2}{2}$

$= \dfrac{q}{2}\left(\dfrac{r_1^2 + r^2}{r^2 r_1^2}\right) = \dfrac{q}{2r^2 r_1^2}\{(r_1 - r)^2 + 2r_1 r\} = \dfrac{2r_1 r.q}{2r^2 r_1^2}$; $(r_1 - r)^2$ being negligible compared to $2r_1 r$ $= q/rr_1$.]

The work done against electrical forces in moving a unit positive charge from P_1 to P = average force × distance $= \dfrac{q}{rr_1}(r_1 - r) = q\left(\dfrac{1}{r} - \dfrac{1}{r_1}\right)$.

Similarly, the work done in moving the unit positive charge from P_2 to $P_1 = q\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$; and so on. In like manner, the work done in moving the unit positive charge from P_n to $P_{n-1} = q\left(\dfrac{1}{r_{n-1}} - \dfrac{1}{r_n}\right)$.

Adding all these terms, the total work done in moving the unit positive charge from P_n to $P = q\left(\dfrac{1}{r} - \dfrac{1}{r_1}\right) + q\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right) + \ldots\ldots + q\left(\dfrac{1}{r_{n-1}} - \dfrac{1}{r_n}\right)$

$= q\left(\dfrac{1}{r} - \dfrac{1}{r_n}\right)$.

But, if V_r and V_n represent the potentials at distances r and r_n respectively, the total work done in moving the unit positive charge from r_n to $r = V_r - V_n$.

$$\therefore \qquad V_r - V_n = q\left(\frac{1}{r} - \frac{1}{r_n}\right).$$

But since r_n is infinitely large, $\dfrac{1}{\infty} = 0$, and $V_n = 0$. Hence $V_r = \dfrac{q}{r}$.

That is, **the potential at a point distant r from a charge q**

$$= \frac{q}{r} = \frac{\textbf{charge}}{\textbf{distance}}.$$

24(b). Potential due to Several Charges.—The potential at a point due to several charges q_1, q_2, etc., at distances r_1, r_2, etc., respectively from the point, is the algebraic sum of the potentials due to each charge, *viz.*—

$$V = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots\ldots = \Sigma\left(\frac{q}{r_i}\right).$$

25. Relation between Electric Force and Potential Difference.—Suppose there are two points A and B, having potential V_a and V_b situated in an electric field and placed at a small distance apart. Of these two points, B is at a lower potential, so the electric force acts from A to B and is practically uniform, if the distance between A and B is small If F be the electric force, and d the distance AB, we have, the potential difference between A and $B = V_a - V_b$ which, by definition, is equal to the work done on a unit positive charge in taking it from A to B. But the work done is equal to $F \times d$.

$$\therefore \quad V_a - V_b = F \times d\,; \quad \therefore \quad F = \frac{V_a - V_b}{d}.$$

26. Gold-leaf Electroscope as a Measurer of Potential.—We have seen in Art. 4 that in using a gold-leaf electroscope the tin-foils inside the case should be earth connected so that they would have zero potential. When a charged body is brought near to, or in contact with, the disc, the rod and the leaves acquire a potential due to the charged body. **The leaves diverge due to the difference of potential between the leaves and the case.** Each of the leaves moves towards the wall of the case near it (being free to do so) on account of the difference of potential and a divergence is produced between themselves. This divergence will be a measure of the potential of the disc of the electroscope. There will be divergence both for positive and negative potentials. If the electroscope be placed on an insulating stand and the disc is earthed so that the disc and the leaves are at zero potential, then the leaves will also diverge when a charged body is brought in contact with the tin-foils of the case, due to the difference of potential between the leaves and the case. There will be no divergence of the leaves, no matter what charge is given to the electroscope, when the disc and the case, standing on an insulating slab, are joined together. When an electroscope is used to measure potential, its disc is connected to the charged body, and the case is earthed.

[**Note**—In experiments on electro-statics, each apparatus used should be *well dried* to prevent leakage of electricity along any film of moisture.

An *electroscope* may also leak when it is *too hot*, as the ascending current of hot air will carry a part of the charge of the central rod as it comes in contact with it.]

27. **Proof-plane and Electroscope.**—The difference in procedure in using a proof plane and an electroscope to indicate charge, and potential, of conductors should be very carefully noted. **To test charge** at any point of a conductor, that point is touched with a proof-plane. The proof-plane takes off some charge from that point and the charge is then transferred to the electroscope when it is touched by the proof-plane. The electroscope thus acquires a potential according to the charge on the proof-plane. **To test potential**, however, the conductor and the electroscope are put in direct contact through the metallic disc of the proof-plane.

28. **Potential of a Charged Conductor.**—To examine the distribution of potential over the surface of a charged conductor, the metal part of a proof-plane is connected to a gold-leaf electroscope by a wire, and, by means of the insulating handle, the metal part is slided over the surface of the charged conductor (Fig. 17). The divergence of the leaves will be found to remain constant for all points of the surface, no matter whether the surface density be the same at all points or not. This shows that **all parts on the surface of a charged conductor are at the same potential,** though the surface density of charge may vary from place to place.

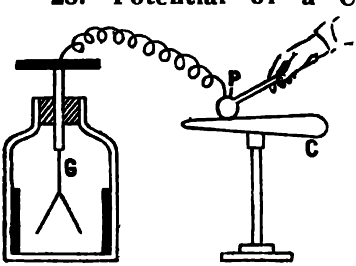


Fig. 17

In the case of a **charged insulator**, differences of potential usually exist between various points on the surface.

Potential Inside a Hollow Conductor.

It has been already seen that there is no charge on the inner surface of a charged hollow conductor and hence there is no line of force there. For this reason no work is done if a charged body is moved within the conductor, and because work is done due to difference of potential, there is no difference of potential within it, or, in other words, the **potential is constant** *at all points within a hollow conductor.*

The following experiment proves that *the potential throughout the inside of a closed hollow conductor is the same as that on the outside.* It should be noted that *there is* **no charge** *on the inside surface,* **but there is a potential**—*the same as that on the outside.*

Expt.—(1) Place a fairly deep metal can A on an insulating stand and charge it positively. Take a glass rod R and wrap one end of a wire round it and connect the other end of the wire to the disc of an electroscope placed at a considerable distance away from the charged can so that the can may not have any inductive influence on the electroscope E, the case B of which is earthed and so is at zero potential. Now holding the glass rod with hand, introduce the coiled end of it gradually within the can and notice that the leaves of the electroscope gradually diverge, which reaches a constant value when the coiled end of the rod is well within the can. Moving the rod into various positions within the vessel at this stage, it will be observed that the divergence remains unaltered. This shows that *the potential throughout the inside of the can is the same.* If the bare end of the coiled wire at that stage is made to touch the can, the divergence of the leaves does not alter. This proves that the potential in the hollow is the same as that of the can.

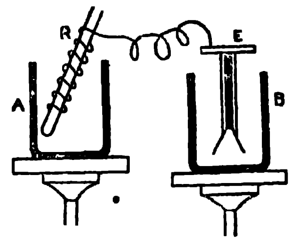


Fig. 18

(2) To prove that the potential in the hollow is the same as that on the outside of the can, place an uncharged electroscope inside the can so that its metallic base connected to the tin-foils inside is in contact with the can and its disc is well within the vessel. It will be seen that there is no divergence of the leaves proving that the potential of the disc, which is in the hollow of the can, is the same as that of the base. But the base is in contact with the can and so the potential of the base is the same as that of the can itself. So *the potential of the hollow space is the same as that of the outside surface of the can.*

[**Note.**—If an uncharged body is put well inside the charged hollow can, the body, although quite neutral at all parts, will have the same potential as the charged can. *This is the* **only instance** *where* **an object can have a potential,** *positive* or negative, **and yet be free from any charge.**

If the opening of the hollow can is small, the potential inside the can will be found to be the same as that of the can itself ; but this

does not hold good in the case of a can with a large opening, because in that case the inner space is not completely surrounded by the electrified conductor.]

29. **Equipotential Surface.**—An equipotential surface is a surface on which the potential is the same at all points. From the definition of potential it is clear that there cannot be any flow of electricity along an equipotential surface, or, conversely, a surface along which there is no flow of electricity is an **equipotential surface**. Thus in statical electricity, the surface of an insulated charged conductor is an equipotential surface. For, if this is not the case, there would be a difference of potential between certain points on the surface, and then a current would flow from the points of higher to those of lower potential and the electricity would no longer be statical. Since every point is at the same potential, no work is done in moving a charge from one point to another on an equipotential surface. So, there is no component of electric intensity along the surface. Hence, *the lines of force* (which start normally from a charged conductor) *cut an equipotential surface everywhere at right angles.*

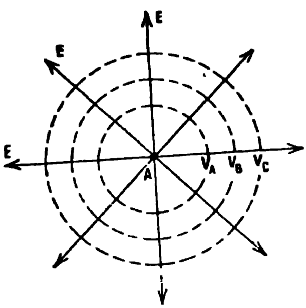


Fig. 18(a)

In the case of a single charge concentrated at a point, or a charged spherical body, equipotential surfaces are spherical surfaces concentric with the point or the sphere. In Fig. 18 (a), A is a point charge round which concentric equipotential surfaces, having potentials V_A, V_B, V_C, etc., have been drawn at successively greater distances. The lines of force, marked E, radiate out in all directions uniformly and are at right angles to the equipotential surfaces. In other cases, the equipotential surfaces have no definite form.

29(a). The following experiment can be performed to show that **the surface of an electrified conductor is an equipotential surface.**

Expt.—Take any insulated pear-shaped conductor (as in Fig. 17) charged with electricity and make the experiment as described in Art. 28. The gold-leaves will indicate the same divergence as the proof-plane is slided along the different points on the surface inspite of the fact that the surface density at the pointed end is greater, *i.e.* there is more electricity at the pointed end than at the rounded end. The equal and constant divergence shows that, after the leaves have diverged, there is no transference of electricity between the conductor and the electroscope. Hence the potential of every point of the con-

ductor and that of the charged electroscope must be the same, or, in other words, the surface of the conductor is equipotential.

29(b). A Line of force at any point is Perpendicular to an Equipotential surface passing through that point.—Let OE represent a force through a point O on a charged conductor AB, where the intensity of the field is F [Fig. 18(b)]. Suppose a unit charge is moved through a distance l in the direction OD which makes an angle θ with OE. Then the work done = force in the direction $OD \times$ distance $= F \cos\theta \times l$. The work done will be zero, when $\theta = 90°$, *i.e.* in a direction normal to OE. The trace of the surface normal to OE, along which the work is zero, gives, according to definition, the equipotential surface through O. That is, a line of force and an equipotential surface are perpendicular to each other.

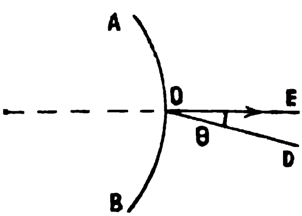


Fig. 18(b)

30. Explanation of Electric Induction by Potential: Free and Induced Potential—Suppose A is a positively charged body (Fig. 19). Then the conductor A has a positive potential due to its own charge, and it is spoken of as its **free potential**. The space surrounding A acquires a positive potential which diminishes quickly near about the charged body A, and then more slowly as the distance increases from A. Now consider two points B and C [Fig. 19(a)] in the surrounding medium (air); both of which will acquire positive potential, the potential at B being greater than that at C, (*cf.* $V = q/r$). The difference of potential, or electric pressure, between B and C will tend to move electricity from B towards C, but the medium being non-conducting there cannot be any flow of electricity in it as in the case of a conductor. The medium will set up an opposing force against the

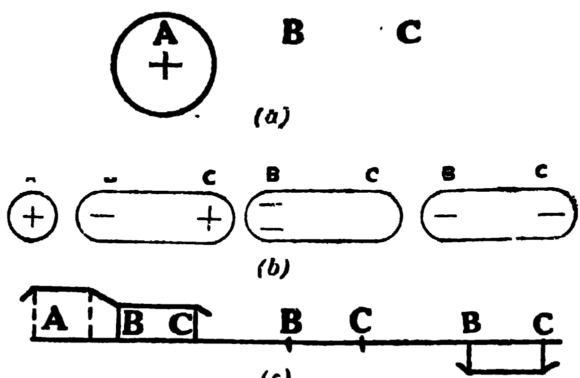


Fig. 19

forward electric pressure and will balance it. Thus *the insulating medium is strained.*

If now an insulated conductor takes up the position BC [Fig. 19(b), 2], electricity will at once flow from B to C, due to the difference of potential between B and C, until the potential is uniform over the whole conductor. This is because *a conductor cannot set up any opposing force like an insulator* and so there is a flow of electricity. The result is that there will be a deficit, or negative charge, at the end B and surplus, or positive charge, at C. The phenomenon is known as *inductive displacement.*

It should be noted that BC has no charge as a whole but is at a positive potential due to the presence of the charged body A. This is called an **induced potential** produced by the inductive influence of A acting through the medium air (di-electric). The induced potential of BC will be positive or negative according as the potential of A is *positive or negative.*

If now BC is earthed, positive charge flows to the earth and the potential becomes zero, though it has a negative charge [Fig. 19(b), 3]. It will be found that a greater negative charge appears at B when BC is earthed, as then electricity flows out of the conductor to the earth until it will have such a negative charge that the negative potential on BC due to this negative charge becomes exactly equal to the positive potential due to A, so that the actual potential of BC is zero. If A be now removed and BC disconnected from the earth, the negative charge of BC is distributed on its surface and it is at a *free negative potential due to its own negative charge* [Fig. 19(b), 4].

30(a). Explanation by Electron Theory.—It should be said that, due to difference of potential, electrons move from points of lower potential to points of higher potential, *i.e.* from the end C to the end B [Fig. 19(b)2]. So the end B having a surplus of electrons becomes negatively charged and the end C having a deficit of electrons becomes positively charged. Again when BC is earthed [Fig. 19(b), 3], electrons flow from the earth to it and the potential becomes zero. If A be now removed and the earth-connection is disconnected the surplus electrons which have come from the earth are distributed on BC, which consequently becomes charged negatively and acquires a free negative potential. [Fig. 19(b), 4].

Potential Diagrams.—The facts stated above are graphically represented in Fig. 19(c), where the horizontal line represents the *zero* potential. The potential at other points are plotted according to their values which are represented by ordinates at different distances. It should be noticed that *the potential of a conductor is uniform and there*

is a quick reduction of potential near a conductor, after which the reduction is gradual. Negative potentials are represented by ordinates below the horizontal line.

31. Explanation of Electric Induction by Lines of Force.— *Case I*—Suppose a positively charged glass-rod A is brought near one end B of an insulated uncharged conductor BC [Fig. 20(i).] In the absence of the conductor BC electric lines of force will proceed from the positively charged rod A and will travel through air up to the walls of the room, but as soon as BC is brought near it, some of the lines of force will travel through BC, as a conductor provides a path of much less resistance than air. The end B, where the lines of force enter, acquires a negative charge and the end C, where they leave the conductor, acquires a positive charge ; and since BC as a whole is neutral, as many lines of force leave the end C as enter at B. So the induced positive and negative charges are equal.

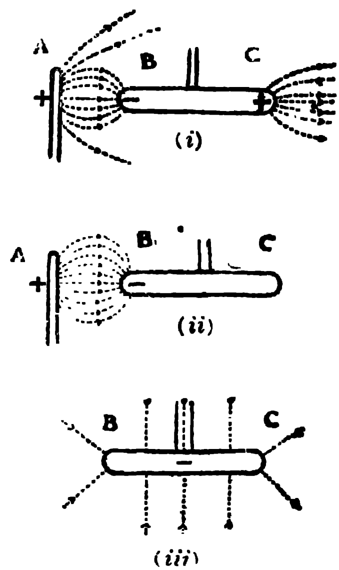


Fig. 20

Notice, in Fig. 20(i), that all the lines of force proceeding from A do not terminate on B ; some of them proceed and end on the walls of the room. This shows that *the total positive charge on A is greater than the induced negative charge at the end B.* This is always the case unless the inducing body is completely surrounded by the induced body (see Art. 14).

Case II—When BC is earthed [Fig. 20 (ii)], its potential is reduced to *zero* and the lines of force from C disappear, that is, the free positive charge is neutralised, and many lines of force, which formerly passed direct to the walls of the room, which are at zero potential, now traverse a shorter distance to the end B, which is also at zero potential, thus increasing the number of lines of force entering the conductor at B. So *the negative charge at B is slightly increased.*

Case III—When the earth connection is cut off and the charged glass-rod is removed to a distance, the lines of force terminating at B spread uniformly over the surface of the conductor, that is, the nega-

tive charge is distributed all over the conductor [Fig. 20(*iii*)]. Note that in this case lines of force proceed from the walls of the room towards *BC*.

31(a).—Explanation of Charging a Gold-leaf Electroscope by Induction in terms of Lines of force.—The charging of a gold-leaf electroscope by induction can be explained in terms of lines of force proceeding as in Art. 31. The figure 21 gives the three cases corresponding to that article leading to the charging of the electroscope negatively by induction, the two tin-foils within the electroscope having been shown to be permanently connected to the earth.

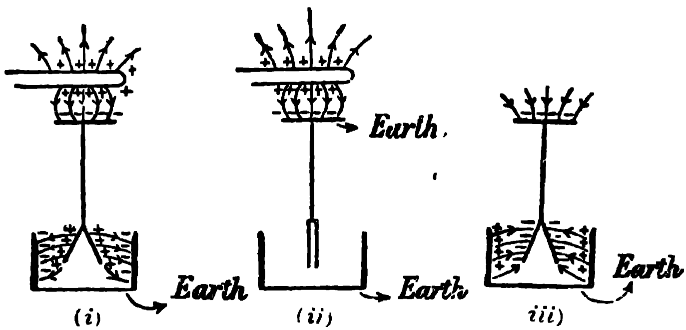


Fig. 21—Charging a Gold-leaf Electroscope by Induction

Explanation.—When the positively charged rod is brought near the disc of the uncharged electroscope, a number of lines of force starting from the rod terminates on the disc. As the disc, the brass rod, and the leaves of the electroscope form one conductor, the number of lines of force emerging out of the positively charged leaves and terminating on the earth-connected tin-foils within the electroscope is the same as the number ending on the disc. The tin-foils facing the leaves are thus negatively charged by induction, the positive being neutralised by flow of electrons from the earth. These lines of force which are supposed to have a force of tension drag the leaves apart as in Fig. 21(*i*). In Fig. 21(*ii*), the disc is touched and the positive charge on the leaves is neutralised and the lines of force between the leaves and the tin-foils vanish, and so the leaves now having no tension collapse. In Fig. 21(*iii*), the charged rod is taken away and the negative charge on the disc is distributed all over the conducting parts. This time the leaves being negatively charged, the tin-foils become positively charged by induction. The lines of force starting from the tin-foils terminate on the leaves and draw the leaves apart.

Explanation of Faraday's Ice-pail Experiment by Lines of Force :—As the charged ball A is gradually introduced within the can, more and more of the lines of force started from the charged ball will end on the can. If the ball is not sufficiently inside, the induction

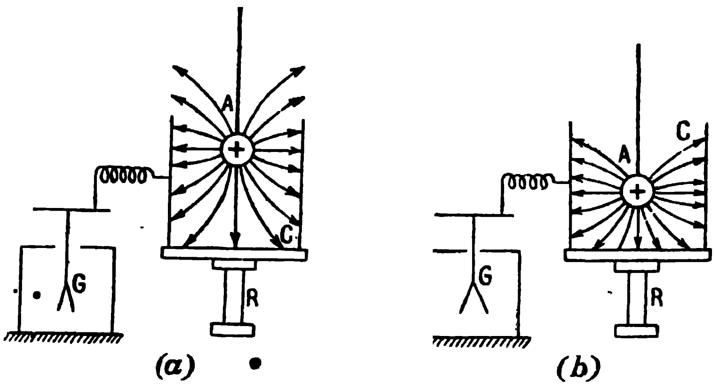


Fig. 22

in the can is bound to be incomplete, for all the lines of force will not terminate on the can [Fig. 22(a)]. When the ball is almost surrounded by the can, all the lines of force emanating from it must end on the inside of the can [Fig. 22(b)], and so the induction will be maximum. Since a line of force starts from a positive charge and end on an equal opposite charge, the inducing charge is equal to the induced opposite charge.

Examples.—*1. Charges of 3, —4, 5 and 6 units are placed at the corners A, B, C, D respectively of a square, each of whose sides is equal to 10 cms. Find the potential of the middle point of the square.*

Distance of the middle point from each of the corners of the square $=10/\sqrt{2}$

$$\therefore \quad V=\frac{3\sqrt{2}}{10}+\frac{-4\sqrt{2}}{10}+\frac{5\sqrt{2}}{10}+\frac{6\sqrt{2}}{10}=\frac{10\sqrt{2}}{10}=\sqrt{2}=1{\cdot}414 \quad \text{units.}$$

2. *A hollow spherical conductor, whose radius is one decimetre is charged with 10 units of electricity. Find the potential (a) at the surface of the sphere, (b) inside it, and (c) at a point 25 cms. from the centre.* (*C. U. 1934*)

(a) One decimetre = 10 cms. Potential $V=\frac{Q}{r}=\frac{10}{10}=1$ E. S.U.

(b) The same as that on the outside, *i.e.* 1. E. S. U.

(c) $$V=\frac{Q}{r}=\frac{10}{25}=0{\cdot}4 \text{ E. S. U.}$$

3. *Find the work done when a charge of -10 units is removed from any point A, at a distance of 10 cms., to another point B, at a distance of 20 cms., from a charge of 80 units.*

Potential at $A = V_a = \frac{80}{10} = 8$; Potential at $B = V_b = \frac{80}{20} = 4$.

Potential difference $= V_a - V_b = 8 - 4 = 4$

$\therefore$ The work done in removing -10 units $= 4 \times -10 = -40$ ergs.

The work is negative because the electric attraction resists the movement of the charge as it is moved from a higher to a lower potential.

32. Electrostatic Screens.—If it is necessary to shield any delicate instrument from the disturbing effects of other charged bodies, the instrument may be placed inside a hollow conductor, because there can be no induced charge inside the hollow conductor due to exterior charges. A hollow conductor serves as a screen whether insulated or earthed.

In order to protect a delicate instrument, like a gold-leaf electroscope, from the action of a charged body, an earthed metallic plate may be interposed between them as in Fig. 22(c). The lines of force from the body are all intercepted by the plate and thus the instrument is screened.

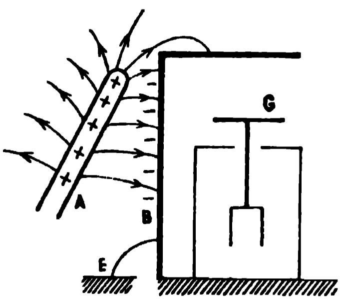


Fig. 22(c)

33. Magnetic and Electric Phenomena Compared.—

(1) **Generation.**—Artificial magnets can be prepared by rubbing a rod of iron or steel with lodestones or powerful artificial magnets. Electricity can be developed by rubbing two dissimilar substances, but in this case the rubber and the rubbed both get electric charges in equal and opposite quantities.

(2) **Distribution.**—The magnetism of a magnet is almost entirely localised at the two poles, but electric charge is distributed all over the surface of any charged conductor ; the distribution, however, depends upon the shape of the conductor. There is no relation between the shape of a magnet and the distribution of magnetism on it.

(3) **Laws of Force.**—The laws of attraction and repulsion are almost similar in the two cases.

(4) **Magnets and Charged bodies.**—(*a*) A magnet attracts magnetic substances only ; a charged body attracts any light object. (*b*) A freely suspended magnet turns in a particular direction, a

charged body does not. (c) Each magnet has two kinds of magnetism, but a charged body has one charge only (except in the case of induced charge); that is, magnetic poles cannot be separated (see Art. 12, Part V.), but we can separate positive electricity from negative electricity. (d) Magnetism of a magnet is not lost by touching it with the hand, but a charged body is discharged in this way.

(5) **Magnetic and Electric Induction.**—For the comparison of magnetic and electric induction, see Art. 12.

(6) **Lines of Force.**—The general behaviour of the magnetic and electric lines of force is almost the same, though there are some important differences (vide Art. 17).

Questions

Art. 17.

1. Explain clearly the meaning of an electric line of force. How does it differ from a magnetic line of force ? (Pat. 1936).

Art. 18.

2. State the laws of action between charges.

A and B are two small spheres charged with $+9$ and $+16$ units of electricity respectively. The distance between them is 28 cms. How far from A along the line AB will the intensities due to the charges be equal. (C. U. 1947)

[*Ans* : 12 cms. from A towards B ; as also 84 cms. from A away from B]

2. (a) State carefully Coulomb's laws of force between electric charges and hence show how the idea of an electrostatic unit of charge is derived. (Pat. 1944 ; cf. All. '44)

3. Two small equal spheres carrying charges of 5 and 10 units are placed 20 cms. apart. Find the force they exert on each other, (a) before, (b) after they have been connected for a moment by a fine wire. (L.M.)

[*Ans* : $\frac{1}{8}$, $\frac{1}{64}$ dyne.]

4. Two small pith-balls hanging by silk fibres 30 cms. long from the same point are given equal charges and repel each other to a distance of 6 cms. If the pith-balls weigh 10 milligrams each, find the charge on each. (L. G. S.)

[*Ans.* : $\pm$ 6 C. G. S. units.]

5. Two small metallic spheres each weighing 5 gms. are suspended from a point by two strings of negligible weight and each of length 30 cms. When the spheres are charged with equal quantities of electricity, the two strings make an angle of 30° with each other. Find the amount of charge on each of the spheres. (Pat. 1927)

[*Ans.* : 568·22 units.]

Art. 20.

5 (*a*). Define "Electric Field at a point". A, B and C are the three corners of an equilateral triangle whose sides are each 5 cms. in length. Two point charges of $+100$ and -100 e.s. units are placed at A and B respectively. Find the direction and magnitude of the resultant electric field at C.

[*Ans.* : 4 units ; 60° with the base CB]. (All. 1946)

Art. 22.

6. What do you mean by potential of a conductor ?
(C. U. 1920, '26, '31 ; Pat. 1928, '43, '48)

6. (*a*). Explain "electric potential". (Pat. 1946)

Art. 24.

7. Prove that the electric potential at a point due to a charge concentrated at a point is inversely proportional to the distance from the centre.
(Cf. Pat. 1939)

8. A hollow spherical conductor, whose radius is one decimetre, is charged with 10 units of electricity. Find the potential (*a*) at the surface of the sphere, (*b*) at a point 25 cms. distant from the centre. (C. U. 1934)

[*Ans.* : (a) 1 E. S. U. ; 0·4 E. S. U.]

9. Obtain the value of the potential at a point due to a single electric charge $+q$ at a distance r from it. (C. U. 1934 ; Pat. 1948)

9 (*a*). $ABCD$ is a sq. of 20 cms. side. Positive charges 6, 12 and 24 e.s. units are placed at the points A, B and C. Calculate the work reqd. to transfer a unit positive charge from D to the centre of the square.

[*Ans.* : 0·75 units]. (Pat. 1946)

Art. 28.

10. Charges of 10 E. S. U. of positive electricity are placed at the four corners of a square each side of which is 8 cms. Calculate the potential at the point of intersection of the diagonals.

[*Ans.* : $5\sqrt{2}$ E. S. U.] (Pat. 1943)

11. You are given an insulated charged hollow pear-shaped conductor in which a hole has been drilled at the top. How would you proceed to investigate (i) the surface density of the charge on (a) the outside, (b) the inside surfaces of the conductor ; (ii) the potential of (*a*) the outside, (*b*) the inside surfaces ? What results would you expect ? Give your reasons.
(Pat. 1924 ; cf. C. U. 1917)

12. Describe expts. to show that the potential is the same throughout the whole space in the interior of a hollow charged conductor, and that it is the same as that of the conductor itself. (Pat. 1948)

13. An insulated ice-pail and an insulated brass ball are both charged with positive electricity, the pail to a high potential, the ball to a low potential. The ball is then brought close to the pail and lowered into it without touching

until the bottom is reached. After contact the ball is removed. Describe the changes in the potential both of the ball and the pail, (*a*) before contact, (*b*) on contact, and (*c*) after removal. (Pat. 1930)

[**Hints.**—(*a*) The potential of the ball increases until it reaches the bottom; (*b*) the potential is the same, (*c*) the potential of the ball is zero and that of the pail increases still more.]

14. Describe an experiment to show that the surface of an electrified conductor is an equipotential surface. (C.U. 1926 ; Cf. Pat. '39)

Art. 30.

15. What is meant by electro-static induction? Explain this phenomenon by considering potential. How would you proceed to prove that the total charge induced is always equal to the inducing electrification. (Pat. 1936)

Art. 31.

16. How would you charge a gold-leaf electroscope positively by the method of induction? State generally the condition of electroscope during the different stages of the above experiment regarding the following points :—(*a*) the total charge on the electroscope ; (*b*) its potential ; (*c*) the divergence of its leaves. (*See also* Art. 11). (Pat. 1929)

Art. 32.

17. An electroscope is surrounded by a cylinder of wire gauge which is put to earth. If an electrified body is brought near to it, how will the leaves behave?

[**Hints.** If the wire gauge cylinder completely surrounds the electroscope, then, on bringing the electrified body near to it, opposite charge will be induced on the outside and similar charge will escape to the earth. The electroscope is unaffected.]

CHAPTER IV

Capacity : Condensers

34. Capacity and Potential.—We know that every body has a capacity for receiving heat, known as the thermal capacity of the body, which is the quantity of heat required to raise the temperature of the body through one degree. Similarly, *electrical capacity of a conductor is measured by the quantity of charge required to raise the potential of the conductor by one unit.* Thus, if C be the capacity of a conductor

which is raised to a potential V by a quantity of electricity Q, we have, $C = Q/V$; or $Q = CV$; or, expressed in words,

$$\text{Capacity} = \frac{\text{Quantity of Charge}}{\text{Rise of Potential}}.$$

Unit of Capacity.—In the above relation, if $Q = 1$ and $V = 1$, then $C = 1$. Thus, *a conductor is said to have a capacity of one electro-static unit* (e. s. u.) *when one electro-static unit of charge raises its potential by unity.*

The practical unit of capacity is a **Farad**. *A conductor has a capacity of 1 farad, if a charge of one coulomb of electricity raises its potential by 1 volt.*

A farad is rather too large a unit for ordinary purposes ; so the usual unit is one-millionth (10^{-6}) of a farad, which is called a **micro-farad** ; 1 farad $= 10^6$ micro-farads.

It should be noted that the farad is founded on the electro-magnetic system of units, being derived from the *volt* and *coulomb* (see Part VII., Ch. IV).

1 farad $= 9 \times 10^{11}$ electro-static units of capacity.

1 micro-farad $= 9 \times 10^5$,, ,, ,,

35. Capacity of a Conductor depends upon its Surface Area.—This can be shown by taking a sheet of tin-foil T attached to a roller of glass G, at the lower end of which a small load L is provided in order to keep the sheet stretched (Fig. 23). The sheet can be rolled up or down. Connect the lower end of the sheet with an electroscope E by means of a wire and charge it when unrolled. Notice the amount of divergence, which indicates the potential of the sheet due to the charge given to it. As the sheet is rolled up, the *area* of the exposed surface is *diminished* and the divergence of the leaves will be found to increase indicating rise of potential. This shows that the *capacity has diminished*, for the quantity of electricity remained the same. It is also clear from the relation $C = Q/V$ that when V is greater, C is less, Q remaining constant. As the sheet is rolled down, *area* of the surface will be *increased*, and the divergence of the leaves will be found to diminish indicating fall of potential. This means that the *capacity has increased*, the quantity having remained the same. Thus *the*

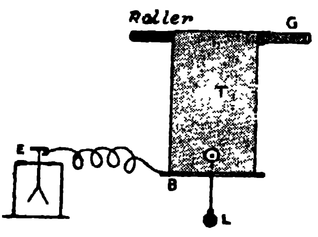


Fig. 23

capacity of a conductor depends upon the area ; it increases as the area increases and decreases as the area decreases.

36. (a) Capacity of a Sphere.--Let a sphere of a radius r [Fig. 23 (a)] be charged with Q units. The potential at any external point due to the sphere at a distance of d cms. from the centre of the sphere $= Q/d$. Therefore the potential of points on the surface of the sphere—that is, the potential of the sphere,

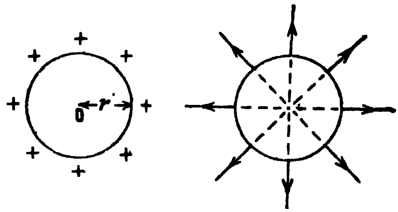


Fig. 23(a)

$V = \frac{Q}{r}$; So, the capacity $C = \frac{Q}{V}$

$= \frac{Q}{Q/r} = r.$

Or, **the capacity of a sphere is equal to its radius.**

When it is said that the capacity of a conductor is 1 cm., it means that its capacity is equal to the capacity of a spherical conductor of radius 1 cm.

The capacity of a sphere of 1 inch radius is about 0·0000028 micro-farad.

(b) Surface Density of Charged Spheres—Consider two uniformly charged spheres of radii r_1 and r_2, placed at a considerable distance apart, charged with Q_1 and Q_2 units respectively and joined by a long fine wire. If C_1 and C_2 be their capacities and V their common potential, we have $Q_1 = C_1 V$ and $Q_2 = C_2 V$;

$\therefore$ $Q_1 : Q_2 = C_1 : C_2 = r_1 : r_2$, *i.e. the charges of the spheres, like the capacities, are directly as their radii.*

Now, denoting the surface densities of the spheres as ρ_1 and ρ_2, we have, $\rho_1 : \rho_2 = Q_1/4\pi r_1^2 : Q_2/4\pi r_2^2$ (see Art. 16) $= r_1/r_1^2 : r_2/r_2^2$.

$$= r_2 : r_1,$$

i.e. **the surface densities are inversely as the radii.**

37. Potential Energy of a Charge.—The potential of a conductor placed at a point in an electric field due to a given charge should be clearly distinguished from the potential energy of a charged conductor. The potential of a charged conductor is the work necessary to bring a unit positive charge from infinity up to a point close upon the conductor, but the *potential energy of the charge is the total electrical work done in charging the conductor.*

If a conductor is charged with Q units and has got a potential V, the work done in bringing a positive unit of electricity from infinity up to a point close upon the conductor is denoted by V, and, in bringing Q units, the work done will be denoted by QV, *supposing that V all along remains constant.* But this is not the case when a conductor is charged to Q units beginning from its uncharged state. With charging, the potential gradually rises from O to the final value V, just as the level of water in a tank is not attained all at once but gradually from zero to the final value with addition of water.

Suppose the total charge Q is given to the conductor by a large number of small charges, so the potential of the conductor rises from O to V in proportion to the charge carried ; so the average value of potential during the process of charging $=(O+V)/2=V/2$. Hence the work done in charging the conductor $=Q\times V/2=\frac{1}{2}QV$; and this represents the potential energy of a charge Q at potential V. The energy E can be expressed in three ways :—

(*i*) $E=\frac{1}{2}QV$ ergs. For a conductor whose capacity is C, we have $Q=CV$. $\therefore$ (*ii*) $E=\frac{1}{2}\,CV\times V=\frac{1}{2}CV^2$ ergs.

Again, (*iii*) $E=\frac{1}{2}C.\dfrac{Q^2}{C^2}=\frac{1}{2}\dfrac{Q^2}{C}$ ergs.

The work done in the process of charging a conductor is stored up as the potential energy of the charge, and the spark which usually accompanies the discharge of a conductor proves the existence of this energy which is thus dissipated in the form of heat, sound, etc.

38. Principle of Condensers.—The capacity of a conductor depends not only on the **shape** or **size** of the conductor or the medium in which it is placed but also on its **position** with respect to other conductors. Let an insulated metal plate A be connected to an electric machine (Fig. 24), and let the plate be fully charged to a potential $+V$. On bringing a similar metal plate B near it, induction will take place, the near side will be negatively charged, and the far side positively charged [Fig. 24(*a*)]. Now the negative charge on B tends to diminish the poten- of A, while the positive charge tends to increase it. But the negatively charged surface being nearer, the potential of A on the whole is

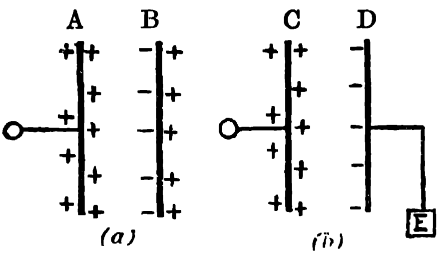


Fig. 24—Action of Parallel Plate condenser

lowered a little, which means that the capacity of A is increased a little, for it is evident from the mathematical relation $C = Q/V$, that as V diminishes C increases. The capacity of A having increased, it is now able to take a slight additional charge.

If B is now earthed, the positive charge on the far side of it disappears [Fig. 24 (b)], and the opposing influence being absent, the potential of A is further lowered, so the capacity of A is also further increased and, consequently, it will now receive a much greater charge from the machine. Hence, it is seen that though it is not essential that the condensing plate B should be earthed, *it is better to have an earthed condensing plate.*

Such an arrangement, by which the capacity of an insulated charged conductor is increased artificially by bringing an earthed conductor near it, is called a **condenser**.

The above condenser is known as a **Parallel Plate Condenser**. Similarly, a condenser can consist of two concentric spheres (one of them being earth-connected) having an insulating medium between them. Such a condenser is called a **Spherical condenser**.

Experiments on Capacity.—(i) Connect the insulated metal plate A (Fig. 24) by means of a fine wire with a gold-leaf electroscope. Remove B , charge A positively and observe the amount of divergence of the leaves Hold in the hand a metal plate much smaller than A at a distance, say, 10 cms. apart in front of A. Notice the diminution of divergence. Now remove the small plate and replace it by an uncharged and insulated plate B which is much larger in size. The divergence is still more diminished, which shows that the potential of A has decreased. But the charge on A is the same as before, so it shows that by bringing an insulated conductor near it the capacity of the conductor has been increased $(V = Q/C)$, but the potential has diminished.

(ii) Now connect B to earth and notice that the divergence decreases still further. The quantity of charge on A remains unaltered, and so this shows that *the capacity of A has increased further due to the presence of the plate B when connected to earth.*

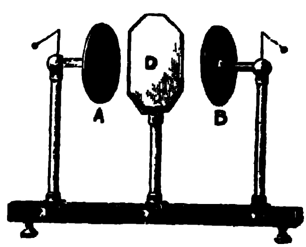


Fig. 25—Parallel Plate Condenser

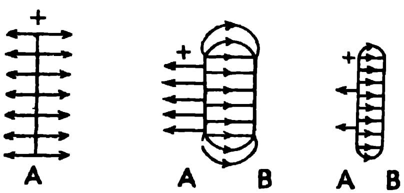
[**Parallel Plate Condenser** —Two conducting plates A and B placed parallel to each other at a small distance between them, which can be conveniently adjusted, is known as a **parallel ptate condenser** (Fig. 25). Suppose A is insulated and connected to a gold-leaf electroscope while B is adjustable and connected to earth].

(iii) Bring B nearer and notice that the divergence goes on decreasing. Now slowly increase the distance of B from A and notice the increase of divergence of the leaves. Thus we see that by decreasing the distance between the plates, the potential is decreased, *i.e.* the capacity is increased, the charge on A remaining unaltered. Hence, **the capacity of a condenser is inversely proportional to the distance between the plates.**

(iv) Now keeping the distance between the plates A and B fixed, carefully insert between the plates a square slab of glass D and notice the diminution of divergence. By inserting between the plates slabs of different materials, such as paraffin, mica, ebonite, shellac, etc., decrease in divergence by different amounts will be noticed, which shows that **the capacity depends much upon the medium (di-electric) between the plates.**

Thus, the capacity of a condenser depends upon (*i*) *the* **area** *of the two conductors* [vide Art. 35 or expts. (i) and (ii) above] ; the larger the plate the larger will be its capacity, (*ii*) the **distance** *between the plates*, and (*iii*) *the* **medium** *(di-electric) between them.*

38(a). Field between the Plates of a Parallel Plate Condenser.—Fig. 26 represents the field of force of a parallel plate condenser. Fig. 26(a) represents the distribution of lines of force on the charged plate A alone. Fig. 26(b) shows the distribution when the condensing plate B is brought near A. At the edges a few lines of force are bent due to lateral pressure of the lines of force. Fig. 26 (c) shows the distribution when the plates are closer. The closer the plates, the less is the bending of the lines of force. In this case the lack of uniformity of the field between the plates is slight, *i.e.* the effective area of the plate is increased.



(a) (b) (c)

Fig. 26—Lines of force diagram of a Parallel Plate Condenser

39. Specific Inductive Capacity—It has been found that glass, paraffin, mica, ebonite, shellac, etc., used as di-electrics, instead of air, increase the capacity of a condenser, and so they are said to have a higher **Specific Inductive Capacity** (S. I. C.), *which is defined as the ratio of the capacity of a condenser with any di-electric other than air to its capacity with air as the di-electric, i.e.*—

Specific inductive capacity (S. I. C.) of any di-electric x

$$= k = \frac{\text{Capacity of any condenser with any di-electric } x}{\text{Capacity of same condenser with air as di-electric}}.$$

Capacity of condenser with di-electric x } = **k × Capacity of a similar air-condenser.**

Thus, the capacity of a condenser is proportional to the value of the S. I. C. of the material used. For this reason the capacity of a condenser is greatly increased with glass (S. I. C. = 8·45 to 10) or mica (S. I. C. = 6·64) as the di-electric instead of air (S. I. C. = 1).

Determination of S. I. C. of a Di-electric.—

Faraday's Experiment.—In order to determine the S. I. C. of a di-electric, Faraday took two exactly similar spherical condensers, one of which is shown in Fig. 27. Each condenser had an inner metallic sphere A fixed concentrically within an outer metallic shell B, which is made of two hemi-spheres separable at E about the horizontal plane. The inner sphere is connected by a metallic collar to the external knob K. The collar passes through a thick plug C of shellac or sulphur by which it is very well insulated from the outer shell. By taking the two halves apart, the di-electric concerned is packed within the inner space between the two spheres. In case of a gaseous di-electric, the air in the intervening space is first pumped out through the stop cock T and then the gas is pumped in. The method is suitable for solids in the form of powder, liquids and gases. The space between the two spheres in one of the condensers contained air, while that in the other was filled with the di-electric whose S. I. C. was to be determined. The air condenser was charged with Q units of electricity and its potential V was determined with the help of a gold-leaf electroscope connected to the knob K It was then connected with the other condenser, and the common potential V' was determined as before.

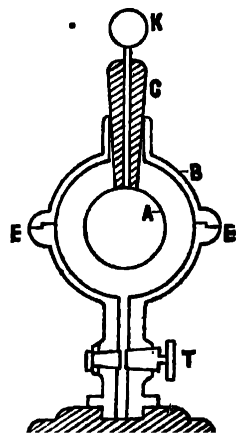


Fig. 27

Now, if C_1 be the capacity of the air condenser and C_2 that of the other, we have, $C_1 V = Q = (C_1 + C_2)V'$

$$\therefore \quad \frac{C_1 + C_2}{C_1} = \frac{V}{V'}; \quad or \quad \frac{C_2}{C_1} = \frac{V - V'}{V'} = \text{S. I. C.} = \frac{d - d'}{d'}, \text{ where}$$

d and d' are the divergences of the electroscope before and after sharing of charges.

By filling the second condenser with different di-electrics the ratio C_2/C_1 will be different.

Insulators and Di-electrics.—It should be remembered that all di-electrics are insulators. We call them insulators when we refer to the fact that they do not allow electricity to flow along them, and we call them di-electrics when we refer to the fact that they allow the transmission of electrical influence to take place through them, and, in fact, *the di-electrics themselves play an important part in the action.*

40(a). Factors determining Capacity of a Conductor.—It is evident from the experiments of Art. 38 that the capacity of a given conductor is the least when it is isolated, that is, when there is no other conductor in its neighbourhood. The capacity of a conductor may be increased in the following ways :—

(1) By increasing its area (see Art. 35), **(2) By bringing near it**, *(i) an insulated conductor* [see expt. (i), Art. 38] ; *(ii) an earth-connected conductor* [see expt. (ii), Art. 38] ; *(iii) It is also increased by decreasing the distance between the plates* [see expt. (iii) Art. 38] ; *(iv) an uncharged di-electric* [see expt. (iv), Art. 38].

40(b). Factors determining Potential of a Conductor.—

(*i*) The potential of a conductor increases or decreases proportionately with the *amount of charge* given to it or taken from it. For a given amount of *charge*, the potential is inversely proportional to the capacity of a conductor, which is evident from the equation, $C = Q/V$.

(*ii*) The potential of a conductor is diminished (but capacity increased) by bringing *another conductor, insulated or earthed, near it* [see expt. (i), Art. 38].

(*iii*) A definite amount of charge being given to a conductor, its potential will depend upon *the size of the conductor.* If the size increases, its potential diminishes (but capacity increases), and if the size diminishes, its potential increases (but capacity diminishes, $V = Q/C$).

(*iv*) The potential of a charged conductor depends upon *the the nature of the di-electric* surrounding it [see expt. (iv) of Art. 38].

41(a). Distribution of Charges at the same Potential.—Suppose two conductors A and B of capacities C_1 and C_2 are in electrical

contact [Fig. 27(a)]. If a charge $+Q$ be given to any of them, the charge will be distributed between them according to their capacities. Because the conductors are in contact, they acquire the same potential. Let the common potential be V, and let q_1, q_2 be their charges. Then the total charge $Q = q_1 + q_2$.

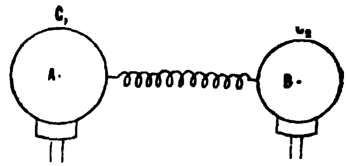


Fig. 27(a)

We have, $V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{Q}{C_1 + C_2}$;

$\therefore\ q_1 = Q \frac{C_1}{C_1 + C_2}$, and $q_2 = Q \frac{C_2}{C_1 + C_2}$.

If the conductors are two spheres of radii r_1 and r_2, we have,

$q_1 = Q \frac{r_1}{r_1 + r_2}$ and $q_2 = Q \frac{r_2}{r_1 + r_2}$.

Note.—It is to be noticed that in this case, the two spheres have the *same potential, but different charges*, on them.

41(b). Sharing of Charges between two Conductors at Different Potentials.—Suppose two conductors A and B have capacities C_1 and C_2 and charges q_1 and q_2 respectively [Fig. 27 (a)]. The potential v_1 of $A = q_1/c_1$; the potential v_2 of $B = q_2/c_2$. Let $q_1 + q_2 = Q = c_1 v_1 + c_2 v_2$.

Connect A and B by a fine long metallic wire. Suppose v_1 is greater than v_2. So some charge flows from A to B until the two conductors attain a common potential v. Now $v_1 > v > v_2$. Let q' and q'' be the charges now possessed by A and B respectively, after sharing of charges. Then, supposing there is no loss of charge on sharing, we must have $q_1 + q_2 = q' + q'' = Q$.

$$\text{Now, } v = \frac{q'}{c_1} = \frac{q''}{c_2} = \frac{q' + q''}{c_1 + c_2} = \frac{Q}{c_1 + c_2} = \frac{c_1 v_1 + c_2 v_2}{c_1 + c_2} \quad \ldots \quad \ldots \quad (1)$$

$$\therefore\ q' = c_1 \times \frac{Q}{c_1 + c_2};\ q'' = c_2 \times \frac{Q}{c_1 + c_2} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The amount of charge that will flow from A to B will be

$= q_1 - q' = c_1 v_1 - c_1 v = c_1(v_1 - v)$

= charge lost by A = charge gained by B. ... (3)

42. Leyden Jar.—The Leyden Jar (Fig. 28), so called as it was

first constructed at Leyden, is a very well-known type of condenser. It consists of a glass jar having inner and outer coatings of tin-foil. The coatings cover the bottom and the side upto 2 or 3 inches from the edge. The inner coating is connected through a metallic chain with a vertical brass rod passing through an insulating cover of the jar and terminated above by a knob. The inner and outer coatings are like the two plates of a parallel plate condenser with glass as di-electric separating them.

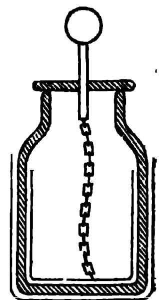


Fig. 28
Leyden Jar

The capacity of an ordinary-sized Leyden jar is about 0·0025 micro-farad.

Charge of Leyden jar.—**To charge the jar with positive electricity**, the inner coating is connected with the prime (positive) conductor of an electric machine (Art. 52), and the outer coating is connected to the earth. The inner coating receives positive charge, and negative charge is induced on the inner surface of the outer coating, the induced positive charge of the outer surface being neutralised by electrons from the earth. **To charge the jar negatively**, the inner coating or the knob is connected with the negative terminal of the machine, the outer coating being connected to the earth.

To discharge a Leyden jar, the usual way is to connect the two plates by means of a special discharger, called the **discharging tongs** or **discharger** (Fig. 29). This consists of a pair of bent brass rods with a brass knob at each end, having an insulated handle and working on a hinge, so that the distance between the knobs can be adjusted according to necessity. In Fig. 30, it has been shown how a Leyden jar can be discharged with the discharger referred to in Fig. 29.

Fig. 29—Discharger

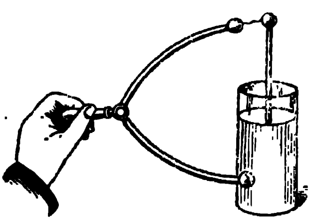


Fig. 30—Method of discharging a Leyden Jar

43. Dissected Leyden Jar : Seat of Charge.—It is the di-electric of a condenser which plays the most important part in the inductive action taking place within it. The opposite charges reside on the surfaces of the di-electric, the plates or coatings acting simply as conductors. This function of the di-electric can be illustrated by taking a Leyden jar with movable coatings (Fig. 31).

After charging the jar in the usual way, the inner coating A is lifted

by means of an insulating handle and placed on a sheet of glass. The

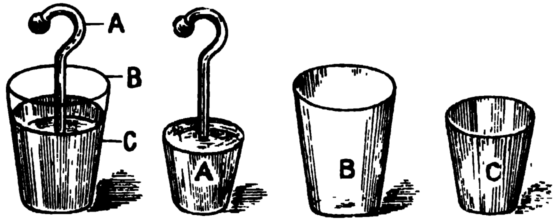


Fig. 31 -Dissected Leyden Jar

glass jar A is then lifted out of the outer coating C and laid on the glass sheet. On now testing the two coatings A and C by an uncharged electroscope, no divergence of leaves is produced. If the separate parts are then replaced and the condenser is built up as before, a powerful discharge may be obtained by discharging it. This shows that **the real seat of the charge is on the surface of the di-electric** which here is glass. That the di-electric is charged is easily proved by inverting the glass jar over the disc of an electroscope when the leaves will at once diverge.

44. **Discharge and Residual Charge.**—The charge cannot flow over the surface of a di-electric which is an insulator, and **the di-electric remains in a state of strain on account of the charge.** *For this reason an electric field can persist in a di-electric, i.e. in an insulating medium, but not in a conductor*, which cannot support the strain ; so charges flow on the surface of a conductor until the same potential is attained everywhere. When the strain in a di-electric becomes too great, the di-electric breaks down and a spark passes across it, which is seen in the discharge of a Leyden jar, or in a frictional machine

The discharge in the case of a Leyden jar does not occur in one spark. If the coatings are again connected together by the discharging tongs, after waiting for a short time, another smaller spark will be obtained, and, in this manner, often several successive sparks will be observed. It is because the di-electric, which is in a state of strain, does not recover itself all at once, but takes a little time. So, after a time, a charge appears on the coatings and another spark is obtained. The charge, which remains stored up in the di-electric after the first discharge, is called the **residual charge** and the successive discharges are called secondary or **residual dis-**

charges. These effects are not obtained in condensers having air as di-electric.

45. Uses of Condensers.—A condenser is a convenient arrangement to store up a quantity of electricity at a low potential. An application of this will be seen in the Wimshurst machine (Art. 52); two Leyden jars are used there to increase the capacities of the discharges.

Condensers are greatly used in Wireless Telegraphy and Telephony (see Ch. X., Part VII), where two types are used—*fixed* and *variable*. **Fixed Condensers** are usually made of sheets of tin-foil separated by thin sheets of mica or paraffined paper (Fig. 32). Alternate sheets of tin-foil are joined together to make use of both the surfaces of each sheet as a distinct plate. The two alternate sets end in two terminals *P* and *R* of which one is earthed. By this means a condenser equivalent to two flat plates of large area, separated by one thin mica sheet as di-electric, is obtained in a small space.

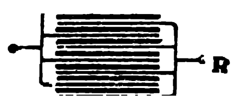


Fig. 32—Fixed Condenser.

Variable Condensers are usually made of two sets of metal vanes, one set fixed and the other set movable, with air as di-electric (Fig. 33). The movable set of parallel plates can be rotated between the fixed sets and this rotation alters the area of the plates in close proximity, and thus alters the capacity of the condenser. These are used in wireless sets as tuning condensers.


Fig. 33—Variable Condenser

46. Condensing Electroscope.—It is an ordinary gold-leaf electroscope whose sensitiveness has been greatly increased by adopting a device. Volta originally used such a device to detect a small difference of potential at the junction of two dissimilar metals (vide Art. 2, Part VII). So it is also sometimes called Volta's electroscope. Potentials, common in electrostatics, being usually very high, there is no difficulty in detecting them by an ordinary electroscope. But a small potential difference like that between the terminals of a battery cannot be so detected. A gold-leaf electroscope can, however, be made to record such a small potential differences by connecting it to a suitable condenser. Such a device is called *a condensing electroscope*. The arrangement consists of a gold-leaf electroscope over the disc of which is placed an insulated metallic disc of the same diameter, which is coated with shellac varnish. The two metal discs now constitute a condenser, the upper one being earth-connected. The shellac varnish between the discs acts as the di-electric in the condenser.

Action.—One end, say, the negative terminal of a battery, is earthed ; the disc of the gold-leaf electroscope acquires the potential of the other terminal, when momentarily connected to it. Let C be the capacity of the electroscope alone, and nc the capacity when another earthed plate is on the disc. Let V be the potential difference between the terminals of the battery ; the disc then has been charged to potential V and a charge ncV rests on it. Now disconnect the battery and remove the earthed plate. The charge is unchanged, but the capacity is reduced from nc to C, and so the potential has now been raised from V to nV. If the original potential V was not too small, the above magnification should cause the leaves of the electroscope to diverge.

A condensing electroscope can not, it should be noticed, be used to detect the potential of a body of small capacity ; for as soon as the condenser of the electroscope will be connected to it, its potential will be considerably affected. Such an electroscope can, therefore, be only used with a source of small, but steady, potential.

47. Capacity of Condensers.—(i) Spherical Condenser.—Let a condenser consist of two concentric spheres of radii r_1 and r_2 cms. respectively, the outer one (of radius r_2) being connected to the earth (Fig. 34). Let a charge of $+Q$ units be given to the inner sphere, then the induced charge on the inner surface of the outer sphere is $-Q$ units.

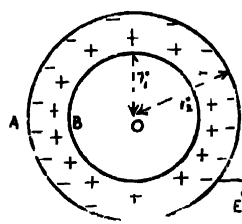


Fig. 34—Spherical Condenser

The potential of the inner sphere due to its own charge is $Q r_1$, and the induced potential of it due to the negative charge on the outer sphere is $-Q/r_2$. Therefore, the actual potential of the inner sphere $= Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = Q\left(\frac{r_2 - r_1}{r_1 r_2}\right)$, and this is also the difference of potential between the coatings, since the potential of the outer sphere is zero (being connected with the earth). We know that,

$$\text{Capacity} = \frac{\text{Charge}}{\text{P. D. between the coatings}} = \frac{Q}{Q\left(\frac{r_2 - r_1}{r_1 r_2}\right)} = \frac{r_1 r_2}{r_2 - r_1}.$$

(ii) Parallel Plate Condenser.—The capacity of a spherical condenser per unit area of the surface of the inner sphere $= \frac{r_1 r_2}{(r_2 - r_1) \times 4\pi r_1^2}$.

Let d be the distance between the spheres, then $r_2 = r_1 + d$. So the expression becomes, $\dfrac{r_1(r_1+d)}{4\pi r_1^2 + d}$ Now, suppose r_1 is very large in comparison with d, then the expression becomes $1\ 4\pi d$.

When the radius of the spherical surface is very large, it can be regarded as a plane surface. Consequently, the capacity of an air condenser consisting of two parallel plates, each of area $A, = A/4\pi d$, where d is the distance between the plates.

If the di-electric be any other substance of S. I. C. (k), the capacity $= \dfrac{kA}{4\pi d}$.

Otherwise thus :—It has already been found that the capacity C of a condenser varies (*a*) *directly* as the size or area A of the charged plate ; (*b*) *inversely* as the distance d between the plates, and (*c*) *directly as the* S.I.C. (k) of the di-electric. Mathematically it can be expressed as,

$$C \propto A\ ;\ C \propto 1/d;\ C \propto k.$$

$$\therefore \quad C = \text{a constant} \times \frac{kA}{d}.$$

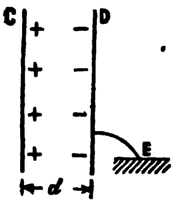


Fig. 34(a)

This constant depends upon the shape of the condenser, and the value of this constant in the case of a parallel plate condenser [Fig. 34(a)] is $1/4\pi$.

$$\therefore \quad C = \frac{kA}{4\pi d}.$$

When air is the di-electric, the value of k is 1 and so the capacity $C = \dfrac{A}{4\pi d}$

(iii) Parallel Plate Condenser with a Compound Di-electric.—Suppose the two plates C and D, each of area A, are placed d cms. apart of which t cms. are of a di-electric having S.I.C. of value K, the rest being air [Fig. 34(b)]. Let Q units of charge be on C while D is earthed ; let δ be the surface density of charge on each plate and F the intensity in air.

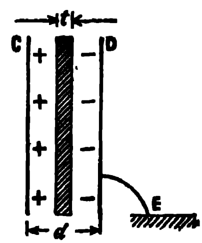


Fig. 34(b)

The p. d. between the plates $= V_C - V_D$.

= intensity in air $\times (d-t) +$ intensity in di-electric $\times t = F \times (d-t) + \dfrac{F}{K} \times t = F\left(d - t + \dfrac{t}{K}\right)$

$=4\pi\delta\left(d-t+\frac{t}{K}\right)$, since, according to a law established by Coulomb, the intensity near a charged conductor is 4π times the surface density.

But Capacity $C=\frac{Q}{V_C-V_D}=\frac{A\delta}{4\pi\delta\left(d-t+\frac{t}{K}\right)}=\frac{A}{4\pi\left(d-t+\frac{t}{K}\right)}$.

From this it follows that, (*a*) When $d=t$, $C=\frac{KA}{4\pi d}$;

(*b*) When $t=0$, $C=\frac{A}{4\pi d}$, (air-condenser).

48. Grouping of Condensers.—Leyden jars and also other condensers can be joined together when a large difference of potential or a large capacity is required. They can be joined in two ways ;—(a) in *series* (or *cascade*), and (b) in *parallel*.

(a) In Series (or Cascade).—In this arrangement the second plate of the first condenser is joined to the first plate of the second condenser and so on (Fig. 35). Here all the plates are insulated except the last one, which is earthed. If a charge $+Q$ is given to the plate A of the first condenser, it induces $-Q$ on the inner side of the other plate B and $+Q$ goes to the first plate C of the next condenser. This is repeated ; so each condenser acquires $+Q$ units on one plate and $-Q$ units on the other. If V be the potential difference of the first plate A and the last plate G of the series, and V_1, V_2, V_3, the potential differences between the plates of the separate condensers, we have, $V=V_1+V_2+V_3$ ... ... (1)

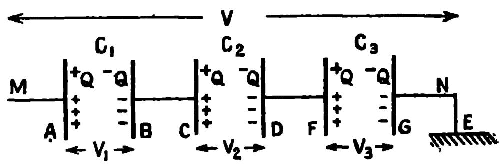


Fig 35.—Condensers in Series

Let C be the combined capacity of the system, and C_1, C_2, C_3 their individual capacities, then $V=\frac{Q}{C}$; $V_1=\frac{Q}{C_1}$; $V_2=\frac{Q}{C_2}$; etc.

Hence, $\frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}$ from (1).

$\therefore \quad \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$.

Thus, *the reciprocal of the combined capacity of a number of condensers in series is the sum of the reciprocals of the capacities of the separate condensers.*

It should be noted that the resultant capacity is always less than that of any individual condenser, but this arrangement is used when **a large potential difference** is required.

(b) In Parallel.—In this arrangement the insulated plates are joined at the common point M which is connected with the source of potential, and similarly all the other plates are joined at N which is earthed (Fig. 36). It is clear that all the condensers, being directly connected to the source and the earth, have the same potential difference V. When a charge is given at M, the charge is distributed to the condensers according to their capacities. If Q_1, Q_2, Q_3 be the charges of the condensers, the total charge

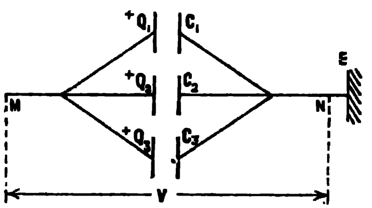


Fig. 36—Condensers in Parallel

$Q = Q_1 + Q_2 + Q_3$ ... ... (1)

If C_1, C_2, C_3 be the individual capacities and C the combined capacity, we have, $Q = VC$; $Q_1 = VC_1$; $Q_2 = VC_2$; etc.

$\therefore$ From (1), $VC = VC_1 + VC_2 + VC_3 = V(C_1 + C_2 + C_3)$.

i.e. $C = C_1 + C_2 + C_3$.

Thus, *the combined capacity of a number of condensers in parallel is the sum of the separate capacities.*

It should be noted that this arrangement is used when **a large capacity** is required.

Examples.—*1. Fifty thin tin-foils, each of area 20 sq. cms. are placed one above another, each foil being separated and insulated from the next by a piece of paraffined paper of thickness 1 mm. and of specific inductive capacity 2. The ends of 25 foils alternating with the remaining 25 foils are joined together. So also the ends of the other 25 foils are joined together. Calculate the capacity of the condenser.*

(Pat. 1927)

When $2n$ plates are arranged one above another in this way, the two outside surfaces are uncharged ; the electricity spreads itself uniformly over all the other surfaces so that, with the exception of the outside plates, each plate has a charge on both sides. If n be the number of plates in each set, we have on the whole, $(4n-2)$ or $2(2n-1)$ sides, so that the whole system is

equal to $(2n-1)$ separate plate condensers, the capacity of which is given by $\frac{k \times (2n-1)A}{4\pi d}$. Here $n=25$; $A=20$ sq. cms. : $d=0\cdot1$. cm. ; $k=2$;

$$\therefore \quad \text{Capacity} = \frac{2 \times 49 \times 20 \times 7}{4 \times 22 \times 0\cdot1} = 1559\cdot09 \text{ cms.}$$

2. Two leyden jars are exactly alike, except that in one the tin-foil coatings are separated by glass, and in the other by ebonite. A charge of electricity is given to the glass jar and the potentials of its inner coating is measured. The charge is then shared between the two jars, and the potential falls to 0·6 of its former value. If the specific inductive capacity of ebonite be 2, what is that of glass ? **(Pat. 1931)**

Let C_1 and C_2 be the capacities of the glass and the ebonite jar, and k_1 and k_2 be the specific inductive capacities of glass and ebonite respectively, then $C=\frac{k_1 A}{4\pi d}$; $C_2=\frac{k_2 A}{4\pi d}$, where A is the area and d the distance between the coatings of each jar. $\therefore \ \frac{C_1}{C_2}=\frac{k_1}{k_2}$.

If Q be the charge given to the glass jar and V be its potential, $C_1=\frac{Q}{V_1}$.

Then, when the charge is shared between the two jars, $C_1+C_2=\frac{Q}{0\cdot6V}$;.

$$\therefore \quad \frac{C_1+C_2}{C_1}=\frac{1}{0\cdot6}\ ; \ or \quad 1+\frac{C_2}{C_1}=\frac{1}{0\cdot6}\ ; \ or \quad 1+\frac{k_2}{k_1}=\frac{1}{0\cdot6}$$

$$or \quad \frac{k_2}{k_1}=\frac{1}{0\cdot6}-1=\frac{0\cdot4}{0\cdot6}=\tfrac{2}{3}\ ; \ or \quad \frac{2}{k_1}=\tfrac{2}{3}\ ; \quad \therefore \ k_1=3.$$

3. Two spheres of 2 and 6 cms. radius are charged respectively with 80 and 30 units of electricity ; compare their potentials. If they are connected by a fine wire how much electricity will pass along it ? **(C. U. 1932)**

The capacity of a sphere is equal to its radius. Again, $C=Q/V$.

Let C_1, V_1, Q_1 and r_1 be the capacity, potential, charge and radius of the first sphere, and C_2, V_2, Q_2, r_2 those of the second sphere, then

$$\frac{Q_1}{V_1}=C_1=r_1\ ; \ \text{and} \ \frac{Q_2}{V_2}=C_2=r_2.$$

$$\therefore \quad V_1=\frac{Q_1}{C_1}=\frac{80}{2}=40\ ; \ \text{and} \ V_2=\frac{Q_2}{C_2}=\frac{30}{6}=5. \quad \therefore \ \frac{V_1}{V_2}=\frac{40}{5}=\frac{8}{1}.$$

When the spheres are connected by the wire. some electricity will pass from the first sphere to the second (as the first sphere is at a higher potential), and they will have a common potential. After the redistribution of charges, let q_1 be the charge on the first and q_2 that on the second, and let V be the common potential ; then total charge $=q_1+q_2=80+30=110$ units.

$$\text{We have,} \quad V=\frac{q_1}{r_1}=\frac{q_2}{r_2}=\frac{q_1+q_2}{r_1+r_2}.$$

$$\therefore \quad q_1 = r_1 \times \frac{q_1 + q_2}{r_1 + r_2} = \frac{2 \times 110}{2+6} = 27{\cdot}5 \text{ units.}$$

But the original charge on the first sphere was 80 units. Therefore $(80 - 27{\cdot}5) = 52{\cdot}5$ units of electricity will pass along the wire.

4. *A brass sphere of 10 cms. radius is electrified to potential 80. It is then made to share its charge with another brass sphere and the potential is found to fall to 20. What is the radius of the second sphere?* *(Pat. 1929)*

The capacity of a sphere is equal to its radius. From the relation, $C = CV$, we have $Q = 10 \times 80 = 800$ units.

This charge is shared with another, and the two spheres are then at the same potential 20. If Q_1 and Q_2 be the respective charges on the two spheres, total charge $Q = Q_1 + Q_2$.

Q_1 = its capacity × potential $= 10 \times 20 = 200$; $\therefore \quad Q_2 = 800 - 200 = 600$.

But Q_2 = its radius (r) × potential ; hence $600 = r \times 20$. $\therefore \quad r = \frac{600}{20} = 30$ cms.

5. *Two equal soap bubbles, equally and similarly electrified, coalesce into a single larger bubble. If the potential of each bubble while at a distance from the other was P, what is the potential of the bubble formed by their union?* *(Pat. 1931)*

The volume of the united bubble will be equal to the sum of the volumes of the individual bubbles ; it will, therefore, be equal to twice the volume of any of the two bubbles. But the volume of a sphere $= \frac{4}{3}\pi r^3$.

If r_1 be the radius of each small bubble, r_2 that of the larger bubble after they coalesce, we have, $\frac{2 \times 4\pi r_1^3}{3} = \frac{4\pi r_2^3}{3}$; $\quad r_1 : r_2 :: \sqrt[3]{1} : \sqrt[3]{2}$.

But the capacity C of a sphere is equal to the radius.

$\therefore \quad C$ of small bubble : C of large bubble :: $\sqrt[3]{1} : \sqrt[3]{2}$ ... (1)

Since quantity of charge = capacity × potential, total quantity of charge on the two bubbles before contact $= r_1P + r_1P = 2r_1P$.

After contact, charge = potential V × radius = potential $V \times r_2$;

$$\therefore \quad 2r_1P = V \times r_2 \; ; \quad \therefore \quad V = \frac{2r_1P}{r_2} = 2P \times \frac{r_1}{r_2} = \frac{2P}{\sqrt[3]{2}} \qquad \text{from (1)}$$

Multiplying both numerator and denominator by $\sqrt[3]{(2)}$].

we have, $$V = P\frac{2 \times \sqrt[3]{2^2}}{2} = P\sqrt[3]{4}.$$

6. *ABCD is a square of 1 metre side of a non-conducting material. Four metallic spheres of 4, 5, 8 and 10 cms. diameters are placed at the corners. All of them are connected by a very fine metallic wire and a charge of 540 units is imparted to the system. What is the potential at the centre of the square?* *(Pat. 1932)*

If the total charge Q is distributed as Q_1, Q_2, Q_3 and Q_4 in the four spheres having diameters 4, 5, 8 and 10 respectively, *i.e.* radii 2, $\frac{5}{2}$, 4 and 5 respectively, we have $540 = Q_1 + Q_2 + Q_3 + Q_4$ ... ... (1)

As all the spheres are connected together, they are at the same potential, say V. Then since the capacity of a sphere is equal to the radius,

$$V=\frac{Q_1}{2}=\frac{2Q_2}{5}=\frac{Q_3}{4}=\frac{Q_4}{5}. \quad \therefore \quad Q_1=2V\ ;\ Q_2=\tfrac{5}{2}V\ ;\ Q_3=4V\ ;\ Q_4=5V.$$

Substituting these values in (1), $\quad 540=2V+\frac{5}{2}V+4V+5V=\frac{27V}{2}.$

$$\therefore \quad V=\frac{540\times 2}{27}=40. \quad \therefore\ Q_1=80\ ;\ Q_2=100\ ;\ Q_3=160\ ;\ Q_4=200.$$

Each side of the square being 100 cms., the distance of the centre of the square from the corners $=\frac{100}{\sqrt{2}}$.

$$\therefore \quad \text{Potential at the centre}=\frac{80\sqrt{2}}{100}+\frac{100\sqrt{2}}{100}+\frac{160\sqrt{2}}{100}+\frac{200\sqrt{2}}{100}$$

$$=\frac{540\sqrt{2}}{100}=\frac{27\sqrt{2}}{5}\ \text{E. S. units.}$$

7. *Two condensers of capacities 6 and 10 units are charged respectively to 16 and 13 units of potential. What is the common potential when they are connected in parallel?* (*C. U. 1941*)

Before connection: Let Q_1 and Q_2 be their respective charges, then $Q_1=5\times 16=80$ units ; $Q_2=10\times 13=130$ units. After connecting them in parallel the total capacity $C=5+10=15$ units and total charge $P=Q_1+Q_2=80+130=210$ units ; and if V be the common potential, we have, $CV=Q$;

$15\ V=210. \qquad \therefore \quad V=\frac{210}{15}=15$ units.

Questions

Arts. 34 & 36.

1. Two conductors of capacity 10 and 15 respectively are connected by a fine wire and a charge of 1000 units is divided between them. Find the potential of either conductor and the charge on each. (C. U. 1920 ; Cf. '33)

[*Ans :* Potential = 40 units ; charge on the first, 400, second, 600].

2. Define potential and the capacity of a conductor, and obtain from the definitions an expression to connect them with the quantity of charge. (All. '45)

A conductor A has a capacity of 10 and potential 50 ; another conductor B is of capacity 6 and potential 65. Calculate the charges on each conductor after they have been connected by a very thin long wire. (Pat. 1934)

[*Ans :* $Q_a=556{\cdot}25$; $Q_b=333{\cdot}75$]

3. Explain what is meant by the capacity of a condenser. Upon what factors does the capacity of a Leyden jar depend ? (All. 1932 ; Pat. '44)

(*See* Art. 40)

4. Show that the capacity of a spherical conductor is numerically equal to its radius. (C. U. 1937)

5. Two equal metal spheres connected by a long fine wire are insulated and electrified. What change would be produced in the relative amounts of these charges, if one of the spheres were to contract so as to have a quarter of its original surface ? (Pat. 1936)

[*Ans* : $Q_1 : Q_2 :: r_1 : r_2 :: 1 : 2$]

Art. 38.

6. What is a condenser ? Explain the principle underlying it. Mention the factors on which the capacity of a condenser depends, and describe expts. in support of your statement. (Pat. 1948)

7. Two plates, A and B, of brass are supported on glass handles and placed facing each other.

(*a*) One of the plates, A, is connected to a frictional machine and the other, B, to a gold-leaf electroscope. The machine is worked for some time. (*b*) The plate A is disconnected from the machine, and (*i*) it is moved nearer the other, (*ii*) a plate of glass is interposed between them. (*c*) The plate A being disconnected from the machine, the plate B is momentarily connected to the earth, and then (*i*) the plate A is moved nearer to B. (*ii*) a plate of glass is interposed between them. Explain what happens in each case. (C. U. 1912 ; cf. '46 Pat. 1929)

8. Two parallel plates form a condenser, one plate of which is charged and connected to a gold-leaf electroscope and the other is earthed. Indicate what will happen if the distance between the plates is increased or decreased. (Pat. 1942)

9. Explain the following : "A condenser is an arrangement by which the capacity of an insulated conductor is artificially increased." (C. U. 1241)

Art. 39.

10. Explain how Faraday determined the fact that different substances have different specific inductive capacities. (C. U. 1933 ; Pat. 1927)

11. Define capacity and specific inductive capacity. (Cf. Pat. 1942, '46)

What do you mean by the statement that the specific inductive capacity of paraffin is 2·19 ? (Dac. 1928)

12. What do you understand by the specific inductive capacity of a di-electric ? Explain how with two exactly similar spherical condensers and a gold-leaf electroscope you can find S. I. C. of sulphur. (C. U. 1940)

Art. 40.

13. Describe a condenser, and demonstrate experimentally how the capacity and potential can be altered. Explain the terms 'capacity' and 'potential' fully. (See also Art. 38) (All.)

Art. 41.

14. When a charge of 50 units is given to a sphere, it is found to have potential 20. After being connected to a second sphere, the potential falls to 8. Find the radius of the second sphere. (Pat. 1938)

[*Ans* : 3·75]

15. A spherical conductor of 10 cms. radius is charged positively with 100 units of electricity and it is connected with another spherical conductor of 5 cms. radius carrying a negative charge of 50 units. What will now be the charge on each sphere? Find also the potential of each sphere before and after contact. (Pat. 1940)

[*Ans* : Potential before contact $= +10 : -10$; $\left\{\begin{array}{l} Q_1 = 100/3 \\ Q_2 = 50/3 \end{array}\right\}$
,, after contact $= \frac{10}{3}$

16. Four metallic spheres of 4, 5, 8, 10 cms. diameter are joined together by a very fine metallic wire, and a charge of 810 e. s. units imparted to the system. Find the charge on each sphere, and their common potential.

[*Ans* : $v = 60$ e. s. units ; 120, 150, 240, 300 respectively]. (Pat. 1948)

Art. 42.

15. Describe the construction of a Leyden jar.

In charging a Leyden jar, the outer coating is (*a*) insulated, (*b*) connected to the earth. What difference does it make?

(C. U. 1912, '14, '19, '24, '29 ; Cf. All. '23 ; Pat. '31)

16. Define di-electric constant. The inner coating of a Leyden jar is connected to a gold-leaf electroscope. If the jar rests on a piece of ebonite, one charge from an electrophorus produces a large divergence of the leaves of the electroscope. If the ebonite be removed, and the jar is held in the hand, several charges of the electrophorus are needed to produce the same divergence. Explain this. (Punjab)

Art. 44.

17. What is meant by the residual charge of a condenser? (C. U. 1929)

Art. 47.

18. What is meant by the statement that the electric potential at a point is 10? Find an expression for the capacity of a parallel plate condenser.

(Pat. 1927)

19. Describe the construction and action of a simple parallel plate air-condenser, and obtain an expression for its capacity. How will the capacity be affected if a slab of ebonite is introduced between the parallel plates?

(See also Arts. 38 & 39) (Pat. 1944)

Art. 48.

20. Three condensers of capacity 1, 2 and 3 micro-farads are connected with the second and third in series and the first in parallel with them. Calculate the resultant capacity. (Pat. 1937).

[*Ans* : 11/5]

CHAPTER V.

Electric Machines

49. Electric Machines.—It is a mechanical device for the rapid production of electrical charges. Electric machines may be divided into two classes ;—(*i*) the **Frictional machines**, and (*ii*) the **Induction** or **Influence machines**.

The frictional machines, such as the **Glass-Cylinder machine** in which electricity is produced by friction against a cushion of leather, or the **Ramsden machine**, where a circular glass plate is rubbed against two pairs of silk pads, are now practically obsolete.

50. Electrophorus.—This is the simplest form of an induction machine devised by an Italian, Volta, about 1775. By this instrument a series of charges may be obtained from a single charge by induction. It consists of a circular slab of shellac or ebonite C, called the **cake**, a circular *metal* **disc** P of slightly smaller diameter, provided with an insulating *handle* H, and a metal base S, called the **sole**, on which the cake is placed (Fig. 37).

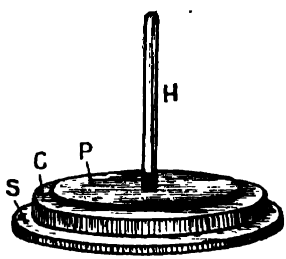
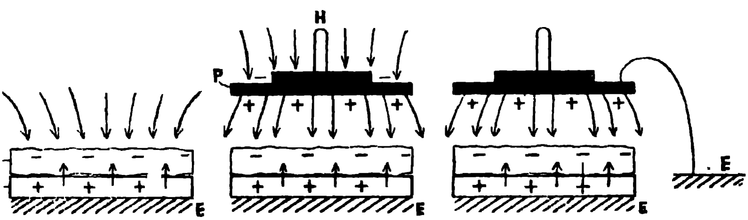


Fig. 37—Electrophorus

Action.—The different parts are first warmed, and a negative charge is developed on the cake C by rubbing it with a piece of flannel or catskin. It is then placed on the sole S [Fig. 38(a)]. The

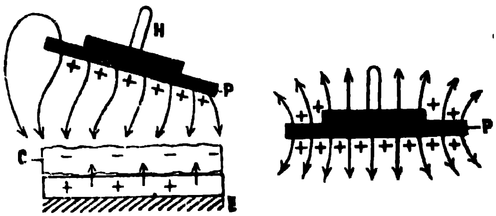


(a) (b) (c)

Fig. 38—Charging an Electrophorus

disc is then placed on the cake holding it by the insulating handle [Fig. 38(b)]. The negative charge on the cake acts inductively on the disc, attracts positive charge towards the lower surface, and repels negative charge to the upper surface of the disc. The arrowed lines show electric lines of force in the three steps. The disc is then

touched with the finger, *i.e.* connected to the earth, so that its potential becomes zero [Fig. 38(c)]. Now there will be no electric lines of force to the top of the disc from the earth. On now lifting up the disc by the handle, the lines of force are stretched, the remote ends attach themselves to the table or to the walls, etc. [Fig. 38(d)]. On now removing the disc P, the inductive influence of the cake is removed and so the induced positive charge spreads itself on both sides of the disc, which may be communicated to another body [Fig. 38(e)]. Replacing the disc on the cake, the whole series of operation may be repeated several times without again exciting the cake.



(d) Fig. 38 (e)

It should be noted that (*a*) by *increasing the area* of the electrophorus the charge obtained in each operation can be proportionately increased ; (*b*) when the charged disc is brought before an insulated conductor, the conductor can take charge until its potential becomes equal to that of the disc after which the conductor can take no more charge. So there is a *limit to the amount of charge* which can be given to an insulated conductor by an electrophorus.

The **function of the sole** may be explained thus :—The negative charge produced on the cake by rubbing induces positive charge on the inner surface of the sole, and the induced negative charge on the outer surface of the sole passes to the earth. The positive charge on the sole attracts the negative charge on the cake and draws it a little within the cake ; and thus the tendency of the cake to lose its charge by leakage is reduced.

Since the substance of the cake is a non-conductor, its charge does not pass into the disc ; the disc, when placed on the cake, touches the surface of the cake at a few points only due to the roughness of the surface. Hence there is a thin layer of *air* between these surfaces which *acts as the di-electric*. Therefore conduction can be neglected, and the phenomena may be treated as governed by induction.

51. Electrophorus and Energy of the Charge.—After charging the ebonite cake once, charges can be obtained from it as many times as necessary without exciting the cake again, *i.e.* we receive a series of charges from an initial small charge. This appears, at first sight, to violate the principle of conservation of energy; but, in fact, it does not.

The **energy of the spark** produced by an electrophorus, when the charged disc is brought before an earth-connected or insulated conductor, is derived *from the mechanical work* done in raising the plate, *i.e. in overcoming the attraction* between the negatively charged ebonite slab and the positively charged disc when separating the two ; thus the electrical energy is obtained by the transformation of mechanical energy in accordance with the **principle of conservation of energy.**

Changes in the Potential of the Disc.—When the disc is placed on the cake, it acquires a negative potential due to the induced *free negative charge.* When connected to the earth, its potontial is raised to zero and next when the disc is gradually lifted, after disconnecting from the earth, it acquires a greater and greater positive potential until finally the positive potential becomes maximum when it is completely free from the influence of the negatively charged cake.

52(a). **Wimshurst Machine.**—It acts on the principle of induction. The machine consists of two varnished circular glass-plates, placed close to each other and capable of rotation in opposite directions about a horizontal axis as shown by the arrows (Fig. 39). A number of **metal sectors** are fixed on the outer surface of each plate. These sectors serve both as **inductors** and **carriers.** The inner broken circles (thick lines) represent the sectors on the front plate, and the outer broken circles represent those on the back plate. Two diagonal conductors AB, CD lie across each plate at right angles to each other. The diagonal conductors ending in metallic brushes graze the metal sectors as the plates rotate.

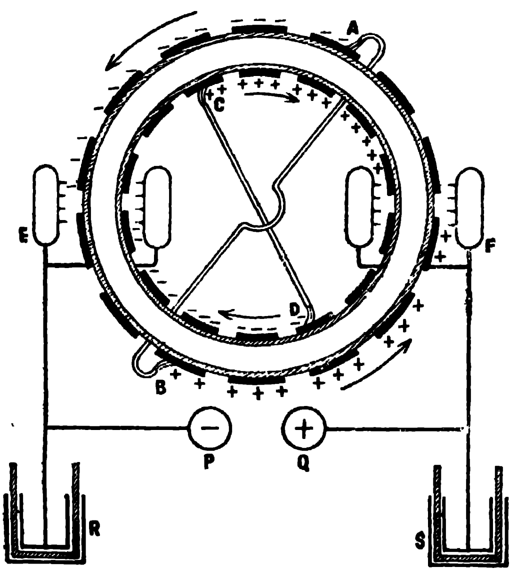


Fig. 39—Wimshurst Machine

Facing the plates there are two rows of sharp points, called the collecting combs, E **and** F, **at opposite ends of a horizontal diameter. These**

collecting combs are connected to the *dischargers* P, Q of the machine, and to the inside coatings of two Leyden jars, R and S, placed on the base-board of the machine.

Action.—Suppose one of the back sectors possesses a slight negative charge. When it comes opposite the sector C touching the brush of the diagonal conductor, it induces a positive charge on the sector C and a negative charge on the sector D at the other end. The sectors C, D leave the brushes with their induced charges, rotate and reach the positions opposite the sectors A, B on the back plate, where they induce negative and positive charges on A and B respectively. Now, the back sectors A and B will retain these charges while leaving the brushes. So, after one or two revolutions all the sectors approaching the collecting-comb on the left-hand side will acquire negative charges, and all those approaching the right-hand collecting-comb will acquire positive charges. The sectors on the left-hand side discharge their charge to the points of the collecting-comb causing it to be negatively charged, and the attached discharger P charged *negatively*. And the charged sectors, therefore, come out uncharged Similarly, the other discharger of the machine acquires *positive* charge.

The difference of potential between the two knobs of the dischargers becomes so great in course of operation of the machine that the air di-electric can no longer support the strain and ultimately "breaks down", if the knobs are not too far apart, and a spark passes across the air gap (see Art. 44). The charges on the knobs disappear due to this discharge, and a small interval is necessary before the charges can again accumulate on the knobs to acquire a sufficiently high potential to cause another spark discharge. For this reason *the discharge is an intermittent one.* If the knobs are brought nearer, a smaller potential difference is sufficient to produce a discharge, and so the frequency of the discharge increases.

The **object of the Leyden jars,** which are nothing but condensers, is to increase the capacities of the dischargers to enable them to accumulate heavy charges in order to give strong and loud sparks.

We have assumed a small charge on one of the sectors in explaining the action of the machine, but in practice no actual charging is necessary, as a very minute charge left in the machine is sufficient to start action.

Though a very large amount of electricity cannot be obtained with a Wimshurst machine, yet what little it produces is at *very high pressure* A machine of average size will produce a potential difference of 40,000 to 50,000 volts*. Larger machines give very high voltages indeed.

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*Volt is the unit of potential difference or electric pressure (see Ch. IV. Part. VII).

(b) The Voss Machine.—It also acts on the principle of induction. It consists of two co-axial circular plates of glass or ebonite placed vertically parallel to each other (Fig. 40). The back plate is larger and is fixed in position while the front plate can be rotated about the common horizontal axis by means of a handle.

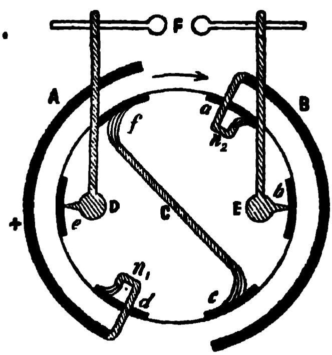


Fig. 40—Voss Machine

A set of six to eight *carriers*, *a*, *b*, *c*, *d*, etc., of metallic strips are distributed at regular intervals along the periphery of the revolving disc. Two large metallic strips *A*, *B*, called the *field plates* or *armatures*, are attached to the opposite sides of the back of the fixed plate. A centric metallic rod *c*, ending in two metallic brushes, called the *neutralising brushes*, placed on the revolving plate, touch a pair of carriers *c*, *f*, at diametrically opposite positions. There are two collecting combs *E*, *D*, each provided with pointed spikes, which face the carriers at the opposite ends of the horizontal diameter. The plates *A*, *B* have *appropriating brushes* n_1, n_2 connected to them, and these brushes make contact with the *carriers* as the same pass under them in course of their rotation. The collecting combs are connected to the adjustable prime-conductors *F* which end in two knobs. Usually the insulated plates of two Leyden jars are connected to the combs, the outer plates being earth-connected.

Action.—Suppose one of the field plates, say, *A* has a small positive charge and the smaller disc is revolving in the clock-wise direction. When the carrier *f*, in course of its rotation, passes out from under the plate *A*, it meets the neutralising brush connected to the conductor *C*. By induction of the charge on *A*, the carrier *f* is negatively charged and the carrier *c* at the far end of the rod positively charged. In course of its forward motion, the carrier *f* occupies the position *a* and delivers a part of its negative charge to the field plate *B* through the *appropriating brush* n_2 and the balance is collected by the comb *E*. At the same time, the positively charged carrier *c* passes on to the position *d* and gives a portion of its positive charge to the field plate *A* through the *appropriating brush* n_1 and the balance is discharged to the comb *D*. As the disc revolves, the field plates *A* and *B* receive greater and greater charges and, as a consequence, the carriers delivering charges to the collecting combs receive also increasingly greater and greater amount of opposite charges. Thus the two combs are charged up oppositely. The two Leyden jars connected to them increase the

capacity of the prime conductors F for accumulation of heavy charges in order to give strong and loud sparks.

(c) **The Van de Graaff Generator.**—It is a device for producing very high voltages. The discharging action of points and the collecting action of a hollow spherical conductor have been utilised in the construction of this machine.

A belt C of insulating material is made to travel continuously in the anti-clockwise direction round pulleys, as shown in Fig. 41 with the help of a motor. At the bottom of its path while it passes near the pointed end of a conductor A, which is maintained at a, say, positive potential of $(10-20) \times 10^3$ volts by means of a suitable electric machine, charge leaks from the pointed end to the belt on which it remains localised. The charge travels round and, as it passes near a set of pointed spikes, attached to the inside of the hollow spherical conductor M, the negative charges from the latter leak to the belt to neutralise the positive charges there, whereby the metal sphere is itself positively charged up. In course of time as more and more positive charges develop on M, it acquires a high voltage. To stop discharge from M, the generator is placed inside an earth-connected tank (provided with stop-cocks T_1 and T_2) which is filled up with air at high pressure.

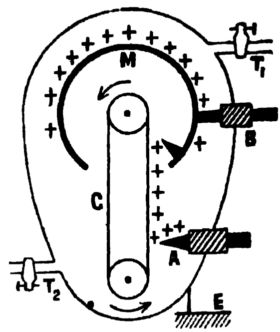


Fig. 41

A 5 million volt generator of this type erected at the Carnegie Institute of Washington in 1937 is housed in a tank, 55 ft. high and 37·5 ft. in diameter, filled with air under a pressure of 50 lbs. sq. in.

53. **Action of Points.**—It has already been stated that the distribution of electricity over the surface of a conductor is uniform only when the surface is uniformly shaped. *The density of the charge at any part of a conductor is inversely proportional to the radius of curvature of the surface of that part.* It is greatest on those parts of the surface which have the greatest curvature (least radius of curvature). So the density of charge at the pointed parts of a conductor is very great. Air particles, or dust particles, adjoining sharp points on a charged conductor on which the density of charge is great, acquire by contact a portion of the charge and are electrically repelled. This action, called the **discharging action of points**, discharges the conductors. This is the reason of the loss of electric charge from sharp points ; and this is why *points and sharp edges are avoided in electrical machines.*

Experiments to show the Discharging Action of Points.—

(1) This can be shown by holding a lighted candle before the pointed end of a piece of metal joined to one of the poles of a Wimshurst machine when the flame of the candle will be seen to be blown aside by air current produced by the discharging action of the point (Fig. 42). Such a continuous stream of the particles of a medium moving away from the sharply pointed parts of a charged conductor constitutes what is called the **electric wind.** To test the charge carried by the stream, the disc of an uncharged electroscope may be held against the stream when the leaves would be found to diverge. The nature of the charge which causes such divergence, on examination, would be found to be similar to that of the conductor.

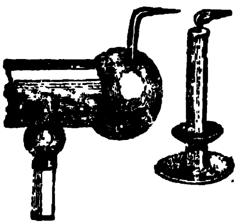


Fig. 42

(2) The action of the **Hamilton's Mill** (electric whirl) is an example of the discharging action of points. The mill (Fig. 43) consists of a brass disc B pivoted at its centre, having a number of conductors C, C fixed around it, the ends of the conductors being all bent at right angles the same way round. When the mill is joined up to the prime conductor of a Wimshurst machine, the disc with the needles rotate in a direction opposite to that of the bends of the needle, as shown by the arrows in the figure. Air particles on contact with the pointed ends of the needles are charged up similarly and are electrically repelled, and the reaction caused thereby makes the wheel rotate in the opposite direction.

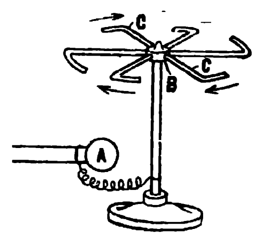


Fig. 43—Hamilton's Mill

54. Causes of Atmospheric Electricity.—The lightnings and thunders are natural phenomena which show that the atmosphere contains a heavy amount of electricity of both kinds. In the polar regions heavy discharges of electrified particles, lasting even for hours, causing display of colours, often times occur—the phenomena being known as **Aurora Borealis.** They also point to the same conclusion. But the causes of the electrification of clouds are even at present not very clear.

One theory supposes that water vapour arising from the water on the earth's surface carries a positive charge, while the water and the earth remain charged up negatively.

Elster and Geitel's idea is that the ultra-violet rays of the sun are the cause of electrification of the air. These rays break up the mole-

cules of the atmosphere into positively and negatively charged systems. The action may be due to the Cosmic rays [Art. 87 (a), Part VII] also.

Others suggest that this electricity is due to radio-active emanations from radio-active elements in the earth's crust. These radiations (Ch. IX, Part VII) also break up the air molecules and charge them with different kinds of electricity.

55 Lightning and Lightning Conductors.—It is assumed that during thunder-storms enormous differences of potential exist between the neighbouring clouds or between a charged cloud and the earth. A charged cloud and the earth may be regarded forming a very big condenser with air as di-electric between them. When the difference of potential between them becomes very high, the air di-electric breaks down followed by a discharge of electricity in the form of a flash of lightning. The air in the path of the lightning is heated by the discharge and so expands suddenly. This sudden expansion again cools the air due to which there is also sudden contraction producing a partial vacuum, and, as a result of this, the surrounding air rushes there with tremendous force. *The report of the thunder* is due to these sudden expansion and contraction of the air.

Lightning Conductor.—The action of the lightning conductor depends upon the discharging action of points and its conducting property. The lightning conductor was suggested by Benjamin Franklin (1749). It consists of a long rod or a strip of metal (iron or copper) running from the top of a building to be protected from destruction by lightning. The upper end of the rod is furnished with sharp points and the lower end is fixed to a metal plate well buried in wet earth.

During the thunderstorm, when a charged cloud passes over the points of the lightning conductor, induced charge of the opposite kind accumulates at the points. Air particles around the points are charged up by contact, and being electrically repelled, constitute an electric wind directed towards the cloud. The cloud thereby becomes gradually discharged. If, however, the difference of potential between the cloud and the conductor is so great as to produce a discharge, the lightning conductor offers a straight path of least resistance for the discharge to pass to the earth without damaging the buildings.

Precaution.—As a precautionary measure *(a)* one should not hold an umbrella during a storm, as it may act as a lightning conductor ; *(b)* one should not stand near or under a tall tree and also not near any metal fence or barbed wire ; *(c)* one should not remain standing in an open maidan.

Questions

Art. 50.

1. Explain the action of an electrophorus. (C. U. 1913, '16, '18, '21, '23, '26, '28, '42, '44 ; Pat. 1925, '29, '42 ; All. 1928 ; Dac. 1932).

2. What are the changes occurring in the potential of the upper disc of the electrophorus during the process of charging ? (C. U. 1942).

3. Describe a simple form of electrical machine for producing static electricity. (C. U. 1914 ; cf. Dac. 1933)

4. Describe the construction and explain the action of an electrophorus. How would you use it to charge an electroscope (a) positively, (b) negatively ? (Utkal 1948 ; C. U. 1932)

5. Describe the ordinary electrophorus and the method of charging a conductor by means of it, explaining how the energy of the charge on the conductor is obtained. Why is there a limit to the amount of charge that can be given to an insulated conductor by the electrophorus ? (*See.* Art. 51). (Pat. 1930 ; cf. C. U. '38, '44)

Art. 51.

6. Show how any amount of charge can be drawn from the electrophorus, when once excited, without violating the principle of conservation of energy. (C. U. 1932 ; Cf. '28 : All. '20 ; Pat. '32)

Art. 52.

7. Describe with neat sketches the parts and working of a Wimshurst machine. What is the function of the Leyden jars ? (Utkal 1948 ; Pat. 1926, '28 ; Cf. '22, '27, '31 ; C. U. 1934)

8. Describe and explain the action of an induction machine. (C. U. 1935)

Art. 53.

9. Explain why a conductor, which is required to retain an electric charge for a long time, should be rounded and without sharp points ? (C. U. 1925)

10. A positively charged conductor is brought near an insulated uncharged brass ball. Is the potential of the ball altered thereby ? Would this alteration (if any) be modified by the ball having a needle sticking out of its surface and would any such modification depend on the position of the needle ? Give reasons. (Pat. 1931)

[**Hints**—The potential of the ball which was zero in the beginning becomes positive when brought near the positively charged conductor. If the needle sticks out of the ball on the side opposite to the charged conductor, the ball will lose some positive charge through the needle and so the potential will be reduced ; but if the needle sticks out on the same side as the charged conductor, the ball will lose some negative charge and potential will be increased].

11. Describe an experiment to illustrate the discharging effect of a sharp conductor and some practical applications of the phenomenon. (Pat. 1937)

Art. 54.

12. Write a short note on "Atmospheric electricity." (Pat. 1937)

Art. 55.

13. What is meant by 'striking by lightning" ? How are high buildings protected against it ? (All. 1928, '31 ; Pat. '29, '31, '33 ; C. U. '35)

PART VII

CURRENT ELECTRICITY

CHAPTER I

Voltaic Cells

1. Historical.—(Discovery of Voltaic Electricity).—It has been already seen that when a conductor with a static charge was connected with the earth, or joined to another conductor, a flow of electricity, usually a discharge, could always be obtained; or, in other words, when two conductors at different potentials are connected, positive charge flows from the conductor at the higher to that at the lower potential until their potentials are equalised. This flow of electricity is called an **electric current**, but the flow just described lasts only for a moment. According to the *modern electronic theory*, it is not the positive charge which flows (from the higher potential to the lower), but it is the negative charge (electrons) that flows from the conductor at lower potential to the conductor at the higher potential (see Fig. 3).

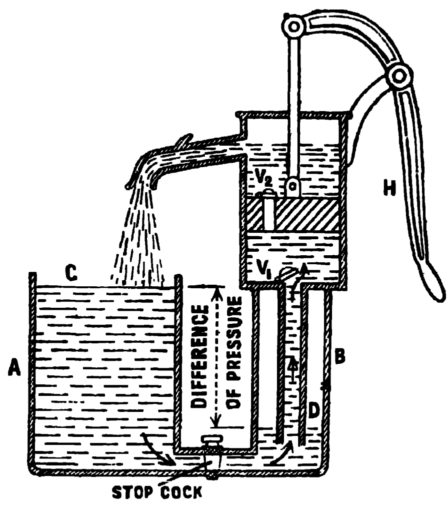


Fig. 1

If the difference of potential between the conductors could be kept constant by any arrangement, a continuous current could be obtained.

The following analogy will make the issue more clear. Suppose the two vertical cylinders A and B (Fig. 1) are connected near the bottom by means of a pipe fitted with a stop-cock. The water level in A is at C, while that in B is at D. If the stop-cock is opened, water will flow from A to B, but the flow will stop as soon

as the water attains a common level in both the cylinders. In similar manner electric current also stops flowing when there is no longer a difference of electric pressure. It is possible to put a pump in the circuit, as shown in the figure, and keep pumping water from B to A just as fast as to maintain the same difference of pressure as in the beginning. Thus a steady flow of water can be maintained at the expense of mechanical energy used at the pump. So also, in the electrical case, a steady current can be maintained at the expense of some form of energy which will keep the potential difference constant. It will be seen in Art. 3 that chemical energy liberated internally in a primary cell by action between the chemicals used in it maintains the potential difference between the terminals of the cell constant.

The first step towards a steady current was made in 1786 by **Galvani**, an Italian Physiologist and Anatomist. There are many stories of his observations and one of which is that incidentally on some day freshly skinned frog's legs were hanging through a brass hook attached to iron railings. Galvani to his astonishment found that every time the legs touched the iron there was a sudden contraction of muscles. He concluded that the effect was electrical and the source of electricity was the nerves and muscles of the animal body contacting each other.

2. **Volta's Pile.**—The next step of development was due to the Italian Physicist, Volta, who in 1800 showed that the above effect was rather due to the contact of two dissimilar conductors. He showed that the same effect could be obtained by using copper and zinc plates separated by a piece of cloth moistened with acidulated water and that there was a "*contact difference of potential*" between the two dissimilar metals in contact. This effect was multiplied by taking a number of such pairs of copper and zinc and arranged in a similar way in a vertical column. This is known as *Volta's pile.* Applying his discovery Volta set up the simple voltaic cell which is given in Art. 3.

(a). Theory of Contact Difference of Potential.—According to Volta, there is set up a difference of potential at the point of contact between two dissimilar conductors. How this arises in the case of different interfaces is explained below :—

(i) Solid-Solid Interface.—In conductors there are free electrons [see Art. 7(c), Part VI] which are assumed to move about within intra-atomic spaces, almost as freely as gas molecules. Different materials possess different electronic densities, *i.e.* different internal pressures. When two different conductors contact each other, their electronic pressures act through the common interface tending to

equalise the pressure. Electrons of the conductor having higher internal pressure pass into the conductor of lower internal pressure, the former being thereby positively charged and the latter negatively charged. This explains the difference of potential between two solid conductors put in contact.

(ii) **Solid-Liquid Interface.**—The difference of potential between a solid and a solution in contact may be explained almost in the same way as in the case of a solid-solid interface. The only difference is that owing to difference in internal electronic pressure one tends to deposit positive **ions*** on the other instead of causing electrons to flow. If the solid actually dissolves in the solution, it deposits positive ions on the solid and thereby it itself is negatively charged up while the solution gets positively charged, and after a time a stage comes when an electric equilibrium is reached. When this stage is reached, the solid and the liquid interface has two layers of equal but opposite charges. **Lord Kelvin** called it the **electrical double layer.** The reverse process of a solution depositing positive ions on a metal in contact also takes place. In this process the solid is positively charged up while the solution gets negatively charged. Fig. 2 illustrates how zinc is negatively charged up while copper is positively charged when immersed in dilute H_2SO_4.

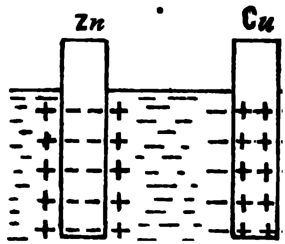


Fig. 2—Electric Double Layer

2(b). Theory of Electrolytic Dissociation.—According to this theory, put forward by **Arrhenius**, the molecules of a solution dissociate, by the very act of solution, into two distinct parts associated with equal but opposite electric charges. These parts are called the positive and negative ions. On setting up an electric field between two points in the solution, the two ions move in opposite directions and the motions of these ions in the electric field between the electrodes constitute an electric current in the solution, just as the motions of free electrons in a conductor are responsible for the current caused in it.

This type of dissociation is quite different from what is called the **thermal dissociation.** In the latter type, dissociation takes places at high temperature, in which always neutral molecules are formed by the splitting up of the more complex molecules. But in electrolytic

* An ion is an atom or group of atoms associated with a charge, positive or negative.

dissociation, only charged ions are produced, which may or may not be chemical molecules. As for example,

Thermal dissociation $NH_4Cl \rightleftarrows NH_3 + HCl$.

Electrolytic dissociation, $\begin{cases} 2NaCl = 2Na^+ + 2Cl^- \\ NH_4Cl = NH_4{}^+ + Cl^- \end{cases}$

3. (a) **Simple Voltaic Cell.**—A plate of copper and a plate of zinc are dipped in dilute sulphuric acid contained in a glass vessel. If the plates are connected by means of a metallic wire, bubbles of hydrogen gas will be seen to be given off from the surface of the copper plate.

The chemical action in the cell may be represented by the following equation.— $Zn + H_2SO_4 = ZnSO_4 + H_2$.

The chemical changes proceeding in the cell will set up a difference of potential between the plates, the copper plate being at a higher potential than the zinc plate. Due to this difference of potential, transference of electricity will take place along any wire connecting the two plates, and this constitutes what is called an **electric current.**

The difference of potential between the plates is known as the **Electro-motive force** (usually written, E. M. F.) which causes a fairly steady current to flow from zinc to copper inside the liquid and copper to zinc through the wire outside the liquid (see Art. 7). The direction so indicated is the direction of the **conventional current** and will be followed in this book, though the direction of the actual current, if current is due to flow of electrons, is just in the opposite direction (vide Art. 8). The E. M. F. of a simple voltaic cell is about 1·08 volts. The copper plate from which the current is said to start is called the **positive**, and the zinc plate, through which current enters the liquid, is called the **negative pole** of the cell.

(b) **Electronic Theory of the Simple Voltaic Cell.**—When two electrodes, *Cu* and *Zn*, are immersed in dilute H_2SO_4, electric double layer (see Fig. 2) is formed around each in no time, the difference of potential between *Cu* and the solution being +0·46 volt, while that between the solution and Zinc being −0·62 volt, so that the difference of potential between the *Cu* and *Zn* electrodes is 0·46−(−0·62)=1·08 volts. This explains the E. M. F. of the cell. When the *Cu* and the *Zn* electrodes are joined up by a conductor, electrons from the *Zn* electrode travel through the conductor to the *Cu* electrode to equalise the difference in potential between them. Thereby, the equilibrium at the electric-double-layer around both is upset. So the positive layer in front of the *Zn* electrode, and

the negative layer in front of the Cu electrode will predominate, as a result of which negative (SO_4^-) and positive (H^+) ions embedded in the solution will be urged towards the Zn and the Cu electrode respectively to restore the equilibrium. Thus the potentials of the two electrodes will be maintained by an internal action in the cell and a steady current will flow in the circuit of the cell. The concentration of the acid in the cell will decrease with action in the cell, and Zn will go into solution combining with SO_4 as follows,

$$Zn^{++} + SO_4^{--} = ZnSO_4.$$

H^+ after delivering its charges to the Cu plate will escape by bubbling.

4. **Defects of the Simple Cell.**—With a simple voltaic cell, as described above, the strength of the current gradually diminishes after some time. This defect is mainly due to two causes (i) **Local action,** and (ii) **Polarisation.**

(i) **Local Action.**—If commercial zinc be used in a simple cell, bubbles of hydrogen are seen to evolve from both the plates after the generation of the current, and even on open circuit, they may be evolved at the Zn plate. By the action of sulphuric acid, zinc sulphate is formed, and the acid is neutralised as more and more zinc is dissolved. So the chemical action decreases, and hence the strength of the current diminishes. This is one aspect. Again commercial zinc contains impurities like C, As, Pb and Fe They together with zinc form miniature local cells on the body of the zinc plate with the aid of the acid. These local cells produce local currents using up the zinc plate which do not contribute to the main current (see Fig. 3). Hydrogen bubbles may, therefore, be evolved at the zinc plate even on open circuit. This effect with commercial zinc is known as **local action.**

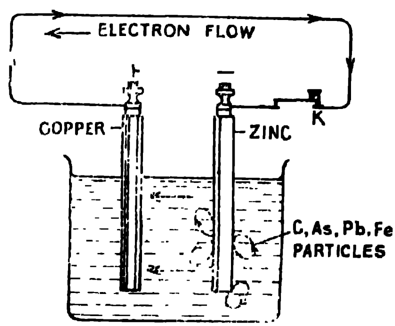


Fig. 3

The defect can be remedied by amalgamating the zinc plate. To amalgamate it, it is first washed with dilute H_2SO_4 or HCl and is then gently rubbed with clean mercury with the help of a brush or a piece of cloth. Mercury dissolves zinc forming an amalgam, but it

has no action on the impurities. Hence on the surface of the plate there is a shining layer of a mixture of mercury and pure zinc. The impurities remaining under the layer cannot come in contact with the acid and thus local action is stopped. The acid has no action on the mercury but acts on the zinc on the surface layer, which generates the current.

(ii) **Polarisation.**—As current flows, bubbles of H_2 evolve at the copper plate on which they gradually form a thin layer. Due to this the current strength falls and finally stops altogether. The effect is called the *polarisation* of the cell. It is explained as follows.—

(*a*) As the two plates are connected by an external wire, positive H^+ ions travel to the copper plate, deliver the charges and bubble away. In such a process a film of neutral hydrogen is bound to develop on the copper plate after some time. With thickness increasing, it offers greater and greater resistance to the current which thereby diminishes accordingly.

(*b*) As the action of the cell proceeds, the incoming H^+-ions are partially deposited on the film of neutral hydrogen which is non-conducting. Due to non-delivery of their charges to the *Cu* plate, the strength of the current may further diminish. Not only that, these H^+-ions also act in another way They create a new pole there, which is **electro-positive relative to zinc** and an electric field is set up in the cell which tends to send current in the opposite direction. This is called **back-electromotive force** or **polarisation e. m. f.** As a result the current considerably decreases. A stage is soon reached when the back e. m. f. altogether stops any H^+ -ion from moving towards the *Cu* plate. The current falls to zero at this stage and the cell is said to be completely polarised.

The **polarisation** in simple volatic cells is prevented by three principal methods :—(i) **Mechanical**—*by brushing off the hydrogen from time to time from the copper plate ;* (*ii*) **Electro-chemical**—*by using two solutions such that the hydrogen meets with a second solution, from which ions of the same metal as that of the positive plate are liberated (Cf. Daniell Cell) or some such gas is liberated as does not cause polarisation (Cf. Bunsen Cell).* (*ii*) **Chemical**—*by using some strongly oxidising substance such as chromic acid (or bichromate of potash with sulphuric acid) as in the Bichromate Cell, Nitric acid, Manganese dioxide, etc., to convert the hydrogen to water.*

The first method, however, is not at all satisfactory and for this reason many cells (as described below) have been devised in which polarisation is removed by the use of either of the latter methods. The chemicals used to remove polarisation are called **depolarisers.**

5. Voltaic Cells.—Different types of volataic cells are given below, each of which is provided with an excitant liquid and a depolariser.

(a) One-Fluid Cells.—

(i) The Bichromate Cell.—This cell consists of two carbon plates with a zinc plate Z placed between them (Fig. 4). The plates are immersed in dilute sulphuric acid in which crystals of *potassium bichromate* are added, which act as a **depolarising agent**. The two carbon plates form the positive, and the zinc plate the negative pole of the cell.

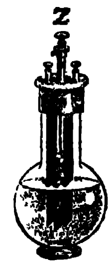


Fig. 4—Bichromate Cell

Action—By the action of sulphuric acid on the potassium bichromate, potassium sulphate and chromic acid are formed according to the following equation.—

$$K_2Cr_2O_7 + 7\ H_2SO_4 + H_2O = 2H_2CrO_4 + K_2SO_4 + 6H_2SO_4.$$

The sulphuric acid acts on zinc to form zinc sulphate liberating H^+-ion, and the chromic acid (H_2CrO_4) acts on hydrogen to form water, and chromic oxide formed thereby is converted into chromic sulphate, $Cr_2\ (SO_4)_3$, by the action of sulphuric acid thus,

$$6\ H_2SO_4 + 3Zn + 2H_2CrO_4 = Cr_2(SO_4)_3 + 3ZnSO_4 + 8H_2O.$$

The colour of $Cr_2(SO_4)_3$ is green and so, with the action of the cell, the colour of the liquid changes from red to green.

The E. M. F. of this cell varies from 1·8 to 2·2 volts. The current of this cell soon falls off, the internal resistance being low ; so it is useful where **strong current** is wanted **for a short time**.

In a modified type of the cell, chromic acid is used directly instead of potassium bichromate in order to avoid formation of crystals of chrome-alum which deposit on the plates and affect the chemical action of the cell.

(ii) Leclanche′ Cell.—This cell consists of an amalgamated zinc rod Z immersed in a strong solution of ammonium chloride (NH_4Cl) contained in an outer glass vessel (Fig. 5). In this vessel a porous pot is placed with a gas-carbon rod C at the centre surrounded by a mixture of broken carbon and powdered manganese dioxide (MnO_2).

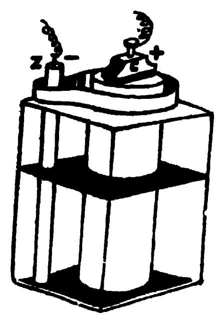


Fig. 5—Leclanche′ Cell

The Zn-rod acts as the negative plate, carbon-rod as the positive plate, NH_4Cl as exciting liquid and MnO_2 as depolariser. The charcoal powder is mixed up to make the depolariser an electrical conductor. By the action of Zn on NH_4Cl, NH_3 is liberated

which escapes through the mixture while the hydrogen ion liberated is oxidised by the MnO_2 into water according to the following chemical equations.—

Reaction : $Zn^{++} + 2NH_4Cl = ZnCl_2 + 2NH_3 + 2H^{++}$

$2H^{++} + 2MnO_2 = Mn_2O_3 + H_2O.$

Here MnO_2 is the **oxidising agent**, but as it is a solid its action is slow ; so if the cell is allowed to act for some time, polarisation sets in, and the current falls off. This is an obvious **disadvantage** with this cell. If, however, some rest is allowed, the cell regains its strength. So the cell is suitable **for intermittent work**,—*e. g.* telephone, telegraph, electric bell, etc. The E. M. F. is about 1·4 volts.

The **advantage** of this cell is that it lasts for a very long time. The electric bell system of a home can be worked for years with a battery of Leclanche′ cells with only occasional addition of water to ammonium chloride to replace losses by evaporation.

The ordinary **dry cells** used **in electric torches** are usually Leclanche′ cells where ammonium chloride is used in the form of a paste. For *high tension wireless batteries* also a number of Leclanche′ dry cells are used.

(b) Two-fluid Cells.—

(i) Daniell Cell.—This cell consists of a copper vessel A in which stands a porous earthen pot B containing dilute sulphuric acid and an amalgamated zinc rod Z. The outer copper vessel contains a concentrated solution of copper sulphate which acts as the "**depolariser**". In order to keep up the strength of the copper sulphate solution, crystals of copper sulphate are placed in two perforated shelves P and Q, which are partially immersed in the solution (Fig. 6). With fall of concentration, the E. M. F. falls and the internal resistance increases.

Zn reacts with H_2SO_4 liberating positive hydrogen ions according to the following equation.—

Reaction:—$Zn^{++} + H_2SO_4 = ZnSO_4 + 2H^{++}$.

The hydrogen ion diffuses through the porous pot and acts on the copper sulphate forming sulphuric acid and liberating Cu^{++}-ion.

$$2H^{++} + CuSO_4 = H_2SO_4 + Cu^{++}.$$

This copper ion is deposited on the copper plate and so there is no polarisation.

The E. M. F. is 1·08 volts. The internal resistance is rather high. So this cell is useful for **small but constant current**.

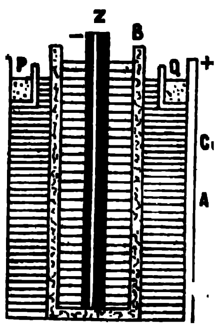


Fig. 6—Daniell Cell

The **advantages** of a Daniell cell are—

(a) The E.M.F. of this cell remains so nearly constant that it may be used as a comparison cell.

(*b*) It will give a constant small current for some time without polarisation and with practically no expense.

The **disadvantages** are :—

(*a*) When it is left standing, diffusion takes place through the porous pot in both directions and the amalgamation of the zinc tends to wear off.

(*b*) Immediately after using, the cell should be dismantled and it should be reset everytime before use.

(*c*) Sometimes it takes as long as half an hour after it is reset in order to settle down and give its final E. M. F.

(ii) Bunsen Cell.—This consists of an earthen vessel P containing dilute sulphuric acid, in which is placed a cylindrical porous pot B containing strong nitric acid. A carbon rod C is placed in the porous pot, and an amalgamated cylindrical zinc plate Z is placed in the earthen vessel around the porous pot.

Reaction :—
$$Zn^{++} + H_2SO_4 = ZnSO_4 + 2H^{++}$$
$$2H^{++} + 2HNO_3 = 2H_2O + 2NO_2$$

The nitric peroxide gas (NO_2) is soluble in strong nitric acid.

The E. M. F. is about 1·9 volts. This cell is useful for **strong and constant current**. The cell has got the disadvantage of disagreeable fumes of nitric peroxide. Some polarisation is present. Cautious handling is required.

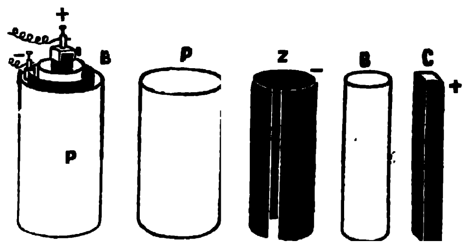


Fig. 7—Bunsen Cell

(iii) Grove Cell.—This cell is similar to the Bunsen cell except that the carbon rod is replaced by a platinum foil, which is very costly.

Dry Cell.—These cells are simply modified forms of Leclanché cells rendered portable by dispensing with liquids. The negative plate (Zn) is a zinc cylinder which forms the walls and the bottom of the cell (Fig. 8). The positive plate is a carbon rod C placed in the centre of the cylinder. The carbon rod is fitted with a brass cap which forms the terminal of the positive pole. Instead of a solution

like that of the Leclanche′ cell, a paste made of sal-amoniac NH_4Cl, MnO_2, C (coke or graphite) is used and a little water is added in the dry cell. The space between the carbon rod and the zinc cylinder is filled with the paste. A paper lining (*P.L.*) separates the paste from the walls of the cylinder and the NH_4Cl acts through this. The carbon rod is effectively insulated from the bottom of the cylinder by means of tar paper washer (*T. P. W.*). The top of the cell is filled up with a layer of saw-dust (*S.D.*), followed by a layer of sand (*S*), then by a layer of pitch or wax (*P*) to prevent loss of water by evaporation and short-circuiting of the poles. There is a pin-hole in the pitch or wax through which the gases escape. The $ZnCl_2$, formed by the action of Zn on NH_4Cl, absorbs the NH_3 gas. The coke or graphite reduces the internal resistance. The *E. M. F.* of the cell is about 1·5 volts. On continued use it may polarise, but it recovers if allowed to remain on open circuit for a while.

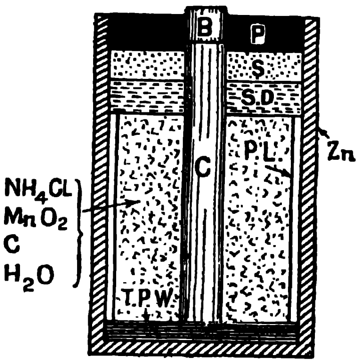


Fig. 8—Dry Cell

Standard Cells.—The E. M. F. of the cells described above undergoes a little change when continuous current is drawn from them, but for accurate scientific purposes it may sometimes be necessary that the E. M. F. of a cell should remain constant. Such cells, called *standard cells*, are used as standards of E. M. F. for determining the E. M. F. of other cells by comparison, *but they are not used for supply of current.* The E. M. F. of these cells *changes very little with temperature.* The *Weston Cadmium cell* and the *Latimar Clarke cell* are two standard cells.

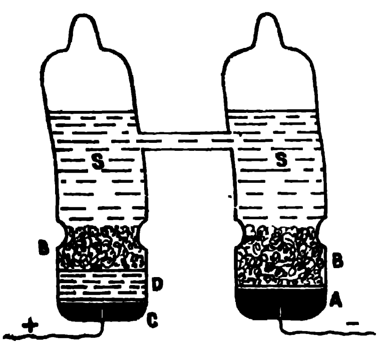


Fig. 9—Cadmium Cell

At a conference held in London in 1908 the **Weston Cadmium cell** was accepted as a standard one. It is made in the form of a *H*-shaped glass tube (Fig. 9) with two platinum wires sealed through the bottom which terminate at two binding screws. Pure mercury *C* is placed at the bottom of one of the tubes which serves as the positive pole.

Upon this, there is a paste of mercurous sulphate D. At the bottom of the other tube an amalgam of mercury and cadmium A is placed which forms the negative pole. For completing the electrical circuit a saturated solution of cadmium sulphate S is then placed in both the tubes which reaches a little above the horizontal connecting tube. Crystals of cadmium sulphate B are placed in the solution to keep the strength of the solution constant. The E. M. F. of the cell is 1·0183 volts at 20°C., and the following relation is given for calculating the E. M. F. at any other temperature, $E = 1{\cdot}0183 - 0{\cdot}0000406\ (t - 20)$ volts.

Latimar-Clarke Cell—In this cell the arrangement is Hg/Hg_2SO_4: $ZnSO_4/Zn$. A pool of mercury at the bottom of the cell forms the positive electrode. This is covered with a paste of mercurous sulphate in which the negative electrode, a zinc rod, is supported. To ensure saturation, crystals of zinc sulphate cover the paste. The cell thus differs in construction from the cadmium cell in the use of *pure zinc* throughout in place of cadmium. It is often shaped like a dry cell. At 15°C., its E. M. F. is 1·433 volts and at any other temperature $t°C$. it is $1{\cdot}433 - 0{\cdot}0012\ (t - 15)$ volts.

Only a very small current should be taken from a standard cell, otherwise it will be damaged, and that is why a very high resistance is always joined in series with such cells.

6 Distinction between Primary Cells and Secondary Cells (or Accumulators).—All the volatic cells considered above are called **primary cells** as they supply the current direct from the chemical energy liberated inside them. These cells have now been largely replaced, except for special purposes, by another type of cells, called **secondary** cells, which do not actually generate current, but a strong current can be obtained from the cells after a current is previously passed through them for some time from another source. The charging current is converted into chemical energy within the cells and this energy is utilised for supplying current afterwards. In primary cells, the constituents of the cell have to be supplied afresh after they are used up, but in secondary cells, they need not be replaced. The theory of the secondary cells is described in detail in Chapter VI.

7. Electric Circuit.—When the two poles of a cell are joined by a wire, the path of the current through the wire and through the liquid of the cell is called the **complete circuit**, the portion of which through the wire is called the **external circuit**, and the portion through the liquid of the cell, the **internal circuit**. The current passes from the

positive to the negative pole through the *external* and from the *negative to the positive pole* through the *internal circuit* (see Fig. 10).

When the two poles are connected by a wire, or and other conductor, **the circuit is said to be 'closed'** *and then only current can flow.* As long as the two poles are not connected by a wire, *i.e.* the path of the current is not complete, no current can flow, and **the circuit is then said to be 'open'.**

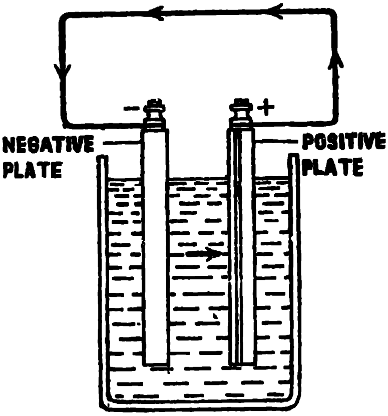


Fig. 10

It is to be remembered that **the value of current strength,** which depends on the E.M.F. of the cell and resistance of the circuit (Art. 31), **is the same everywhere in the circuit, both external and internal.**

8. Direction of an Electric Current (Modern View).—It has been stated in Art. 7 of Part VI, that the electrons in an atom are all in the *shell* of the atom. The same electrons may not always remain confined in the same atoms, but they—specially those near to the outermost boundary of an atom,—may become easily detached, and wander about in all directions for a while before they will get attached to other atoms. The substances in which the number of wandering electrons is great, conduct electricity well ; and because the number is *great in metals*, metals are generally good conductors of electricity. Just as sand particles, suspended in air, can be driven onwards by a breeze, so, if an electric potential, or **electric pressure,** as it is called, is set up between any two points of a metal—such as can be done by joining the two ends of it to the two terminals of a cell—the wandering electrons will be driven onwards along the wire *from the negative to the positive pole* of the cell. *This onward motion of the electrons along a wire from the negative to the positive pole is, according to the modern electron theory, called an* **electric current.**

It is to be noted, however, that it is only an *accepted mode of speech* to say that electricity flows from the positive to the negative pole, or from the higher to the lower potential, and it has become so firmly established that still this mode of describing the above phenomena is universally adopted. But, if an electric current is to be regarded as a flow of electrons (*i e.* of units of negative electricity), then it will travel in the *opposite direction* to the 'conventional' current.

Questions

Art. 2.

1. A strip of copper and a strip of zinc are dipped into a vessel containing dilute sulphuric acid. The strips are attached to the two terminals of a galvanometer, the needle of which is observed to be deflected. This deflection decreases considerably after the strips have remained for some time in the acid. Why is this? What methods have been adopted in practice to avoid this effect in voltaic cells? Give examples. (C. U. 1909, '16 ; Pat. 1932)

2. Explain clearly the phenomenon of polarisation and its causes in simple cells, and show how these are removed in various cells.
(Pat. 1930, '32, '38 ; Cf. C. U. '23 ; All. '18)

3. Explain 'local action' and 'polarisation' and show how they are avoided in a Daniell cell. (C U. 1930, Cf. '41, '46, '48 ; Dac. '34 ; Cf. Pat. 42)

Why is it necessary, for the good working of the cell, that the copper sulphate solution be kept concentrated? (C. U. 1946)

Art. 3.

4. Describe any two arrangements for maintaning a steady current of electricity in a given wire. Explain the mode of supply of energy for maintaining it in two cases. What becomes of the energy as it continues to flow?
(C. U. 1911)

[**Hints.**—Steady current can be maintained in a circuit containing any of the voltaic cells, say a Daniell, or a Bunsen. The energy is derived from the chemical action going on inside the cell, which is dissipated by heating the wire.]

5. Describe a Daniell cell and explain its action.
(C. U. 1911, '14, '21, '26, '28 ; Cf. Pat. 1919, '21, '27, '38)

6. What do you understand by polarisation in a voltaic cell? Describe a Leclanche' cell. What are the means taken to obviate polarisation in this cell? How far is this object attained? What properties make this cell a suitable one for electric bells? (C. U. 1924, Cf. '31, Cf. Pat. '39)

7. Describe a Bunsen cell and explain its action. (C. U. 1934)

8. Describe a Daniell cell. Explain the uses of its various components. What chemical changes take place in a cell when current is taken from it?
(Dac. 1927)

9. Explain the relative advantages and disadvantages of (i) a Leclanche' cell, (ii) a Daniell cell, and (iii) a storage cell or accumulator. (Pat. 1937)

Arts. 4 & 5.

10. Account for the chemical and other changes, if any, which occur before and after joining the terminals of a double fluid cell by a wire. What advantage has a double fluid cell over a simple cell? (Pat. 1936)

11. What is a standard cell, and why is it so called? (C. U. 1926)

Art. 6.

12. Describe a storage cell. What is its E. M. F.; How would you measure such an E. M. F.? (C. U. 1938)

In what respects does an accumulator differ from a Daniell cell?
(C. U. 1935)

CHAPTER II

Magnetic Effects of Currents

9. Oersted's Experiment.—In 1812 Oersted, of Copenhagen, made the following experiment which established a relationship between magnetic and electric phenomena.

Expt.—If a stretched wire $A\ B$ carrying an electric current is held over a pivoted magnetic needle NS with the length of the wire parallel to the axis of needle (Fig. 11), the needle is deflected and tends to set itself at right angles to the length of wire, but being influenced by two forces,—*one due to the magnetic effect of the electric current* and the *other due to the earth's magnetic field*—it takes up an intermediate position. It is observed that (*i*) **the direction of deflection of the needle depends upon the direction of the current**; (*ii*) **the deflection increases with the strength of the current.**

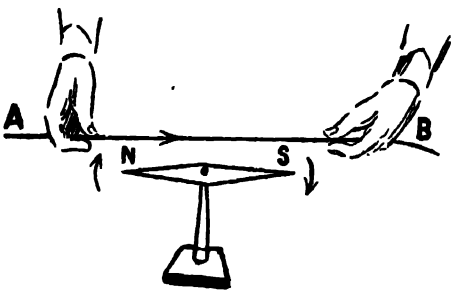


Fig. 11

10. Direction of the Magnetic Field.—To determine the direction in which the north pole of a magnetic needle is deflected due to an electric current, the following are the two convenient rules.—

(1) *Ampere's Swimming Rule.—*If a man be supposed to be swimming in the wire in the direction of the current with his face turned towards the needle, then the north pole is deflected towards his left hand* **(Fig. 12).**

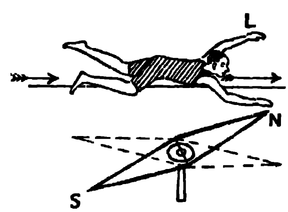


Fig. 12

N. B. Thus the direction of the current flowing through a wire can be detected by observing the direction of deflection of the N-pole of a needle pivoted underneath the wire.

*Andre Marie Ampere, a French pioneer in Electricity (1775-1836).

(2) Maxwell's Cork-Screw Rule.—*If a right-handed cork-screw be screwed along the wire so that the point of the screw travels in the same direction as the current, then the thumb indicates the positive direction of the magnetic line of force—i.e.* the direction in which the north pole of a magnetic needle will be deflected (see Fig. 13.) This law enables one to draw the lines of force around a current indicating their directions.

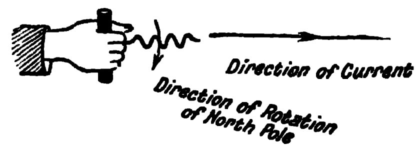


Fig. 13

Thus, if a magnetic needle is pivoted between two parallel wires, one above and the other below the needle, carrying currents in opposite directions (see Fig. 28), the current in the wires will tend to deflect the needle in the same direction, *i. e.* additive magnetic effects will be produced on the needle, and more turns of the wire will obviously increase the effect, just as it is done by increasing the current strength. *So, by using sufficient turns of wire, a very weak current is enabled to produce an appreciable deflection of the needle.* This is applied in many galvanometers—instruments for detecting and measuring electric currents (see Chapter III)—where the plane of the coil of several turns of wire is in the magnetic meridian. **The deflection of the needle depends upon the number of turns of wire and the current strength.**

11. Magnectic Field due to a Linear current.—Since a wire carrying a current deflects a magnetic needle, as done by a magnet, it indicates that just like a magnet it also has got a magnetic field near it. This magnetic field due to the current can be plotted (as done in the case of a bar-magnet) by means of a compass needle or iron filings.

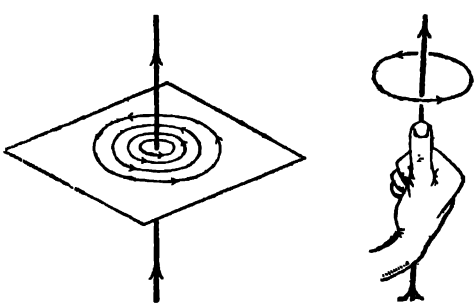


Fig. 14

If a strong current be passed through a vertical wire passing centrally through a horizontally placed card over which iron filings are sprinkled, then, on gently tapping the card, it will be observed that (*a*) *the iron filings (and so the lines of force) surrounding the straight wire set themselves in concentric circles (not, spirals) round*

the wire as their common centre (Fig. 14) ; *(b) the planes of the circles are at right angles to the direction of the wire.* Hence a magnetic needle placed just below a horizontal wire carrying a current tends to set itself along the lines of force surrounding the wire and so experiences a force at right angles to the wire ; but under the action of this force and also the force due to the earth, the needle takes up an intermediate position. Now, placing a small compass needle on the card near the wire in different positions and marking its ends, the direction of the lines of force can be determined. Observe that *(c) the direction is reversed by reversing the direction of the current.* On looking along the wire conveying a current away from the observer *(d) the positive direction of the lines of force will appear to be clockwise* (Cf. Cork-Screw Rule). It will also be noted that *(e) the strength of the magnetic field due to the current increases with the strength of the current,* and *(f) decreases as the distance from the wire increases.*

(a) Rotation of a Magnet round a Current.—A line of force indicates the direction in which a free north pole should move round a straight wire carrying a current. But as a single pole cannot be practically obtained, the above fact can not be demonstrated in practice. **Faraday,** however, devised an apparatus by which the motion of a magnet pole round a current can be exhibited.

In the arrangement of his apparatus [Fig. 14(a)], the current produced by a battery passes up the vertical hollow brass stand *A* which is fitted at the top with a brass cup containing mercury. Two cylindrical bar magnets *n*, *s* with their *n*-poles downwards are fixed to a brass frame- work *F* and are suspended from a beam *B* by an unspun silk fibre. The frame *F* carries a pointer *P* which dips into mercury contained in an annular cup *C* which surrounds the magnets. As the magnets rotate, the pointer also moves providing a rotating contact in a circle touching the surface of mercury in the annular cup. The current passing up the pillar *A* goes to the pointer *P* through the frame *F* and thus to the mercury in the mercury in the cup. The annular cup *C* is connected by means of a brass connector to the nega- tive of thebattery while the positive is connected to the pillar *A*.

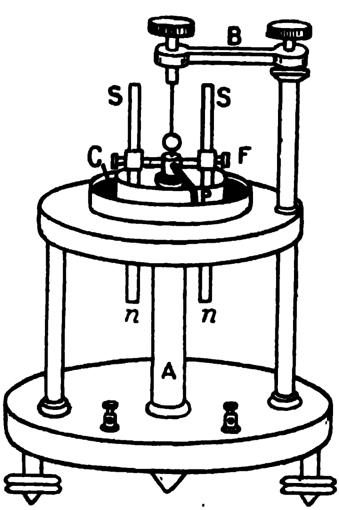


Fig. 14(a)

The magnets rotate on completing the circuit. The direction in which the magnets rotate can be easily traced by the application of Maxwell's Cork-Screw rule. On reversing the current, the direction of rotation is also reversed.

12. Intensity of Magnetic Field due to a Linear Current.— Before passing a current in the wire (Fig. 14) let a small magnetic needle, suspended horizontally, swing under the earth's horizontal field H and let the number of oscillations be n per minute ; then $H \propto n^2$. Now, pass a current up the wire and find the number of oscillations (n_1) per minute at a distance d_1 due magnetic *east* of the wire. On the *east* of the wire the direction of the lines of force due to the current and that due to the earth's field are the *same* ; hence (see Art. 33, Part V) the resultant field, $(F_1 + H) \propto n_1^2$, where F_1 is the magnetic field due to the current at a distance d_1. Repeat the experiment at a distance d_2, and let n_2 be the number of oscillations per minute. Then, the resultant field, $(F_2 + H) \propto n_2^2$.

$$\text{Hence} \qquad \frac{F_1}{F_2} = \frac{n_1^2 - n^2}{n_2^2 - n^2}.$$

It will be found that $(n_1^2 - n^2) : (n_2^2 - n^2) = d_2 : d_1$

$$\frac{\mathbf{F_1}}{\mathbf{F_2}} = \frac{\mathbf{d_2}}{\mathbf{d_1}} \qquad \text{.........................(1)}$$

i.e. the intensity of the magnetic field due to a current varies inversely as the distance.

Note—(*i*) On the *west* of the wire the *direction* of the lines of force due to the current and that due to the earth are *opposite*, so the resultant field would be $(F_1 - H)$, assuming F_1 to be greater, and in that case $(F_1 - H) \propto n'^2$, n' being evidently less than n_1. Hence counting the number of oscillations of the needle per minute on the east and also on the west of the wire, keeping it at the same distance from the wire in both the cases, *the direction of the lines of force due to the current can be known* by knowing the direction of the lines of force due to the *earth's field*, which is from geographical *south to north.*

(*ii*) It should be *remembered* that the rule due to Laplace is that the intensity of the magnetic field at a point due to the current in a *very short element* of current is **inversely proportional to the square of the distance** from that point ; but when this rule is extended to the whole circuit, the results are as stated above in eq. (1). This was verified by Biot and Savart.

(*iii*) Again, by passing a current of double strength in the wire it will be seen that the intensity at the same point near the wire will be twice as great, *i.e.* $F_1/F_2 = 2/1$, *or* **the intensity** *of the magnetic field at any point* **varies directly as the current strength.**

13. Magnetic Field due to a Circular Current.—If a strong current be passed through a circular wire, then, by observing (Fig. 15) the effects as before, it may be noted that, (*a*) *near the wire, the lines of force are approximately circular, and, within the space enclosed by the wire, the lines of force all travel in the same direction ;* (*b*) *near the centre, the lines of force are approximately parallel so that the magnetic field may be taken to be uniform for a small space round the centre ;* (*c*) *at the centre, the direction of the lines of force is at right angles to the plane of the coil. The directions of the lines of force can be assigned by applying the cork-screw rule.*

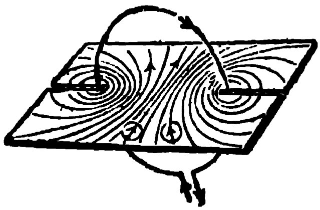


Fig. 15

From Fig. 15 it is clear that when a face of the coil is held before an observer, the lines of force inside the circular wire will point (*i*) **away from the observer** *when the direction of the current is* **clock-wise**, and (*ii*) **towards the observer**, *when* **anti-clockwise.**

The magnetic field due to a current round the circular loop closely resembles the field due to a magnetised disc of steel having the same area as the area of the loop and the thickness equal to the diameter of the wire. Also, like the magnetised disc, the wire-loop will present opposite polarities in the two faces. Thus, *the wire-loop with its current will behave like a magnetic shell the face in which the current seems to pass in a* **clock-wise** *direction, when held perpendicularly to the line of sight, acquires* **south-polarity,** and *the face in which the direction is* **anti-clockwise** *acquires* **north-polarity** (Fig. 16).

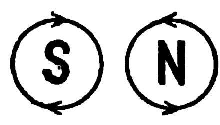


Fig. 16

Ampere's Law.—Thus we have got the following important Law stated by **Ampere.**—*A current flowing in a closed circuit of any shape behaves exactly like a magnetic shell, the edges of which coincide with the wire carrying the current, the moment of the shell per unit area (strength of the shell) being equal to the current strength.*

14. Intensity of Magnetic Field at the Centre of a Circular loop.—The intensity of the magnetic field F at the centre of a circular loop of wire carrying a current (*i.e.* the force in dynes on a unit N-pole) varies directly as the strength of the current C passing round the coil (the stronger the current the greater the effect), *i.e.* $F \propto C$: also $F \propto$ directly as the length (l cms.) of the wire forming the coil, *i.e.* $F \propto l$; again, $F \propto$ inversely as the square of the distance of the wire from the centre, *i.e.* the radius r of the coil ; so $F \propto 1/r^2$.

Thus, $F = k\dfrac{Cl}{r_2}$, where k is a constant ... ... (1)

(a) The Electro-magnetic Unit of Current.—We have seen that the strength of the magnetic field at any point near a wire carrying a current depends upon the following three things :—

(*i*) The length of the wire carrying the current ; (*ii*) the distance of the point from the wire and the direction of flow of the current ; (*iii*) the strength of the current.

So, in order to build up the definition of unit current, unit quantity of each, which goes to build it, must be taken.

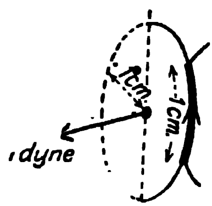


Fig. 17

The eq. (1) enables us to define the unit of current. For, when the length of the wire $l = 1$ cm., radius $r = 1$ cm, the current can be so chosen that the force at the centre $F = 1$ dyne If this current is taken to be unity, then in eq. (1) $k = 1$. Hence—

The unit of current *is defined as the current which flowing in a wire of 1 cm. length bent into an arc of a circle of 1 cm. radius exerts a force of 1 dyne on a unit magnetic pole placed at the centre* (Fig. 17).

This unit is called **the C G. S. Electro-magnetic (or absolute) unit of current** ; it is called electro-magnetic because it is based on the magnetic effect of an electric current.

The **practial unit of current is the Ampere.**

1 ampere $= 1/10$ C. G. S. electro-magnetic unit $= 10^{-1}$ E. M. U.

(In the above case one ampere would exert a force of 0·1 dyne on the unit pole at the centre.)

Then the expression for the intensity of the field becomes,

$$F = Cl/r^2 \quad \ldots\ldots\ldots\ldots (2)$$

(b). Field at the Centre of a Circular Coil.—

If in eq. (1) the whole length of the circular wire is considered, then $l = 2\pi r$, and $\mathbf{F} = \dfrac{2\pi r\mathrm{C}}{r^2} = \dfrac{2\pi\mathrm{C}}{\mathrm{r}}$ dynes (or gauss.)

If there are n turns of wire, $\mathbf{F} = \dfrac{2\pi n\mathrm{C}}{\mathrm{r}}$ dynes.........(3)

If in eq. (3) C is measured in amperes, then C (amperes) will be equivalent to $C/10$ E. M. U. of current.

$$\text{Then} \quad F = \frac{2\pi nC}{10r} \text{ dynes} \qquad \ldots \qquad (4)$$

Examples.—(1) *A compass needle suspended at the centre of a circular coil of wire with its axis in the magnetic meridian makes 10 vibrations per minute in the earth's field alone and 16 vibrations per minute when a current passes through the coil. Find the magnetic field due to the current in the coil when the horizontal component of the earth's field is 0·2 gauss.*

Let H = horizontal component of the earth's field; F = field due to the current in the coil; n = no. of vibrations due to the earth's field alone = 10 per minute; n_1 = no. of vibrations due to the combined field of the earth and that due to the current = 16 per minute.

Two cases may arise due to the direction of the current in the coil.—

(a) When field due to the current and that due to the earth are in the same direction, we have the combined field = $(F+H)$.

$H \propto n^2$; and $(F+H) \propto n_1^2$; $\therefore \frac{F+H}{H} = \frac{n_1^2}{n^2}$; Hence $\frac{F}{H} = \frac{n_1^2 - n^2}{n^2}$

$= \frac{16^2 - 10^2}{10^2} = 1·56$. $\therefore$ $F = 1·56\ H = 1·56 \times 0·2 = 0·312$ gauss.

(b) When the field due to the current and that due to the earth are in opposite directions the combined field will be $(F-H)$, and then we have,

$\frac{F-H}{H} = \frac{n_1^2}{n^2} = \left(\frac{16}{10}\right)^2 = \frac{64}{25}$; or $25F = 89H = 89 \times 0·2$; $\therefore$ $F = 0·712$ gauss.

(2) *A current of 1 ampere flows at right angles to the magnetic meridian in a circular coil of wire of 10 turns and radius 10 cms. Find the magnitude of the magnetic field at the centre of the coil.*

A short magnetic needle suspended at the centre of the coil makes 10 vibrations per minute with no current passing. On passing the current it swings through 180° and then makes 15 vibrations per minute. Explain this. What is the strength of the earth's field?

We have, $F = \frac{2\pi nC}{10r}$......from eq. (4) in Art. 14 (a); $= \frac{2 \times \frac{22}{7} \times 10 \times 1}{10 \times 10}$

$= 0·628$ gauss.

The magnet swings through 180° when the current passes through the coil so that the direction of the magnetic field due to the current in the coil must be opposite to that of the earth's field which has turned the magnet through 180°.

$\therefore$ (As in the last example) combined field $= F - H$.

So $\frac{F-H}{H} = \frac{15^2}{10^2} = \frac{9}{4}$; whence $13H = 4F = 4 \times 0·628 = 2·512$;

$\therefore$ $H = 0·193$ gauss.

15. **Solenoid.**—A long insulated wire wound in the form of a spiral of several turns, with the ends bent along the axis and brought near the middle (see Fig. 19), will form what is called a **solenoid.**

It has been seen in Art. 13 that a single turn of wire carrying a current behaves like a magnetised disc, so when a current is passed through a solenoid, each turn of the spiral will behave like a magnetic disc, but its polarities are destroyed by the opposite polarities of the two neighbouring turns, except at the two extreme faces where only the polarities are exhibited. According to the rules already stated (Art. 13),

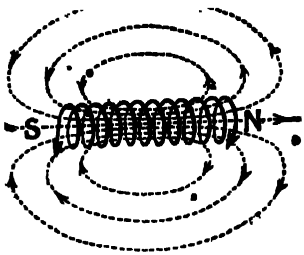


Fig. 18—Solenoid

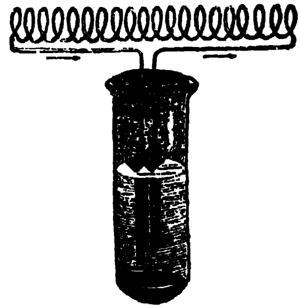


Fig. 19—Floating Battery

the polarities of the faces can be determined. The face in which the current is seen to flow in the **clock-wise** direction will have **south polarity**, and the other face in which the direction is **anti-clockwise** will have **north polarity** ; and the solenoid behaves exactly like a bar-magnet. Fig. 18 represents the distribution of lines of force when a current flows through a solenoid. It will be seen that the external magnetic field due to the solenoid is very similar to that of a bar-magnet. The polarity of the solenoid can be reversed by reversing the direction of the current.

The action of a solenoid is experimentally verified by **De la Rive's Floating Battery** (Fig. 19). The two ends of the wire of a solenoid are soldered to a zinc and a copper plate immersed in dilute sulphuric acid contained in a beaker, which is made to float in water being fixed to a pad of cork or rubber. A simple voltaic cell, thus formed, sends a current through the solenoid, which, like a floating magnet, turns and points towards the north and south pole of the earth. The clock-face rule stated above can be verified by testing the polarities with another permanent bar-magnet.

(a) **Field inside a Solenoid.**—The strength of the magnetic field

inside a solenoid of given shape and size depends upon two factors—(*i*) *the number of turns of wire in the coil*, and *(ii) the strength of the current*.

Moreover, if the number of turns is doubled, the resistance of the wire is doubled and the currrent strength is halved, the intensity of the field inside the solenoid remains unchanged. Hence the field depends upon the product of the above two factors, the current C (in amperes) and the number of the turns n, or, in other words, the number of **ampere-turns**. Thus if F be the intensity of the field inside the selenoid, $F \propto nC$; or $F = a\,nC$, where a is a constant, the value of which for a long solenoid is $0{\cdot}4\pi$, when C is in amperes. $\therefore$ $F = 0{\cdot}4\pi nC$. The quantity nC is called **ampere-turns**.

When C is expressed in C. G. S. electro-magnetic units, the intensity F inside the solenoid is given by $F = 4\pi nC$.

16. Electro-magnet.—If a rod of soft iron be placed inside a spiral of wire through which a current is passed, the rod becomes magnetised with opposite poles at the ends. When the current is turned off, the soft iron rod loses its magnetism. When the current is reversed, the polarity is also reversed. Such an arrangement is called an **electro-magnet.** If the iron bar is in the form of a horse-shoe, then the arrangement is called a horse-shoe electro-magnet (Fig. 20). A very intense field is produced between the pole-pieces of a horse-shoe magnet.

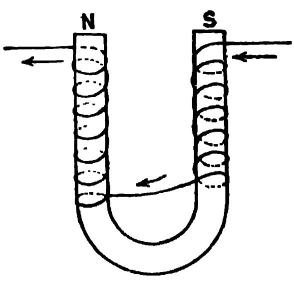


Fig. 20—Electro-magnet

The coil of wire must be wound around the two limbs of such a magnet in such a way that, when held in front, the current would appear to flow in a *clock-wise direction at one end* and in an *anti-clockwise direction at the other end*. To do this, the two limbs of the electro-magnet are wound with the wire in opposite directions as shown in the figure and finally the two terminals are connected to two binding screws. In this way the actions of the two straight solenoids, corresponding to the two limbs, as they are found, assist one another by setting up lines of force in the same direction through the iron. The strength of an electro-magnet depends on the ampere-turns per cm. [vide Art. 15 (*b*)] and the quality of the core. Moreover, to make the field intense and localised in a small space the two poles are brought close and sometimes detachable pole-pieces of soft-iron, varying in shape according to the purpose of use, are mounted on the poles. An interesting application of an electro-magnet is in the *Electric*

Bell (see Chapter VIII). Besides this, induction coils, transformers, telephonic and telegraphic receivers, loud speakers, cranes, etc, are several examples of the various uses of electro-magnets.

(a) Different Uses of Electro-magnets—The electro-magnets can be made much more powerful than permanent magnets, and they are used for several industrial purposes, such as (*a*) for lifting heavy pieces of iron ; (*b*) for separating iron from mixtures containing non-magnetic substances, *e.g.* from clay used in *porcelain manufacture ;* (*c*) for an apparatus that is a magnet only temporarily and can lose its magnetism as desired,—*e.g.* in working electric bells, relays, etc. ; and (*d*) for producing an intense magnetic field as required in electric motors and generators. They are also used in surgery for removing small pieces of iron from the eye. The electro-magnets used for different purposes may differ widely in their constructions and designs, but the principle used in all of them is the same.

Another Use.—Electro-magnets can be used to prepare good permanent magnets. The **methods** are as follows : (*i*) Draw each face of a bar of steel several times across one pole of the magnet, the stroking being always in the same direction. Finally touch each end of the steel bar with an unlike pole of the electro-magnet, and then slide off at right angles to the surface where the contact is made.

(*ii*) Place the bar across the top touching the two poles of the electro-magnet and start and stop the current several times. Switching the current on and off during the process of magnetisation helps to jerk the molecular magnets of the bar, but a few gentle taps, while the current is on, will help the process and thus increase the strength of the new magnet.

(b) The difference in action between an electro-magnet and a natural or artificial magnet lies in the following respects :—(*i*) An electro-magnet is more powerful than a natural or an artificial magnet. The action of an electro magnet increases with the strength of the current in the coil surrounding it until it reaches the saturation stage. (*ii*) The magnetism in an electro-magnet disappears as soon as the current ceases to flow, but in a natural or an artificial magnet the magnetism is more or less permanent. (*iii*) The polarity can easily be reversed in an electro-magnet by reversing the direction of the current, but this cannot be easily done in the case of a natural or an artificial magnet.

17 Action of Magnets on Currents.—We have already considered the action of currents on magnets and observed that a freely suspended magnetic needle brought near a current-bearing conductor is deflected. Here we shall consider the action of

magnets on currents, and the case of the rotation of a current round a magnet. The method is due to **Faraday**.

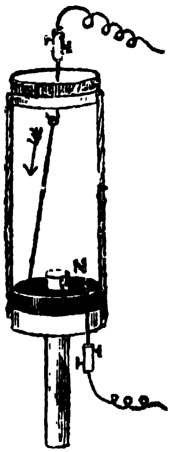


Fig. 21

Expt.—A glass tube is taken which is closed at both ends with corks and clamped vertically (Fig. 21). Through the lower cork the north pole of a bar-magnet projects a short distance into the tube, and through the upper one a wire is suspended from a hook with the lower end dipping in mercury poured into the tube, so that the pole projects slightly above the surface of mercury. On passing a strong current down the wire, the wire will be found to rotate round the pole of the magnet. If the current is reversed, the direction of rotation of the wire is also reversed.

Fleming's Left-hand Rule.—To determine the direction of deflection of a linear current placed at right angles to a magnetic field, the following rule due to Prof. Fleming is useful.—*Hold the thumb and first finger of the left hand as fully extended as possible, and bend the second finger at right angles to the palm so that the three fingers are mutually at right angles to each other. If the first finger points in the direction of the lines of force, and the second finger to that of the current, then the thumb will indicate the direction of motion of the conductor* (see Fig. 22).

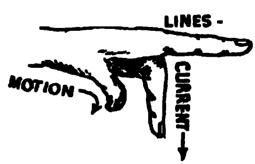


Fig. 22—Representation of Fleming's Left-hand Rule

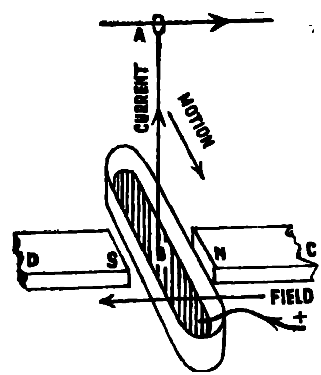


Fig. 23

Illustration of Fleming's Rule.—

Fig. 23 illustrates how Fleming's rule can be verified directly. From a horizontal metallic support a conductor AB is suspended at its upper end which terminates in a ring. The lower end B dips in mercury

kept in a trough which is placed, as shown in the figure, between the opposite poles of two bar magnets DS and CN. As a current is passed between the trough and the horizontal wire, AB moves out of the magnetic field in a normal direction. The direction of deflection is reversed either on changing the direction of the current or of the field, the direction being always in accordance with Fleming's rule.

18. Barlow's Wheel.—The Barlow's wheel is an instance of rotatory motion of a conductor carrying current placed in a magnetic field.

It consists of a star-shaped metal wheel, capable of rotating about a horizontal axle (Fig. 24). It is so arranged that when the wheel rotates, only one tooth of it just dips into a groove (at the base) containing mercury placed between the poles of a strong horse-shoe magnet, the plane of the wheel-disc being at right angles to the field. As one tooth just leaves the mercury, the next is about to make contact. As soon as current is made to pass from the axle to the mercury through a tooth of the wheel, the wheel begins to rotate. On reversing the current, the direction of rotation is also reversed. The speed of rotation depends on the strength of the field and the strength of the current. *The direction of rotation can be determined by* Fleming's left hand rule. *The experiment embodies the principle of the* **electric motor** (Art. 79).

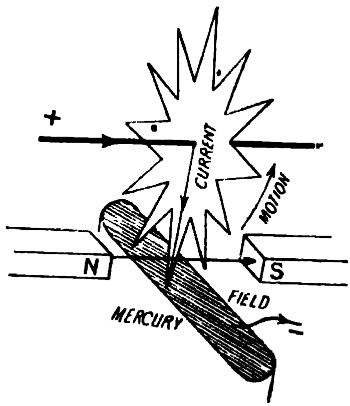


Fig. 24

Explanation of the Direction of Motion in terms of lines of force.—Fig. 25 explains the behaviour of a current-bearing conductor placed in a magnetic field (as in the case of Barlow's wheel) in terms of the properties of lines of force. Fig. 25(A) represents the lines of force passing from the north to the south pole of a magnet, and the circular lines of

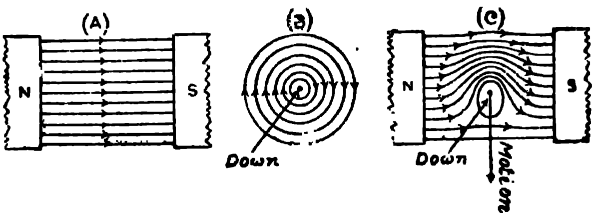


Fig. 25

force in Fig. 25(B) are due to the electric current in a linear conductor flowing downwards perpendicular to the plane of the paper. Fig. 25(C) represents the resultant effect of placing the conductor in the magnetic field, where it may be noticed that on one side of the conductor the two sets of lines of force are in the same direction (upper half) and repel each other, while on the other side (lower half) they are in opposite directions and cancel each other. The resultant effect of mutual repulsion is the movement of the conductor in the direction indicated.

19. Action of Currents on Currents.—It has been found that a magnetic field is always present round a current-bearing conductor. If a second current-bearing conductor be brought within the field due to the former, there will be attraction or repulsion between them according to the following laws :—

(1) Laws of Parallel Currents.—*Two parallel currents flowing in the same direction attract each other* (Fig. 26 *a*), *and flowing in opposite directions repel each other* (Fig. 26 *d*).

Suppose in two parallel wires A and B, suspended vertically, currents are flowing downwards, then the direction of the lines of magnetic force in each wire will be as that shown in Fig. 25(B). It will be noticed that the direction of the lines of force due to one of them, anywhere between the wires, will be opposite to that due to the other. So, if one of the wires is placed within the field of

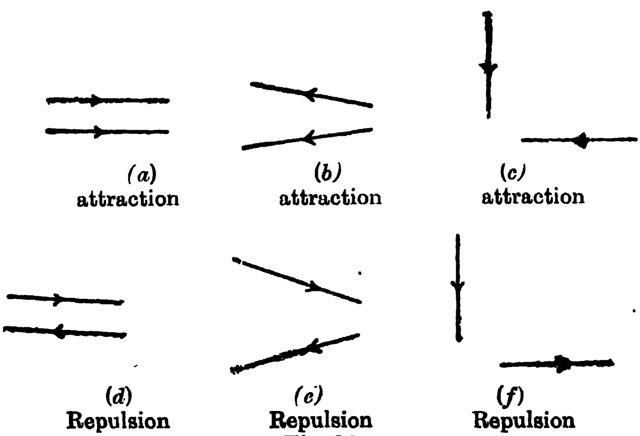


Fig. 26

the other, then they will be attracting each other. If the current flows in opposite directions, the directions of the lines of force

meeting at a point between the wires will be the same, and so they will be repelling each other. This also follows from *Fleming's Left-hand Rule.* Considering one of the wires A to be fixed, and the other B free to move, the direction of the magnetic lines of force at any point on B due to the current in A is perpendicular both to A and to the line joining the point to the wire A at right angles. Now applying Fleming's rule it will be found that B will tend to move towards A. If the current is reversed, A and B will repel each other.

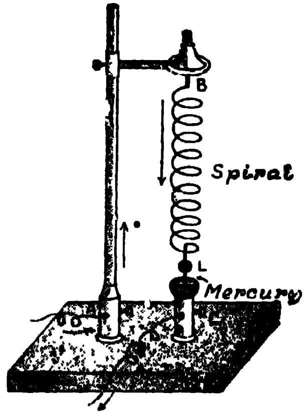


Fig. 27 -Roget's Vibrating Spiral

(2) Laws of Oblique Currents.—*Two oblique currents attract each other when they proceed from* (Fig. 26 *b*), *or to* (Fig. 26 *c*), *their apparent point of intersection, and repel each other if one flows from, and the other towards, that point* (Figs. 26 *e* & *f*).

Roget's Vibrating Spiral.—By this experiment the attraction of parallel currents flowing in the same direction can be verified. It is a close spiral wire which hangs vertically carrying a small metal weight L at the lower end just dipping in a mercury cup placed below (Fig. 27). The upper end of the spring is attached at the top of a stand. When a current is sent through the coil, adjacent turns carrying currents in the same direction attract each other (like parallel currents) and so the small weight is lifted out of mercury. This breaks the current. As soon as the current stops, the attractive force is removed and the spiral drops back in mercury by its own weight. This restarts the current, and the former process is repeated every time.

Questions

Art. 9.

1. You have access to the terminal wires of a hidden battery. How can you find out which wire is connected to the positive pole of the battery ? **(Pat. 1937)**

2. How would you demonstrate the magnetic action of electric current ? **(Dac. 1940)**

3. What is an electric circuit ? Describe two methods by which you could detect the existence, and determine the direction of an electric current flowing in the circuit. **(C. U. 1914 ; Cf. Cal. '46)**

(*See* also Art. 7)

4. A current passing through a long wire is so weak that, when the wire is stretched over and parallel to a suspended magnetic needle, the needle is not perceptibly affected. Describe and explain an arrangement which would enable you to obtain a movement of the needle by the action of the current. (C. U. 1923)

[**Hints.**—The wire should be coiled several times round the needle and the plane of the coil should be set in the magnetic meridian. If there are n turns of the coil, the magnetic effect produced on the needle would be n times as that due to a single turn. (See also Art. 20)].

Art. 11.

5. A strong electric current is passing through a long wire stretched vertically. State clearly how would you detect the direction of the current from its effects by (1) a magnetic needle, (2) the rate of vibration of the magnetic needle, and (3) a flexible wire carrying current. (Pat. 1928)

[**Hints.**—(1) Apply Maxwell's cork-screw rule ; (2) See Art. 12, (*i*) note ; (3) See Art. 20].

6. Describe the magnetic field in the neighbourhood of a long straight conductor carrying a current and show how you would verify your description.

What experiments would you arrange to find out how the conductor carrying an electric current tends to move in a magnetic field ? (Pat. 1932)

7. A small magnetic needle is suspended on a vertical pivot. How would it place itself and why ?

A wire carrying a current is held horizontally (a) along, (b) perpendicularly to the magnetic needle above its centre. Explain the effects observed.

The current is (a) increased in intensity ; (b) reversed in direction ; what will be the effects ? Why ? (C. U. 1919)

[**Hints.**—(See Arts. 9, 11 and 12.) (b) If the current flows from west to east, the direction of the magnetic lines of force due to the current is the same as that due to the earth's field. Hence the needle is unaffected ; but if the direction of the current is opposite, the needle will turn round, when the field due to the current is stronger.

The intensity of the magnetic field increases by increasing the current strength and so the needle is deflected more. Reversing the direction of the current, the needle is deflected on the other side of the meridian.]

8. Give an idea of the magnetic field due to a straight current. From the observation that two parallel currents attract each other. deduce a general rule for the action of a magnetic field on a conductor carrying a current.

[See Arts. 11, 17 and 19 ; (Fleming's Left-hand Rule)]. (Pat. 1927)

Art. 12.

9. A current is flowing in a straight wire four metres long. You are given a magnetised steel needle about 1 cm. long suspended by means of a silk fibre. How would you prove experimentally that the strength of the magnetic field due to the current falls off as the distance from the wire increases.

Art. 15.

10. Describe experiments to show that a circuit carrying a current behaves like a magnet. Under what circumstances will two neighbouring circuits repel each other ? Why ? (See also Art. 19). (All. 1928)

Ara. 16.

11. Describe the construction of an electro-magnet.

(C. U. 1912, '15, '18, '21, '26, '32, '45 ; Utkal 1947 ; Cf. Pat. '49)

How dow does it differ in construction and action from (a) a natural magnet, (b) an artificial magnet ? (C. U. 1918)

Arts. 17 & 18.

12. Describe an arrangement for producing continuous rotation of a wire carrying current when placed in a magnetic field. How is the direction of the current related to the field ? (C. U. 1917, '19)

13. Describe experiments which illustrate the action between two currents, and that of a magnet on a current. (See also Art. 19). (C. U. 1912, '16, '17, '29)

14. Explain the action of Barlow's Wheel or any arrangement for producing continuous rotation by electrical means.

(C. U. 1912, '18, '22, '26, '30 ; Pat. '47)

15. Describe Barlow's Wheel, and explain how it can be used to demonstrate the principle of working of an electric motor and a dynamo. On what factors does the speed of rotation of the wheel depend ? (Pat. 1944)

(See also Art. 77)

Art. 19.

16. Describe and explain the principle of action of the Roget's Vibrating Spiral. (Pat. 1947)

CHAPTER III

Galvanometers

20. Galvanoscope and Galvanometer—An instrument to detect the presence of an electric current is termed **Galvanoscope.** A galvanoscope or a galvanometer depends upon the mutual interaction between the magnetic field due to a current and that due to a permanent magnet.

A simple **galvanoscope** consists of a freely suspended magnetic needle surrounded by a few turns of wire, the plane of which should

be in the plane of the magnetic meridian (Fig. 28). As soon as a current passes in the coil, the needle is deflected (see Art. 9) and tends to set itself at right angles to the plane of the meridian, as the lines of force of the magnetic field due to the coil are perpendicular to the plane of the coil ; but at the same time, the needle is also influenced by the earth's magnetic field, which tries to pull the needle back into the magnetic meridian, *i.e.* the *action of the earth* on the needle is *opposing* that of the current. By applying Ampere's rule it will be evident that currents in the wires, both above and below the needle, which are in opposite directions, will produce a deflection in the same direction, and so additive magnetic effects will be produced on the needle. So the deflection of the needle is due to the resultant of these two forces—the force due to the earth's magnetic field, known as the **controlling force**, and that due to the magnetic field of the coil, known as the **deflecting force**.

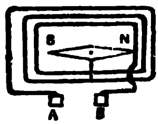


Fig. 28

The *Magnetic field due to the coil depends upon the current strength, and on the number of turns of the coil as each turn is effective in producing a magnetic field* (see Art. 9.). So in this way extremely weak currents can be detected by increasing the number of turns of wire surrounding the needle.

Galvanometer.—This is an instrument for detecting the presence of, and also measuring the strength of, electric currents. There are two types of galvanometers :—(i) **suspended-needle type (moving magnet type)**—in which the coil is fixed and the magnetic needle is movable, *viz.* astatic galvanometer, tangent galvanometer, sine galvanometer, etc. ; (*ii*) **suspended-coil type** (or **moving-coil type**), in which the coil moves and the magnet is fixed.

21. Astatic Pair of Needles : Astatic Galvanometer.—If two magnets are so chosen that their resultant moment is zero, the pair will remain at rest, when suspended in any manner in a uniform field. Such a pair of magnets is said to be an **"astatic"** pair. It is practically obtained by taking two magnets NS and $N'S'$ of equal length, and of almost equal pole strength, and placed parallel to each other with opposite poles towards the same end, the two being rigidly fixed together in a frame (Fig. 29).

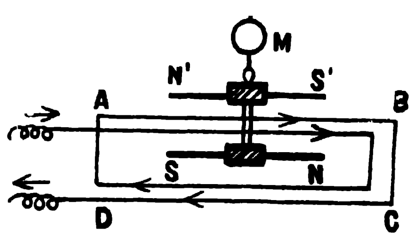


Fig. 29—Astatic Pair of Needles

An astatic pair, free to rotate horizontally, would come to rest in any direction, not necessarily in the magnetic meridian. In practice, however, it is not possible to secure two needles of exactly the same strength, and so the combination behaves as a weak magnet on which the earth's turning effect is very small.

If m and m', which are nearly equal, are the pole strengths of the magnets of an astatic pair, the force due to the earth's field acting on one of the magnets is mH, and that on the other $m'H$. So the resultant *controlling force* acting on the pair is $(mH - m'H)$.

Astatic Galvanometer.—This consists of an astatic pair of needles, *i.e.* a combination of two needles of equal length and streenth, rigidly fixed parallel, with their opposite poles adjacent, and suspended by means of a single silk fibre (Fig. 29). The lower needle of the pair moves freely within a coil $ABCD$ of many turns of wire wound on a wooden frame, and the upper one lies above the upper layer of the coil. An aluminium pointer, attached at right angles to the pair, moves freely over a graduated circle to show any deflection. The deflection may be measured, with greater accuracy, by a spot of light which is thrown from a lamp on the mirror M attached on the suspension fibre and received on a distant translucent scale after reflection (see Art. 25). Two unlike poles of the magnetic needles being adjacent to each other, the turning effect of the earth's magnetism on one is almost cancelled by an almost equal effect in the *opposite* direction on the other. Thus the controlling (*i.e.* opposing) force due to the earth's magnetic field is very small, and so the deflection of the pair is larger for a very weak current. The only **controlling force** in this case is the **torsion** of the silk fibre.

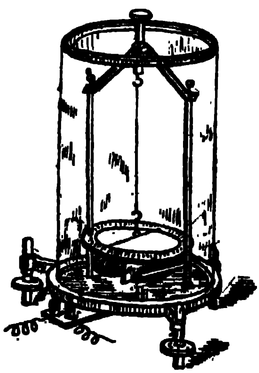


Fig. 30—Astatic Galvanometer

To use the instrument, which is depicted in Fig. 30, it is first levelled by means of levelling screws and is so placed that the coil is parallel to the length of the pair.

The strength of the current is proportional to the angle of deflection as long as it is small (not greater than 10° to 15°).

22. The Sensibility (or Sensitiveness) of a Galvanometer.—*This may be defined as the amount of deflection obtained with a given current.* The galvanometer which gives a considerable deflection with

a. weak current is said to be *sensitive.* The **sensibility** of a galvanometer is measured by the deflection produced by one micro-ampere (10^{-6} ampere) current.

In the case of a reflecting galvanometer, sensitiveness is measured by the current in amperes required to produce a deflection of one scale division (mm.), the scale being placed at a distance of one metre from the coil or needle This is called the **figure of merit** of the galvanometer.

It will be shown presently (see also Art. 24, Part V) that if the needle of a galvanometer comes to rest making an angle θ with the magnetic meridian, then $\tan\theta = F/H$, where F and H are the strengths of the magnetic fields due to the current and the earth respectively. Therefore *the sensibility of a galvanometer can be increased by (i)* **increasing F, the deflecting force**, which may be effected, within certain limits, by increasing the number of turns, [this cannot be increased indefinitely as it will increase the resistance which will reduce the current strength (see Art. 31)] ; and (*ii*) **by diminishing H, the controlling force** which can be done by using an astatic pair of magnetised needles as already described.

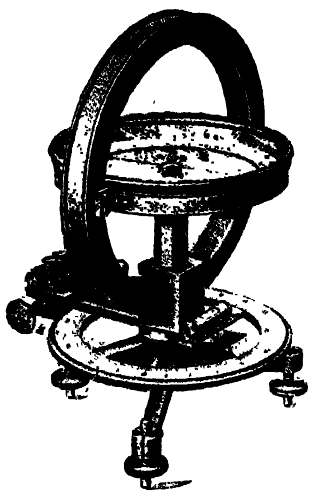


Fig. 31—A combined form of Tangent and Sine Galvanometer

23. Tangent Glavanometer.—The Tangent Galvanometer is a moving magnet, fixed-coil type of galvanometer. This consists of a circular coil of several turns of insulated copper wire mounted vertically on a wooden board provided with levelling screws (Fig. 31). The frame containing the circular coil can revolve about the vertical axis over a circular scale on the wooden board at the base, and any rotation given to the coil can be measured in this scale by means of a light pointer attached to the frame and rotatable with it. At the centre of the coil a horizontal circular scale (graduated in four quadrants reading 0° to 90°, 90° to 0°, etc., successively) is fixed, and a small magnetic needle is suspended or pivoted at the centre. The needle is provided with a long pointer of aluminium wire attached at right angles to its length, which moves over the graduated scale with the movement of the needle.

The needle and the scale are enclosed in a flat casing provided with a glass top. There is a strip of circular mirror at the base of this casing and in it the reflection of the pointer is observed to avoid parallax error in reading the deflection of the needle on the graduated scale. In some instruments, instead of one coil, two or more coils of wire having different diameters and turns, and each having separate binding screws, are set up in the frame and any, or all of them at a time, may be used according to necessity.

Theory and Use.—(*i*) All permanent magnets or magnetic substances must be removed as they will affect the position of the needle. (*ii*) The instrument is levelled so that the needle moves freely and rests exactly over the centre of the scale. (*iii*) The coil is then rotated until it is in the magnetic meridian, *i.e.* parallel to the needle, so that the coil and the needle are in the same vertical plane. The pointer carried by the needle should now point to the zero on the scale; if not, the circular scale, which is rotatable, is turned till the zero of the scale comes against the pointer. Now, when the current passes through the coil, the needle NS is deflected through an angle θ (Fig. 32). If F be the intensity of the magnetic field at the centre of the coil due to the current and m the pole strength of the magnet, the forces acting on both the poles will be mF dynes, which will form a couple. Another pair of forces, each equal to mH dynes, due to the earth's field, will act on the poles of the needle and tend to bring the needle back to the meridian. Due to the opposing action of these two couples, the needle finally comes to rest making an angle θ with the magnetic meridian. At this position, the moments of the couples balance each other (read Art. 24 with 'note'. Part V), whence,

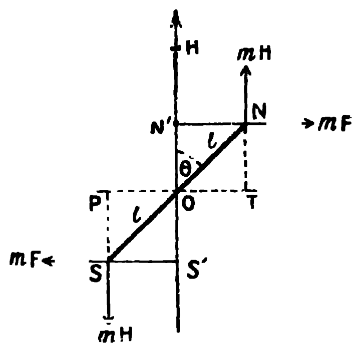


Fig. 32

$$F = H \tan \theta \quad \dots\dots\dots\dots\dots\dots (1)$$

or, the tangent of the angle of deflection is proportional to the deflecting force F, since H is constant for the given place. This is known as the **Tangent law**.

If the number of turns used $= n$: the mean radius of the coil $= r$ cms., and current strength $= C$ in E. M. units, we have, from eq. (3), Art. 14, the magnetic field at the centre, $F = \dfrac{2\pi n C}{r}$.

Hence from eq. (1), $\frac{2\pi n C}{r} = \tan\theta$;

$$\text{or} \quad C = \frac{H}{2\pi n/r} \tan\theta = \frac{\mathbf{H}}{\mathbf{G}} \tan\theta \qquad \text{.................(2)}$$

where $G = 2\pi n/r$, which is constant for the same galvanometer and is called the **Galvanometer Constant.**

Again, for the same place H is constant. So, in a particular place, the quantity H/G is a constant for the same galvanometer.

Putting $H/G = K$, we get C (in E. M. U.) $= K \tan\theta$, where K is called the **Reduction factor** of the galvanometer. **It is the factor by which the tangent of the angle of deflection must be multiplied to give the value of the current strength in e. m. units**

It should be noted that the galvanometer constant is the same for any particular instrument, whereas its reduction factor varies from place to place as the value of H varies.

In the above equations, the value of C is expressed is C. G. S. electro-magnetic units : but if i be the number expressing the strength of the current in amperes, then, since 1 ampere is equal to $\frac{1}{10}$ C. G. S. unit of current, $i = 10\ C$: or $C = \frac{i}{10}$;

$$\text{or} \quad i \text{ (amperes)} = 10\ H/G \tan\theta = \mathbf{10\ K \tan\theta}.$$

This instrument is called the **Tangent Galvanometer** as it obeys the tangent law, that is, the tangent of the angle of deflection produced by the current is proportional to the current passing round the coil. Thus, if a current produces a deflection of 20°, then the deflection, when the current is doubled, will not be 40°, but an angle the value of whose tangent is twice the value of *tan* 20°. In order that the tangent law may be followed, the field due to the current in the coil should be uniform in the region in which the needle moves. As the magnetic field due to the current is fairly uniform over a small region round the centre in the horizontal plane (see Fig 15 and read Art. 13), **the needle of the galvanometer should be very short,** so that the whole of it may move in a uniform field and the relation given in eq. (1) or (2) may hold good.

The following important points should be borne in mind regarding the **use and adjustment** of a tangent galvanometer.

(1) It should be noted that the *galvanometer needle will not at all be deflected*, if the plane of the coil is at right angles to the plane of the magnetic meridian.

(2) Any current from zero to infinity can be measured with a tangent galvanometer, but when using a tangent galvanometer, matters should be so *arranged that the deflection is as near 45° as possible*—at least it should be between 35° and 60° so that any error made in reading the angle may introduce the minimum error in the value of the current.

(3) The *reason for placing* the plane of the coil in the *magnetic meridian* is that the suspended magnet may experience the greatest couple twisting it out of the meridian, while the horizontal component of the earth's field H tends to keep it in the meridian, and this is exactly the condition of the tangent law (see eq. 1, p. 363).

(4) Though, just when the current is allowed to flow in the coil, the deflection may go beyond 90° due to inertia, there cannot be a *permanent deflection of* 90° in a tangent galvanometer owing to the opposing effect of the controlling field due to the earth's magnetism.

(5) Note that the tangent galvanometer is essentially a magnetometer (Art. 29, Part V) in which the magnetic field F is due to a current flowing in a circular coil, the plane of which is in the magnetic meridian. Magnetometer is used for the measurement of magnetic field, and tangent galvanometer is used for the measurement of current.

(6) From eq. 3 (Art. 14), it is clear that the *intensity of the magnetic field*, due to the current, produced at the centre of a tangent galvanometer (*a*) *varies directly* as the *number of turns* (n), and (*b*) *inversely as the radius* (r) *of the coil.* To make the arrangement sensitive, the coil should be of small radius and comprise many turns.

(7) Readings should always be taken vertically from above the pointer, *i.e.* when the pointer is just above its own image in the mirror in order that error due to parallax may be avoided.

(8) If the needle is suspended, then it should be examined whether the needle truly lies in the magnetic meridian or not. If not, there is tension in the suspension fibre, which must be avoided.

(9) In working out the theory of the instrument the magnetic needle has been assumed to move in a uniform field. In order that this condition may be at least roughly fulfilled, the needle should be as short as possible because, for only a small region near the centre of the coil, the field is uniform.

(10) For small deflections near zero, the tangent galvanometer is most sensitive, but it is accurate near about 45°.

(a) Sensitiveness of the Tangent Galvanometer.—The sensitiveness depends upon the amount of deflection for a given current

(Art. 22). So it is clear that the sensitiveness of the tangent galvanometer will increase if (*i*) n is increased, and (*ii*) r and H are decreased (see eq. 2, p. 364). But it should be noted that by increasing n the resistance is increased and so the strength of current is diminished. Hence n cannot be increased indefinitely. Again r cannot be decreased below a certain value as the needle has got some definite length. So the only other suitable method is to decrease the controlling field H by any artificial means. This can be done by fixing a bar-magnet horizontally above the galvanometer needle and adjusting its distance (and so the magnetic field due to it) such that it opposes the horizontal field due to the earth. It thus produces a resultant field which is weaker than the earth's field, and the value of the controlling field being thus reduced, the galvanometer gives a greater deflection for a certain current.

Examples —(1) *A current of 10 amperes produces a deflection of 45° in a tangent galvanometer. What is the value of the current which will produce a deflection of 30° in the same galvanometer ?* (*C. U. 1933*)

A current of 10 amperes produces a deflection of 45°. So from the relation, $C=10K \tan\theta$, we have, $10=10\,K \tan 45°=10\,K$; *or* $K=1$.

$\therefore$ The current (in amperes) required to produce a deflection of 30° :

$$C=10\,K \tan 30°=10\times 1\times 1/\sqrt{3}=5{\cdot}7 \text{ amperes.}$$

(2) *A current is sent through two tangent galvanometers in series. The radius of the coil of one of these is three times that of the other and they have the same number of turns. If the deflection in the latter case be 60°, what is the deflection in the former ?* (*C. U. 1940*)

Let r_1 and r_2 be the radii of the coils of the 1st and 2nd galvanometers respectively and let $r_1=3r_2$.

Because the galvanometers are joined in series, the current is the same in each of them, and no. of turns n being the same,

$$C=\frac{Hr_1 \tan\theta_1}{2\pi n}=\frac{Hr_2 \tan\theta_2}{2\pi n}$$

$\therefore$ $r_1 \tan\theta_1=r_2 \tan\theta_2$; but $r_1=3r_2$, and $\theta_2=60°$; $\therefore$ $3r_2 \tan\theta_1=r_2 \tan 60°$

or $3 \tan\theta_1=\sqrt{3}$; or $\tan\theta_1=\dfrac{\sqrt{3}}{3}=\dfrac{1}{\sqrt{3}}$; $\therefore$ $\theta_1=30°$.

24. The Sine Galvanometer.—This galvanometer is similar to the tangent galvanometer with this difference that the coil of the sine galvanometer can be rotated round a central vertical axis, and that a horizontal circular scale (Fig. 31) is provided on the base-board to read accurately the amount of rotation. A pointer rigidly attached to the frame of the coil enables the amount of rotation to be determined from the base scale.

Theory and Use.—As in the case of the tangent galvanometer, the plane of the coil of a sine galvanometer is first placed in the magnetic meridian, but when, after passing the current, the needle NS is deflected, the coil is rotated until the needle lies again in the plane of the coil, making an angle, say, θ with the meridian, shown by the dotted line ns (Fig. 33). The *deflecting force* mF is at right angles to the coil where F is the field at the needle due to the current and, because in this position the needle is in the plane of the coil, mF is perpendicular to the axis of the needle. Therefore, we have, the moment of the deflecting force = the moment of the controlling force at this position of the needle. The controlling field is due to the earth's horizontal component and is equal to mH acting parallel to the meridian.

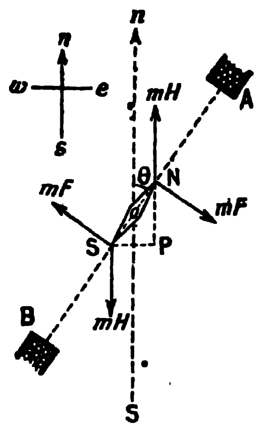


Fig. 33

That is, $mF \times NS = mH \times SP$, where SP is the perpendicular distance between the two controlling forces mH acting at the two ends of the needle ; $F = H\frac{SP}{NS}$ *i.e.*, $F = H \sin\theta$. But $F = \frac{2\pi nC}{r} = H \sin\theta$.

Therefore, we get, $C = \frac{rH}{2\pi n} \sin\theta = \frac{H}{G} \sin\theta = K \sin\theta$.

If C is in amps, $C = 10\ K \sin\theta$.

Advantages over a Tangent Galvanometer :—

(*i*) A current C which gives a deflection of 45° on a tangent galvanometer gives a deflection of 90° on the same instrument, when used as a sine galvanometer. For. let K be the reduction factor of the galvanometer, and θ_1 and θ_2 be the deflections for the tangent and the sine galvanometers respectively, then $C = K \tan\theta_1 = K \sin\theta_2$.

Now, if $\theta_1 = 45°$, we have, $K \tan 45° = K \sin\theta_2$;

or $1 = \sin\theta_2$; $\therefore\ \theta_2 = 90°$.

Therefore the **sine galvanometers are more sensitive,** and so for *feeble currents, sine galvanometers* are more suitable. than the tangent galvanometers, but for *heavy currents, tangent galvanometers* should be used.

(*ii*) Since the needle is in the plane of the coil at the start, as also finally, the suspending fibre is always untwisted, so there is no error due to torsion in the sine galvanometer.

(*iii*) In a sine galvanometer, the coil and the needle are always brought to the same relative positions and so the field of the coil in which the needle lies is always the same. So, for comparative measurements, that is, when two currents are to be compared, the coil may be of any shape and the needle also may be long.

The Disadvantages of a Sine Galvanometer :—

(*i*) A preliminary adjustment should be made before every reading, and (*ii*) heavy currents cannot be measured by it, *i.e.* currrents greater than H/G (for which $\theta = 90°$) cannot be measured by a sine galvanometer.

25. **Lamp and Scale Arrangement.**—This is an arrangement for measuring accurately the angle of deflection of a galvanometer needle or coil in place of the pointer ordinarily used for the same purpose. In this method a beam of light is obtained from a lamp L enclosed in a metal case provided with a fine slit and an adjustable tube, in which a convex lens is fitted (Fig. 34). The beam of light passing through the lens is directed upon the mirror M, usually concave, attached with the suspending fibre of the galvanometer and is reflected back upon a translucent scale placed at some distance from the mirror. Deflections are observed by focussing the image of the slit on the graduated scale $B\ S$ by adjusting the lens.

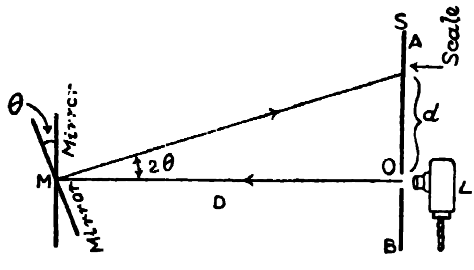


Fig. 34—Lamp and Scale arrangement

It should be observed that if the suspending fibre rotates through an angle θ, the mirror is rotated through the same angle, but the reflected spot of light rotates through an angle 2θ (see Art. 16, Part IV).

If D be normal distance of the scale from the mirror, and d the distance through which the image is shifted, we have

$$\tan 2\theta = d/D.$$

Since 2θ is usually very small, $\tan 2\theta = 2\theta$;

$$\therefore\ 2\theta = d/D\ ;\quad \text{or}\quad \theta = d/2D.$$

Thus $\theta \propto d$, or *the deflection is proportional to the shift of the image on the scale.*

26. Suspended (or moving) Coil Galvanometer.—In this type the coil moves and the magnet is kept fixed, and the rotation of the coil carrying a current in the magnetic field is utilised for detecting and measuring currents. It is often used in delicate experiments where very small currents are required to be measured.

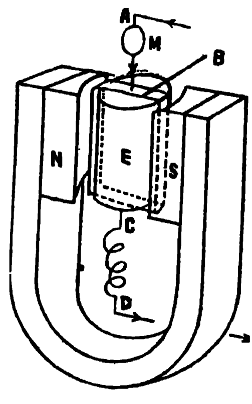


Fig. 35—Suspended Coil Galvanometer

Principal Parts.—In this galvanometer (Fig. 35), a small rectangular coil B of insulated fine copper wire of several turns is suspended between the concave pole-pieces of a large and powerful permanent magnet of the horse-shoe type by means of a fine strip A of some conducting material, usually *phosphor-bronze*—an alloy. Such a suspension fibre is not readily oxidised and can be easily twisted ; but it does not easily break. The strip A leads to one terminal of the instrument. The lower end of the coil is joined by a flexible spring CD of phosphor-bronze, which leads to the other terminal of the galvanometer. The spring serves the purpose of controlling the movement of the coil. A small *mirror* (M), preferably concave, is fixed to the suspension wire and it enables a beam of light reflected from it to indicate the deflection of the coil, which is read by lamp and scale arrangement. Because the pole-pieces N and S (Fig. 35) are shaped in concave form having a cylindrical air-gap in between the lines of force, which start everywhere normally from the N-pole towards the S-pole, all intersect at the axis of the cylindrical space, i. e. they are rendered parallel to the radii of the cylinder. Such a field is called a **radial field.** The advantage of such a field is that the plane of the coil, suspended vertically in it, is always parallel to the same field in all its positions of deflection (vide Fig. 36).

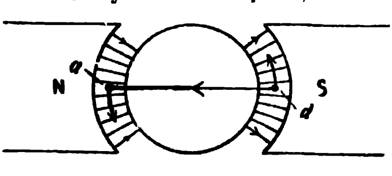


Fig. 36

A soft iron core E (Fig. 35) is fixed in the centre of the coil, without touching it, co-axial with the cylindrical air gap. This core concentrates the lines of force of the magnet within the coil (see Fig. 37).

24 (II)

Action.—It is first levelled so that the coil hangs freely without touching either the pole-pieces or the soft iron core. Before sending current through the coil, the torsion head supporting the suspension fibre, placed at the top of the galvanometer casing, is carefully turned in the proper direction until the spot of light reflected from the mirror M (Fig. 35) is received on the zero-mark of the scale.

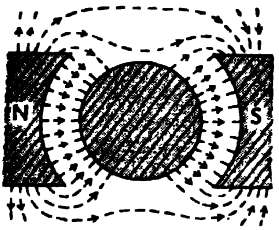


Fig. 37

Now when the current flows through the coil, the coil experiences a couple, which, in this case, is the **deflecting couple**, and it moves to set itself at right angles to the magnetic field and takes up an intermediate equilibrium position owing to the **controlling couple** set up by the torsion in the phosphor-bronze strip which opposes the first couple. The **controlling couple** is proportional to the angle of deflection of the coil, which is indicated by the movement of the spot of light received on a scale which is graduated in mm. When equilibrium is established, *the deflection is proportional to the current in the coil, i.e.* if C be the strength of the current and d the deflection on the scale, $C \propto d$.

Theory.—Let C = strength of the current, H = field strength in the space occupied by the coil, l = vertical length of the coil, b = breadth of the coil ($a\ d$ in Fig. 36). Then the force experienced by each vertical side of the coil at right angles to the plane of the coil $= nCHl$, where n = no. of turns in the coil.

$\therefore$ The total moment of the deflecting coil about the suspension fibre $= n.CHl \times b$. If τ = moment per unit twist of the suspension fibre, which is a constant for phosphor-bronze for small angles of twist, the controlling couple for a twist $\theta = \tau.\theta$. In the position of steady deflection θ, $\quad nCHl \times b = \tau.\theta$.

$$\text{That is, } C = \frac{\tau}{n\,H\,l \times b}\cdot\theta = \frac{\tau}{nHA}\cdot\theta, \quad \ldots \quad \ldots \quad (1)$$

where A = area of the coil ; or, $C \propto \theta$, since τ, n, H, A are all constants. But $\theta \propto d$ (vide Art. 25). $\therefore \quad C \propto \theta \propto d \quad \ldots \quad \ldots \quad (2)$].

This form of galvanometer was invented by **D'Arsonval**, and so a moving coil galvanometer is often referred to as a *D'Arsonval galvanometer*.

In the recent types of galvanometers the permanent magnet is made almost completely cylindrical, two poles of which are brought

very near each other, leaving only a very narrow cylindrical gap in which the coil rotates In this, no soft iron core is necessary to make the field radial. The coil is often put inside a thin silver tube, which, when rotating, produces induced currents which oppose the main current (see Art. 63) and thus the coil is brought to rest quickly.

D'Arsonval galvanometers are very sensitive, and the current measured by these galvanometers should be very small, being of the order of 10^{-6} amp. or less ; otherwise the galvanometers may be damaged.

Sensitiveness of D'Arsonval Galvanometers.—From equation (1) above, it is evident that θ will be large for a small current C, if τ/nHA is very small. Thus the condition for such a galvanometer to be sensitive is that τ/nHA must be as small as possible, i.e. τ small, while n, H and A large. A and n cannot be increased indefinitely without increasing the mass of the coil, the size of the galvanometer and its resistance. So in practice H is sought to be increased as much as possible. For phosphor-bronze, τ is small and again tensile strength is large ; that explains its use for suspension work almost universally.

27. Comparison of the Two Types of Galvanometers.—

Moving Magnet Type	Moving Coil Type
(1) Here the controlling field is the earth's field which is not very strong, so it is affected by the presence of external magnetic field.	(1) Due to the strong field of the permanent magnet the instrument is not affected by external magnetic fields (including the earth's field).
(2) Every time before using the instrument, the coil must be in the magnetic meridian.	(2) The galvanometer may face in any direction, as the controlling force is the torsion of the phosphor-bronze strip and not the earth's field,
(3) The suspended needle takes a long time to come to rest.	(3) There is some arrangement within the galvanometer for stopping the oscillations of the coil.
(4) A current smaller than 10^{-5} amp. cannot be measured with it.	(4) A current even as small as 10^{-9} amp. can be measured with it.
(5) The constant H/G of the galvanometer changes from place to place on the earth's surface.	(5) The constant of the galvanometer is independent of any change of place.

The only disadvantage of a moving coil galvanometer is that the current cannot be calculated from the dimensions of the instrument as in the case of the tangent galvanometer. But this disadvantage is only a minor consideration ; so most of the instruments which are used to measure currents are of the moving coil type. This type of galvanometer cannot be used to measure *strong currents* directly as (*i*) the instrument being sensitive will give a deflection too large to give accurate results, and (*ii*) it may be spoiled. In that case it is used along with a shunt (vide Art. 37).

Questions

Art. 20.

1. Describe the construction and action of a simple galvanometer. (C. U. 1929)

Art. 21.

2. What is meant by an astatic system of two needles ? What is its usefulness ? Explain the principle of the static galvanometer. (C. U. 1921 ; Bom. 1931 ; Pat. 1928 ; All. 1929)

Arts. 23 & 14.

3. Describe and explain the action of a simple form of tangent galvanometer. (C. U. 1913, '22, '24, '31, '31, '39 ; Dac. '32, '34 ; Pat. '18, '29, '46)

Why must the needle be very small ? (Pat. 1941 ; All. '29 ; C. U. '40)

(*See* also Art. 24.)

4. What is meant by 'Reduction Factor' of a tangent galvanometer ? (C. U. 1941 ; All. 1944)

Explain what is meant by (a) the galvanometer constant, (b) the reduction factor. (Pat. 1946)

5. Calculate the strength of the magnetic field at the centre of a coil of single turn and of 34 cms. radius, if this gives a deflection of 45° with a current of 8 amperes. (Pat. 1918)

[*Ans :* 0·15 C. G. S. unit nearly.]

6. The coil of a tangent galvanometer is 10 cms. in radius. How many turns of wire must be wound on it, if a current of 0·01 ampere is to produce a deflection of 45° ? $H = 0{\cdot}18$ C.G.S. unit.

[*Ans :* 287 approx.]

7. Define unit current and establish the working formula of a tangent galvanometer. What modifications would you require to be introduced in a tangent galvanometer in order to convert it into a sine galvanometer ? Which of the two will be more suitable for measuring (*a*) heavy currents, and (*b*) feeble currents ? (Pat. 1925, '30 ; Cf. C. U. '31, '44)

Compare the merits and demerits of these two types of galvanometers. (Pat. 1930)

8. What is a tangent galvanometer and why is it so called? Show how the apparatus is used to obtain the absolute value of an electric current.

(See also Art. 59) (Pat. 1932; Cf. All. '29).

9. A tangent galvanometer gives a deflection of 20° for a current of 0·01 amp. in England, but only a deflection of 12° for the same current in Ceylon. How do you account for it? (S. C.)

10. A tangent galvanometer is found to give a deflection of 15° when a current of 0·5 amp. is flowing through it. What is the current flowing when the galvanometer reading is 20°? (C. L.)

[*Ans*: 0·679 amp.]

11. Describe the tangent galvanometer. When a battery of 10 ohms resistance is connected in series with a galvanometer of 100 turns and of 40 ohms resistance, the deflection is 45°. What would be the deflection if only 50 turns of the galvanometer were connected in series with the battery.

[*Ans*: 39°48′] (C. U. 1939)

12. Describe a tangent galvanometer, and explain how you would use it to measure an electric current. Deduce the necessary formula. (All. 1946; Cf. Pat. '49)

How will the deflection be affected if (*a*) the pole strength of the magnet is increased, (*b*) the galvanometer is carried from Patna to London, the current passed through the galvanometer in each case being the same? (Pat. 1944)

[Read also Art. 24 (note), Part V.]

13. Describe a simple form of tangent galvanometer. Why must the coil be placed with its plane in the magnetic meridian?

A current is sent through two tangent galvanometers in series. The deflection is seen to be the same in both galvanometers. Compare the radii of the coils, if the number of turns is 110 in the first coil and 25 in the second.

[*Ans*: 22 : 5] (C. U. 1947)

Arts. 26 & 27.

14. Describe a suspended-coil type of galvanometer and explain its action. (C. U. 1932; All. '12, '22; Dac. 1931)

What are the special merits of this type? (All. 1932)

CHAPTER IV

Ohm's Law : Resistance

28. The E. M. F. of a cell is the potential difference between its terminals when on open circuit. It is the maximum potential difference developed between the terminals of the cell.

The E. M. F. of a cell depends on the nature of the plates and the liquid used in the cell, and not on the sizes of the plates or their distance or capacity of the cell.

Units for Electrical Quantities

Quantity of Charge.—The C.G.S. electro-magnetic unit (*E. M. U.*) quantity of electricity is that which is conveyed by unit current in unit time.

The practical unit of quantity is the **Coulomb**, which is the quantity conveyed by 1 ampere in 1 second.

[Ampere is the practical unit of current. 1 ampere $= \frac{1}{10}$ C. G. S. electro-magnetic unit (see Art. 14)].

Or **Quantity** (in coulombs) = **Current** (in amperes) × **time** (in seconds).

1 coulomb $= \frac{1}{10}$ C. G. S. electro-magnetic unit of quantity.

Quantity in the Two Systems of Units :—

1 electro-magnetic unit $= 3 \times 10^{10}$ electrostatic units (E.S.U.)

1 coulomb $= \frac{1}{10}$ E.M.U. Hence 1 coulomb $= 3 \times 10^{9}$ (E.S.U.)

Illustration :—

The electron $= 4{\cdot}77 \times 10^{-10}$ *E. S. U.*

$= (4{\cdot}77 \times 10^{-10}) \div (3 \times 10^{10}) = 1{\cdot}59 \times 10^{-20}$ *E. M. U.*

$= (1{\cdot}59 \times 10^{-20}) \div \frac{1}{10} = 1{\cdot}59 \times 10^{-19}$ coulomb.

[The unit of quantity is called the Coulomb after Charles Augustin de Coulomb, a French scientist (1738—1806)].

Current Strength.—*It is the quantity of electricity passing any section of the circuit in one second, i.e. it is the rate of flow of electricity in the circuit.* If C denotes the current strength and Q the quantity passing in t seconds, $C = Q/t$, and $Q = Ct$. In an electric circuit,—*viz.*, the entire path through which a current flows, the strength of the current is everywhere the same, but there is a fall of potential in the direction in which the current is flowing. There can not be any accumulation of electricity in any part of the circuit.

(**Note.**—Considering an electric current as the passage of electrons along a conductor, the strength of a current is measured by the number of electrons moving past a given point per second).

Current in the Two Systems :—

1 electro-magnetic unit $= 3 \times 10^{10}$ electro-static units.

1 ampere $= \frac{1}{10}$ E.M.U. Hence 1 ampere $= 3 \times 10^{9}$ electro-static units.

Potential Difference.—*The Potential Difference (P. D.) between any two points in an electric circuit is one C. G. S. electro-magnetic unit, if unit work (one erg) is done in conveying an electro-magnetic unit quantity of charge from one point to the other.*

The practical unit is one Volt. 1 volt $= 10^8$ E. M. U. of P. D.

This means that when one E. M. U. quantity passes through a P. D. of one volt, 10^8 ergs of work are done. But a coulomb being 1/10th of an E. M. U., the work done will be 10^7 ergs (or 1 Joule) when one coulomb of electricity passes through one volt.

P. D. in the Two Systems :—

$$\left.\begin{array}{l}3\times10^{10} \text{ electro-magnetic units} \\ \text{of potential difference (P. D)}\end{array}\right\} = \left\{\begin{array}{l}\text{1 electro-static} \\ \text{unit of P. D.}\end{array}\right.$$

1 volt $= 10^8$ E. M. U. Hence 1 volt $= \frac{1}{300}$ electro-static units.

[The unit of P. D. is called the **Volt** after the name of Allesandro Volta (1745-1827), the discoverer of voltaic cells.]

Note.—The potential difference in *electro-statics* is the same as the electromotive force in *current electricity* with the only difference that in one case the charge is carried on a small body from one point to the other, and in the other case, the charge is moved along a wire.

One Electro-static Unit of Potential difference = 300 volts.

Resistance.—*Resistance is the name given to the property of a body opposing the flow of electricity through it.* **A conductor has a resistance of one C. G. S. electro-magnetic unit** when *a P. D. of one electro-magnetic unit between its ends sends a current of unit strength through it.*

The practical unit of resistance is the Ohm, *which is the resistance of a column of mercury 106·3 cms. long, 1 sq. mm. in cross-section at a temperature of 0°C., the mass of mercury being equal to 14·4521 gms.*

A conductor has a resistance of one ohm *if a current of one ampere passes through it, when a P. D. of one volt is applied between its ends.*

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} = \frac{10^8 \text{ E. M. U.}}{1/10 \text{ E. M. U.}} = 10^9 \text{ E. M. units.}$$

Remember the Practical units :—

The unit of Current Strength is the **Ampere.**
The unit of P. D. is the **Volt.**
The unit of Resistance is the **Ohm.**

Remember also.—(*a*) **Current** is the *rate of flow* of electricity.

(*b*) **Resistance** opposes the current and regulates its flow.

(*c*) **Voltage** is the moving force which causes the flow.

29. Connection between Static and Current Electricity.—

When the two coatings of a Leyden jar are connected by a discharging pair of tongs, electricity flows from one coating to the other through the tongs. Similarly, if at the time of working a Wimshurst machine, the two knobs of the spark gap are joined by a wire, electricity, instead of passing in sparks, passes from one knob to the other along the wire.

Exactly the same thing happens when two terminals of a battery are joined by a wire. The *difference* lies only in the *numerical values* of the quantities involved, the *character* being the *same*. The other small difference is that the current produced by discharging the Leyden jar, or the two knobs of the Wimshurst machine, lasts only for a very short time. Representing difference of potential by difference of level, the current produced by the electric machine can be compared to a mountain torrent falling from a great height and making a good deal of noise and disturbance with but little water, while electric current produced by the battery is like a broad river flowing slowly and steadily with a difference of level of only a few feet per mile.

Thus, in the case of the Leyden jar, or any such electric discharge, the current is quite **momentary** and the quantity of electricity **small,** but the potential difference (P. D.) is **high** ; while, in the case of the voltaic battery, the current is **continuous** and the quantity of electricity **large**, but the potential difference is **small.**

This is the reason why the poles of a voltaic cell, or even a battery of a few accumulators, may be touched by the two hands without taking any precautions against having shocks, or the connecting wires may be touched without affecting the current passing in the circuit through the body as the leakage of current in this case is very slow, because the P. D. is so small that the loss of charge is immediately made up by generating it by the chemical action of the cell, while, in the case of charged electrophorus or other statically charged bodies, enough precautions have to be taken to avoid touching them by which there will be immediate leakage of charge, as in that case the P. D. is so great that the small quantity of electricity involved leaks away quickly through a high resistance like the human body. There will be danger in these cases when both the P. D. and the quantity of electricity are great, so it will not at all be safe to have the discharge from a Leyden jar, and far more dangerous it is to have it from a Wimshurst machine.

The idea may be clear by taking some actual examples. The maximum potential difference of the terminals of a Leclanche′ cell is about 1·5 volts ; while a glass rod rubbed with silk may be raised to some thousands of volts. The potential difference of nearly 27,000 volts is necessary to make a spark pass in air between two metal balls 1 cm. in diameter, the surfaces of the balls being 1 cm. apart. The smallest potential difference necessary to affect a gold-leaf electroscope is about 100 volts.

It is also seen that there is no real difference between the passage of electricity in the two cases (except the numerical values of the quantities involved) when we consider that, according to the modern theory, **electric current** consists in the movement of a stream of very small negatively charged particles, called electrons, along the wire (from molecule to molecule) from *lower* to *higher* potential.

A **few experiments** will show that statical electricity and current electricity are essentially the same.

Magnetic effect.—We know that a steel knitting needle placed inside a coil of insulated copper wire is magnetised when a current is passed through the coil.

The needle is also found to be magnetised if, instead of passing a current from a battery, a discharge from a Leyden jar is passed through the coil.

Heating effect.—By connecting two insulated metal spheres by a fine copper wire, the wire may be turned red-hot when a discharge from a Leyden jar is passed through the wire, and by passing a heavy discharge the wire may be volatilised.

The same effect is observed when a current is passed from a battery.

Lighting Effect.—Everyone is familar with the lighting effect of current electricity in glow lamps.

In statical electricity the same effect is apparent everytime when a spark passes during the discharge of a Leyden jar or a Wimshurst machine.

Chemical Effect.—If a piece of paper soaked in a solution of potassiam iodide and starch is placed in contact with two poles of a battery, iodine is liberated at the positive terminal due to which the spot round the positive terminal is turned *blue.*

Similar effect can be obtained by placing a piece of thin paper in the path of the discharge from a Leyden jar.

Pole-finding Paper.—A piece of white blotting paper is soaked in iodide-starch solution and allowed to dry. This is a pole-finding

paper and is used (after wetting it) to find the pole of a battery, as explained above (vide Chemical effect, Art. 29).

Take two lengths of insulated wire and connect one to each terminal. Now hold the free end of these wires about half an inch apart and in contact with a strip of red litmus paper, which has been wetted and laid on a sheet of glass. The paper in contact with the negative end is turned *blue*.

30. Resistance.—From the above it is clear that the potential differences obtained by voltaic batteries are much smaller than those obtained by electro-statical methods, the latter being generally very high. The rate of flow of electricity through any substance depends directly on the difference of potential or *electric pressure*, as it may be called. For this reason, a good conductor in electro-statical experiments may not appear to be so in current electricity. In electro-statics, it has been found that metals, wood, human body, all appear to be equally good conductors; there an iron and a copper ball of the same size and similarly charged will behave exactly in the same way; but in current electricity, an iron wire and a copper wire of equal length and cross-section will not conduct electric current equally through them, the resistance offered by each of them being different. This shows that *the resistance of a wire depends upon the nature of the material.* The resistances of two wires of the same metal and the same cross-section, but of different lengths, will be different, the longer one having a greater resistance. Again, two wires of the same metal, having the same length, but of different cross-sections, will have different resistances, the resistance of the finer one being greater.

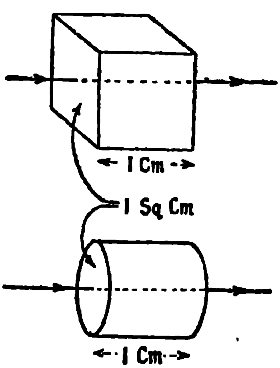


Fig. 38.

Specific Resistance.—If R be the resistance of any wire of length l and cross-section S, then, $R \propto l$; again,

$$R \propto \frac{1}{S}; \quad \therefore R \propto \frac{l}{S}; \qquad or \qquad \mathbf{R} = \rho \frac{l}{S};$$

where ρ is a constant depending on the nature of the material of the wire and its temperature. This ρ is called the **Specific Resistance** (or **Resistivity**) of the substance. When $l = 1$ and $S = 1$, $R = \rho$.

So, **the specific resistance of a substance** *is defined as the electrical resistance between the opposite faces of a one-centimetre cube of the substance* (see Fig. 38).

Laws of Resistance.—

Thus in the case of a conductor the **resistance** depends on (*a*) *the material* ; a copper wire has less resistance than an iron wire ; (*b*) *the length of the wire* ; if the length is doubled, the resistance is also doubled, *i. e.* the resistance of a conductor is directly proportional to its length ; (*c*) *the cross-section* ; if the area of cross-section be twice, the resistance becomes one-half, *i. e.* the resistance varies *inversely* as the cross-section ; (*d*) *the temperature* ; the hotter a metal wire the greater will be its resistance.

Note.—Here we find that the resistance of a wire of given length depends upon its cross-section and not upon its circumference, so a hollow wire will not conduct electricity so well as a solid wire of the same external diameter. Thus *the rule*, observed in statical electricity, *that electric charge resides on the external surface of a conductor does not hold good for current electricity.*

31. Ohm's Law.—In 1826 Dr. G. S. Ohm, a German Professor of Physics at Munich, established a relation between current strength, potential difference, and resistance. The law discovered by him, known as *Ohm's Law*, may be stated as follows :—

"In any wire at uniform temperature the current is directly proportional to the P. D. between its ends.

Thus, if E be the *potential difference*, and C the *current*, we have,

$$E/C = \text{a constant, } R.$$

This constant for any conductor is called its **Resistance**. The quotient, obtained by dividing the net E. M. F. acting in the conductor by the current flowing through the conductor, called the resistance, depends not only on the material forming the conductor but also upon its length, cross-section, and temperature.

In practice, Ohm's Law is usually expressed as,

$$C \text{ (in amperes)} = \frac{E \text{ (in volts)}}{R \text{ (in ohms)}} ;$$

where R is the *resistance* of the wire. If $E=1$, and $C=1$, then $R=1$; thus *a conductor is said to have unit resistance, if unit potential difference between its ends produces unit current in it.*

The reciprocal of resistance, *i.e.* $1/R$, is called **conductance**.

1. Applications of the Ohm's Law.—

Let a circuit contain a source of E.M.F. E, external resistance R and internal resistance r. Then applying the law to the whole circuit,

$$C = \frac{E}{R+r} = \frac{\text{E.M.F.}}{\text{Total resistance}}.$$

Therefore, CR + Cr = E ...(1)
(P. D. used in *external circuit*) (P. D. used in *internal circuit*) (Total *E. M. F.*)

(a) **E M. F. and P. D.**—The above relation means that of the total E. M. F. (E), a portion (Cr) is used in driving the current through the internal resistance of the cell, and the remainder (CR) through the external resistance R. The portion (CR), available for the action of the cell, is known as the **terminal P.D.** or **available volts** of the cell.

$$\text{Or,} \qquad C = \frac{\text{Terminal P. D.}}{\text{External resistance } R}.$$

So, if E_1 volts be the value of the E. M. F. of a cell when no current is taken from it, and E_2 volts be the value when the cell is in a closed circuit and a current is allowed to flow, it will be found in all cases that E_1 *is greater than* E_2. So **the E. M. F. of the Cell = the P. D. between the terminals on open circuit**.

(b) **Lost Volts**—The difference ($E_1 - E_2$), which is the part of the E.M.F. of the cell used up in driving current through itself, is sometimes called the "*lost volts*" in the cell. Thus

$$\textit{Internal resistance} = \frac{\textit{Lost volts}}{\textit{Current}};\ \textit{or}\ \textit{Current},\ C = \frac{\textit{Lost volts}}{\textit{Internal resistance}(r)}.$$

The greater the current, the greater will be the lost volts. Therefore in a cell which is generating current,

$$\text{Total E. M. F.} = \text{available volts} + \text{lost volts}$$
$$E = CR + Cr$$

2. Comparison of E. M. F. and Potential Difference.—

E. M. F. is a motive force (comparable to a mechanical force) developed within a cell due to chemical action. The direction of this force for a cell is fixed. Due to this, a current tends to flow from one pole to another when the poles are externally connected. By Ohm's law,

$C = \dfrac{E}{R+r}$; or, $E = CR + Cr$, where R and r are the external and internal resistances respectively, E = E. M. F. of the cell, and C is the current in the circuit. In the closed circuit, therefore, the E. M. F. is used up partly in sending up the current through the external resistance R and partly through the internal resistance r. This division of the E. M. F. evidently takes place according to the resistances of the different parts of the circuit, and each division of E M.F. is known as

the potential difference ($P. D.$) between the two points considered in the circuit. Since potential falls in the direction of the current, the P.D. depends on the direction of the current. Since $CR = E - Cr$, it is clear that P.D. between the terminals of a cell on closed circuit is less than the E. M. F. of the cell by the amount which falls through the internal resistance of the cell. That is to say, the E. M. F. of a cell is equal to the potential difference between the poles on open circuit.

In a battery of cells, the E.M.F. of each particular cell always acts in its own way whatever might be the direction of the current in the circuit. Thus, in Fig. 38(a), where a circuit has been shown in which two cells act in one direction in series and a third cell in opposition, the potential falls in the clockwise direction depending on the direction of the current, but the E. M. F.'s are acting in their own ways as shown by the arrows in the cells. Thus E. M. F. is like a *cause* and $P. D.$ like an *effect*.

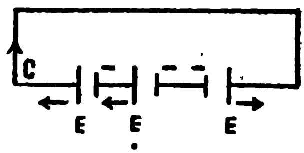


Fig. 38(a)

To apply Ohm's Law in Fig. 38 (a), the *net* E. M. F. in the circuit is to be considered.

That is, $C = \dfrac{2E - E}{R + 3r} = \dfrac{E}{R + 3r}$, where E = E. M. F. of each cell, r = internal resistance of each cell, and C = the current.

Because two cells act in the same direction while the third one is in opposition, the net E. M. F. is $2E - E$.

Example.—*The difference of potential between the terminals of a cell on 'open' circuit is 2·2 volts. When the terminals are connected by a wire of resistance 4 ohms, this difference of potential is reduced to 2 volts. What is the internal resistance of this cell?* (*Pat. 1943*)

We have, (from Art. 31) $E = CR + Cr$; or $2{\cdot}2 = 2 + Cr$; or $C = 0{\cdot}2/r$ ... (1)

But $CR = C \times 4 = 2$; $\therefore$ $C = 2/4 = 0{\cdot}5$

$\therefore$ From (1), the internal resistance $r = 0{\cdot}2/0{\cdot}5 = 0{\cdot}4$ ohm.

3. Voltage drop in a Line.—We have seen that the available voltage (also called terminal P. D.) of a cell is less than the E. M. F. of the cell because some voltage is required to send the current through the internal resistance of the cell. So, when in a *street supply* the electric current is used at a considerable distance from the power house where the current is generated, the voltage at the receiving end is bound to be always less than the voltage of the generator; this drop of voltage in the connecting wire, called *the line*, is equal to the product

of the current and the resistance of the line (CR). Ordinarily the voltage drop for house wiring should not exceed 2 per cent. In every dynamo or motor the current flowing through the resistance of the armature causes a potential drop which does no useful work and which is subtracted from the original E. M. F. in a dynamo, or from the driving P. D. in a motor. It is a dead loss and known as the **"lost volts"**.

Example.—*In an electrification scheme the dynamo is of negligible resistance, but the resistance of the leading wires is 1 ohm per mile. If the voltage of the power house is 220 volts (D. C.), what voltage will be available at a station 20 miles off, when a current of 2 amperes is drawn from the leads? What can be the maximum strength of current available there?* (*Pat. 1939.*)

Total resistance of the two leading wires $= (20 + 20) \times 1 = 40$ ohms.

$\therefore$ Potential drop or lost P. D. = current × resistance $= 2 \times 40 = 80$ volts.

Hence available voltage $= 220 - 80 = 140$ volts

Maximum current available there $= \frac{220}{40} = 5{\cdot}5$ amp.

32. (a) Verification of Ohm's Law.—

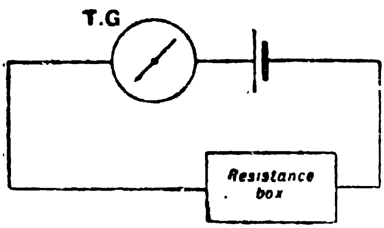


Fig. 38(b)

(1) By Tangent Galvanometer.—A circuit is completed, through a key (not shown in the figure), by a source of constant E. M. F., say, an accumulator, a tangent galvanometer (T. G.), and a resistance box [Fig. 38 (b)]. Let θ_1 be the deflection of the galvanometer needle for the resistance R_1 in the box, when a current is passed through the circuit. Then, neglecting the resistance of the connecting wires,

$$\text{current } C_1 = \frac{E}{R_1 + G + B},$$

where G is the resistance of the galvanometer and B the internal resistance of the cell. Again, we know from the principle of the tangent galvanometer that current $C = 10K \tan \theta_1$ (amp.), Art. 23.,

$$\therefore \quad 10\mathrm{K} \tan \theta_1 = \frac{\mathrm{E}}{(R_1 + G + B)} \quad \ldots\ldots(1)$$

Again, on changing the resistance R_1 to R_2, the value of the current will be changed, and the deflection will be changed to θ_2. So, in this case,

$$\text{current } C_2 = 10K \tan \theta_2 \text{ (amp.)}.$$

Hence, $$10K \tan \theta_2 = \frac{E}{(R_2 + G + B)} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

From (1) we have, $$10K \tan \theta_1 \times (R_1 + G + B) = E. \quad \ldots\ldots\ldots (3)$$

(2) $$10K \tan \theta_2 \times (R_2 + G + B) = E \ldots\ldots\ldots\ldots\ldots (4)$$

It will be seen that the product of the left-hand side of each of the equations (3) and (4) is equal to E, the E.M.F. of the cell. Thus Ohm's Law is verified.

$$\therefore \quad (R_1 + G + B) \times \tan \theta_1 = (R_2 + G + B) \times \tan \theta_2 = \text{const.}$$
$$= R \times \tan \theta_1, \text{ where } R \text{ is the total resistance.}$$

(2) **By Graphical Method.**—This method may be used for the determination of internal resistance of cells or the resistance of a tangent galvanometer.

For a circuit similar to Fig. 38(b), $(R + G + B) \times 10K \tan \theta = E$, [vide equation 3, Art. 32 (a)].

That is, $$\frac{R + G + B}{\cot \theta} = \frac{E}{10K} = \text{a constant} = m \text{ (say).}$$

$\therefore \quad R = m \cot \theta + h$, where $h = -(G + B)$, which is of the type $y = mx + h$, the equation for a straight line.

So, if a graph AB is drawn with R and $\cot \theta$, the graph will be a straight line (Fig. 39). The above equation has been deduced on the assumption that Ohm's Law is true. Hence, *if the graph is a straight line, then the truth of Ohm's Law is verified.*

It is to be noted that the graph does not pass through the origin, but cuts the axis of resistance at some point C on the left of the origin (where $R = O$), the intercept (CO) being the value of the **internal resistance** of the cell plus the resistance of the galvanometer (Fig. 39).

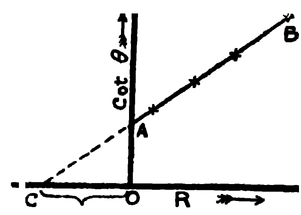


Fig. 39

For, when $\cot \theta = O$,

$$R = h = -(G + B).$$

So if from the graph, the intercept CO $(= G + B)$ be measured off, B can be found when G is known or vice-versa.

(3) **By Potentiometer Method.**—A potentiometer consists of a

wire PQ of uniform cross section of a material of fairly high resistance, say Eureka or Manganin. Instead of a single wire the potentiometer sometimes consists of five or ten or even more wires, each one metre in length stretched parallel to each other and joined in series by thick copper strips, as shown in Fig. 40. A graduated scale is placed alongside the wires, and the instrument is provided with a tapping contact, called the *Jockey*, which slides over the wires from point to point and can make contact with any desired point D. The two ends of the wire are connected to the poles of a cell B through an ammeter A and an adjustable resistance R so that a steady current C flows through the potentiometer wire. In this case, the rate of fall of potential along the wire will be uniform, and the fall of potential $(V_P - V_D)$, measured by a suitable voltmeter V along any length PD (say, l) of the wire, will be proportional to the length l, because the current C is constant and the wire is of uniform cross-section, *i.e.* $(V_P - V_D) = CR_1$; or $(V_P - V_D) \propto R_1 \propto l$, where R_1 is the resistance of l ; or $V_P - V_D / l =$ a constant. This is verified by experiment by sliding the point of contact D.

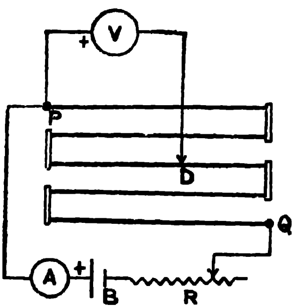


Fig. 40.—Potentiometer

Expts.—Take one accumulator B and connect the positive pole at P and the negative at Q, including one ammeter A and a rheostat R joined in series in the circuit (Fig. 40). Care should be taken to see that the terminal of the ammeter marked (+) is joined to the +ve pole of the battery. Take one voltmeter and connect the positive terminal of it at P and the negative at the tapping contact D.

In order to keep the current constant the resistance of the rheostat R is kept constant. Now move and touch the tapping contact at different points on the wire and note the corresponding readings of the voltmeter and the length of the wire in each case. Tabulate your results and show that $V_P - V_D / l =$ constant. Draw a graph with C and l which will be a straight line.

Comparison of E. M. F's by Potentiometer.—A potentiometer may also be used for comparison of E. M. F.'s. Suppose two cells are given for comparison of their E. M. F.'s, which are, say, E_1 and E_2. Using each cell in the position of the voltmeter V as in Fig. 40, a balance point D is obtained in the potentiometer wire by shifting the Jockey (the positive of the cell being connected to the point P in each case). Let l_1 and l_2 be the balancing lengths. Then $E_1 \propto l_1$, and $E_2 \propto l_2$. $\therefore$ $E_1/E_2 = l_1/l_2$.

(4) **By Voltmeter and Ammeter Method.**—A series circuit comprising the battery B, the key K, the ammeter A, the fixed resistance r of known value, and the variable resistance R (Rheostat) is formed (Fig. 41). The voltmeter V, connected across C and D, gives the P.D. between these points. For a certain value of R, the P.D. between C and D, $(V_C - V_D)$, and the current (C) through the fixed resistance r are recorded from the indications of the voltmeter V and the ammeter A respectively. The ratio $\frac{V_C - V_D}{C}$

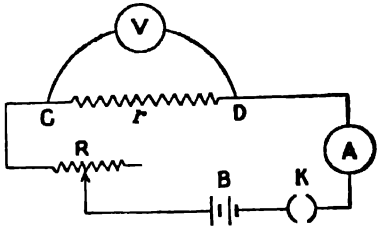


Fig. 41

is calculated. The ratio between these quantities is determined a number of times on changing the value of the current through r by altering the value of R. It will be found that this ratio remains constant for the various values of the current in the circuit, as long as the temperature of the circuit does not change, and the value of this ratio is found to be equal to that of r.

So, $\frac{V_C - V_D}{C} = r$; or $V_C - V_D = rC$. This verifies Ohm's law.

32 (a). **Further Consideration of Ohm's Law.**—It should be remembered that Ohm's Law is true for the whole of an electric circuit as well as for any part of it, provided that the current flowing in it is steady and the temperature of different parts of the circuit does not change appreciably.

(1) *The Law applied to a Part of the Circuit.*—In Fig. 41, the resistance r is only a part of the series circuit consisting of the battery B, the ammeter A, the fixed resistance r, and the variable resistance R.

Applying the law to the part r, $C = \frac{V_C - V_D}{r}$, where C is the current through r, $(V_C - V_D)$ being the potential difference between C and D.

(2) *The Law applied to the Whole of a Circuit.*—If the law is applied to the whole of the series circuit shown in Fig. 41, the current C will be given by,

$C = \frac{E}{R+r}$, where E is the total E.M.F. of the battery B, R being the value of the variable resistance R ; while r is a fixed resistance, the resistance of the ammeter and the connecting wires being neglected.

33. Factors affecting Resistance.—

(1) Variation of Resistance with Temperature.—It should be noted that in dealing with Ohm's Law stress has been laid on keeping the temperature constant. The reason of this is that **the resistance of all metals increases, whilst that of non-metals generally decreases, as the temperature rises.** The resistances of most of the electrolytes and of carbon, decrease with the rise of temperature. For example, the resistance of a metal filament lamp is many times greater when the filament is hot than when cold ; whereas the resistance of a carbon filament lamp is much less when hot than when cold. Besides carbon, the resistances of insulators also decrease when heated.

There are certain metallic alloys which have got high specific resistance but very low temperature co-efficient of resistance, that is, they are almost unchanged in resistance by a change in temperature. These are copper-manganese alloy, called *manganin*, and the copper-nickel alloy, called *Eureka* (or Constantan), Nichrome, German silver, etc. These alloys are, therefore, specially suitable for *the construction of resistance coils.*

(*i*) **Temperature Co-efficient.**—The amount by which a unit resistance increases when heated through 1° is called its *temperature co-efficient.* It varies from substance to substance.

If R_0 ohms be the resistance at 0°*C.*, the resistance at t°*C.* is given by,

$R_t = R_0\,(1 + \alpha t)$, where α is the temperature co-efficient for a moderate range of temperatures.

When the change in temperature is large, the variation of resistance takes place according to the following approximate formula.—

$R_t = R_0\,(1 + \alpha t + \beta t^2)$, where α and β are constants for the material.

(*ii*) The effect of temperature on the resistance of a platinum wire has been made use of in an electrical thermometer, known as the **Platinum-Resistance Thermometer**. A fine platinum wire wound on a mica frame enclosed in a porcelain tube makes such a thermometer. To use it for determination of temperature of a bath, the resistances of the wire at 0°*C.* and at the unknown temperature are determined by means of a sensitive form of Wheat-stone bridge. From the knowledge of the temperature co-efficient, the unknown temperature t is computed.

(2) Effect of Light on Resistance—It has been found that the resistance of the element *selenium* which closely resembles sulphur, decreases when exposed to light. So light falling on selenium included in a circuit can be used to vary the current of the circuit. This remarkable property of selenium has been utilised in the selenium cell, which

is used in several automatic instruments, viz. certain types of burglar alarm etc.

The action of **the photo-electric cell**, without which a Talkie Machine can not work, also depends on the effect of light on metals like sodium, potassium, caesium, etc. The cell is an evacuated bulb fitted with two electrodes, the positive being a metallic ring while the negative is a metallic plate on which there is a thin layer of a salt of any of the metals referred to above. When a beam of light is directed against this plate, electrons are profusely ejected from the plate and fill the evacuated bulb, and as a result the resistance between the electrodes falls. The cell is a part of an electric circuit including a battery and a loud speaker. If the intensity of the beam is modulated, the resistance between the electrodes changes, modulating the current through the loud-speaker. So the loud-speaker speaks.

(3). **Effect of Magnetic Field on Resistance.**—It is found that resistances of some substances, especially bismuth, increase when placed in a magnetic field, and the magnitude of the effect that is produced on bismuth has been utilised in measuring the strength of a magnetic field.

33(a). Resistance of the Human Body.—The resistance of the human body is about 50,000 ohms when the skin is dry. The resistance of the body is mainly due to that of the skin when the current enters and leaves it, and the resistance of the body is much lower—say about 10,000 ohms—if the skin is wet. So it is dangerous to handle any electrical apparatus when the skin is wet, and for the same reason, electric light switch should not be manipulated while standing in a bath, or barefoot on a wet floor. To produce an *electric shock* in the human body a current of about one-thousandth of an ampere must pass through the body ; so a P. D. of less than 50 volts would not produce any shock in the body when touched with dry hands. It is wise to avoid "live" wires if the voltage is over fifty.

A bird can sit on high voltage cable on a street, or a man can hang on it without experiencing any shock, as there is no complete circuit, but if the man touches the other cable or any conductor connected to the earth, the circuit will be completed and a current will flow through him and probably kill him.

Examples.—*(1) A Daniell cell is connected up in series with a tangent galvanometer of 1 ohm resistance and a box of resistance coils. When a resistance of 2 ohms is taken out of the box, the deflection of the galvanometer is 60° ; and when the resistance in the box is increased to 20 ohms, the deflection falls to 30°. Find the resistance of the cell.* (*C. U. 1932*)

Since all the resistances are put in series, the total resistance of the circuit in the first case $=(2+1+r)$ ohms, where r is the internal resistance of the cell.

Therefore, the current $C_1 = \dfrac{E}{3+r}$, (E = E. M. F. of the cell).

In the second case, the total resistance $= (20+1+r)$ ohms. Hence the current $C_2 = \dfrac{E}{21+r}$. $\therefore \dfrac{C_1}{C_2} = \dfrac{21+r}{3+r}$ ....(1)

But $C_1 = K \tan 60^\circ$, and $C_2 = K \tan 30^\circ$; (where K = reduction factor of the galvanometer,

$\therefore \dfrac{C_1}{C_2} = \dfrac{K \tan 60^\circ}{K \tan 30^\circ} = \dfrac{\sqrt{3}}{1/\sqrt{3}} = 3.$ $\therefore$ From (1), $\dfrac{21+r}{3+r} = 3$;

or $21+r = 9+3r$; *or* $2r = 12$; *or* $r = 6$ ohms.

2. The same current passes through a metre of copper wire 1 mm. diameter and two metres of a thinner copper wire. The difference of potential between the ends of the first wire is 1 volt and that between the ends of the second wire 20 volts. Find the diameter of the thinner wire. (*All. 1929*)

Let R_1 be the resistance of the first wire, R_2 the resistance of the second wire, and C the current flowing in each wire, then by Ohm's Law,

CR_1 = P. D. between the ends of the first wire = 1 volt.
and, CR_2 = P. D. between the ends of the second wire = 20 volts.

$\therefore \dfrac{R_1}{R_2} = \dfrac{1}{20}$ ....(1)

Again, if ρ be the specific resistance of copper, $R_1 = \rho \dfrac{100}{\pi \times (\cdot 05)^2}$; and

$R_2 = \rho \dfrac{200}{\pi r^2}$; where 0·05 cm. is the radius and 100 cms. the length of the first wire ; and r the radius, and 200 cms. the length of the second wire.

$\therefore \dfrac{R_1}{R_2} = \dfrac{\rho \times 100 \times \pi r^2}{\rho \times 200 \times \pi (\cdot 05)^2} = \dfrac{r^2}{0\cdot 005}$ ....(2)

Hence, from (1) and (2) we get, $\dfrac{r^2}{0\cdot 005} = \dfrac{1}{20}$ or $r^2 = 0\cdot 00025$;

or $r = 0\cdot 0158$ cm.

$\therefore$ The diameter of the wire $= 2 \times 0\cdot 0158 = 0\cdot 0316$ cm.

3. A battery of 12 equal cells in series, screwed up in a box, being suspected of having some of the cells wrongly connected, is put into circuit with a galvanometer and two cells similar to the others. Current in the ratio of 3 to 2 are obtained according as the introduced cells are arranged so as to work with or against the battery. What is the state of the battery? (*Pat. 1931*)

The resistance of the circuit is not affected even if some of the cells are wrongly connected. So the currents will be proportional to the electro-motive forces. If E be the E. M. F. of the battery, e that of a single cell, C_1 the current when the two cells are arranged to work with, and C_2 when arranged to work against, the battery,

$\frac{C_1}{C_2}=\frac{E+2e}{E-2e}$. Again, $\frac{C_1}{C_2}=\frac{3}{2}$; $\therefore$ $\frac{E+2e}{E-2e}=\frac{3}{2}$; *or* $E=10e$.

Hence, out of the 12 cells of the battery, one cell is wrongly connected and acting in opposition to the others, and thus neutralising the effect of another, and leaving only 10 cells effective.

4. *Two cells, a resistance of 2·5 ohms and an ammeter of negligible resistance are all connected in series, and the ammeter reads 0·8 amp. If the stronger cell has an E. M. F. of 1·8 volts and an internal resistance of 0·5 ohm, find the E. M. F. and the internal resistance of the other cell.* **(Pat. 1942)**

Let e be the E. M. F. and r the internal resistance of the weaker cell.

Case I. Effective E. M. F. $=1·8+e$, and total resistance in the circuit $=2·5+0·5+r=3+r$.

$\therefore$ By Ohm's Law, $0·8=\frac{1·8+e}{3+r}$; *or* $e-0·8r=0·6$..............(1)

Case II. Effective E. M. F. $=1·8-e$, and total resistance $=3+r$.

$\therefore$ $0·1=\frac{1·8-e}{3+r}$; *or* $e+0·1r=1·5$...........(2)

From (1) and (2), we get $r=1$ ohm ;

$\therefore$ $e+0·1\times 1=1·5$; *or*, $e=1·5-0·1=1·4$ volts.

5. *The E. M. F. of a cell is 2 volts, and the P. D. between its plates becomes 1·6 volts when it is connected in series with a resistance of 10 ohms. Find the internal resistance of the cell.*

Method 1.—If r be the internal resistance of the cell,

$$C=\frac{\text{total E. M. F.}}{\text{total resistance}}=\frac{2}{10+r} \quad \text{(Art. 3)}$$

$$\text{Again } C=\frac{\text{terminal P. D.}}{\text{external resistace}}=\frac{1·6}{10}; \quad \therefore \frac{2}{10+r}=\frac{1·6}{10}$$

or $1·6r=10(2-1·6)$, whence $r=2·5$ ohms.

Method 2.—The fall of potential inside the cell $=2-1·6=0·4$ volt.

This is equal to (current $\times$ internal resistance of the cell),

or $0·4=\frac{1·6}{10}\times r$, whence $r=2·5$ ohms.

34. E. M. F. and internal Resistance of Cells.—It should be noted that the E.M.F. of the cell does not at all depend on the size of the cell, *i.e.* on the area of the plates and the distance between them. It depends on (*i*) *the metals and liquids used in the cell*, and (*ii*) *the temperature of the liquid.*

Thus the E. M. F. of a large cell will be exactly equal to that of a cell not larger than a test tube, but constructed of the same metals and liquids. But the advantage of a large-sized cell is that it offers much less resistance to the passage of the current through it. This

resistance, which is known as the *internal resistance* of the cell, depends on (*a*) *the size* or *the area of the plates immersed in the liquid*, (*b*) *the distance between them*, and (*c*) *strength of the electrolyte* (or *electrolytes*) *used in the cell.* Thus in a cell, the E. M. F. is the same when the plates are separated further apart, or when the plates are much nearer, but the *current* in the first case will be *much less* as the internal resistance is much greater. The **great advantage of the accumulator** is its very low internal resistance, and so it can give a steady E. M. F with varying currents. The internal resistance of an accumulator is of the order of 0·01 ohm, whereas that for an ordinary Daniell cell is of the order of 1 ohm. Thus for an accumulator (internal resistance 0·01 ohm) giving a current of 10 amperes fall in voltage through its liquid is only ($10 \times 0{\cdot}01$), *i.e.* 0·1, whereas that for a Daniell cell will be 1×1, *i.e.* 1 volt for a current of 1 ampere only.

The reason of the low internal resistance of an accumulator is that the plates are very close together and that they have a large area. In accumulators the effective area is further increased by having several plates usually connected together to form each element. (*Vide* Ch. VI).

35. Grouping of Cell.—There are three ways of arranging a number of cells to get a strong current.—

(1) Cells in Series.—Cells are said to be joined in *series* when the positive terminal of the first cell is joined to the negative terminal of the second, and the positive of the second to the negative of the third ; and so on (Fig. 42). If there are n cells, each of E. M. F. (E) volts and internal resistance r ohms, then the total E. M. F. will be nE, and the total internal resistance nr. If R ohms be the external resistance of the circuit, then, by Ohm's Law, the current

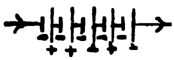


Fig. 42—Cells in Series

$$C = \frac{nE}{nr + R}.$$

(*i*) If the external resistance R be *very large* compared with nr, then $C = nE/R$, *i.e.* the current is increased n times that of a single cell.

(*ii*) If R be *very small* compared with nr, then $C = E/r$;

or the current is the same as that of a single cell and the arrangement has no advantage.

Hence for **a strong current,** *series connection should always be used when the external resistance R is large in comparison with the internal resistance r of each cell.*

(a) Special case.—If one of the n cells is wrongly connected so that it sends current in the opposite direction, there will be $(n-1)$ cells

tending to send a current in one direction having a total E. M. F. $=(n-1)E$, and the remaining one cell tending to reverse the current with an E. M. F. equal to E. So the resultant E. M. F.

$$=(n-1)E-E=(n-2)E.$$

Hence, the current, by Ohm's Law, $C=\dfrac{(n-2)E}{nr+R}$.

Pocket Torches.—The batteries of small torches usually have two or three dry cells in series. These cells are placed end to end, the bottom of the zinc case of one touching the copper cap on the top of the carbon rod of the cell below it.

(2) **Cells in Parallel.**—Cells are said to be joined in *parallel*, when all the positive terminals are joined together and all the negative terminals are joined together (Fig. 43). The arrangement amounts to forming a big cell of plates m times as large as those of a single cell. The E. M. F. will be the same as that of a single cell but the internal resistance will be r/m, the effective area of the plates being m times increased.

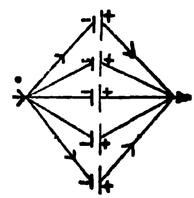


Fig. 43—Cells in Parallel

Hence, by Ohm's Law, $\mathbf{C}=\dfrac{E}{r/m+R}=\dfrac{\mathbf{mE}}{\mathbf{r+mR}}$.

(i) *If R be very small*, $C=mE/r$, *i.e.* the current is m times that of a single cell.

(ii) *If R be very large*, $C=\dfrac{mE}{mR}=\dfrac{E}{R}$, *i.e.* the current is the same.

Hence, for **a strong current** *parallel connection should always be used when the external resistance is small in comparison with the internal resistance of a single cell.*

(3) **Mixed Circuit.**—Cells are arranged in a *mixed circuit* when they are divided into several rows in parallel, each row containing several cells in series (Fig. 44). Let the external resistance be R, and the E. M. F. of each cell E, the number of rows m, and the number of cells in each row be n. Then the total E. M. F. is nE, and the total internal resistance is nr/m. Hence, by Ohm's Law, the total current C passing through the external resistance, R,

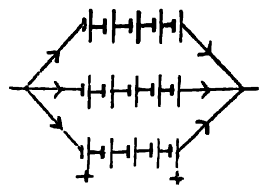


Fig. 44—Cells in a Mixed Circuit

$$\mathbf{C}=\frac{nE}{nr/m+R}=\frac{\mathbf{mnE}}{\mathbf{nr+mR}}\quad\ldots\ldots\ldots\ldots(1)$$

(4) **Best Grouping for Maximum Current.**—To get the maximum current out of a number of cells, N, let them be divided into m rows having n cells joined in series in each row. Then $\mathbf{N = m \times n}$.........(2)

We have, from (1), $C = \dfrac{mnE}{nr+mR} = \dfrac{NE}{nr+mR}$.

The numerator being a constant, current will be maximum when the denominator $(nr+mR)$ will have a minimum value.

Now, $(nr+mR) = (\sqrt{nr} - \sqrt{mR})^2 + 2\sqrt{mnrR}$. The last term is constant.

$\therefore$ C will be a maximum, when $(\sqrt{nr} - \sqrt{mR})^2$ has a minimum value.

But, as a square quantity cannot be negative, the minimum value of $(\sqrt{nr} - \sqrt{mr})^2$ is 0. $\therefore$ C will be a maximum, when $(\sqrt{nr} - \sqrt{mR})^2 = 0$,

or $\sqrt{nr} = \sqrt{mR}$; or, $nr = mR$; or, $\dfrac{nr}{m} = R$..............(3)

That is, *the current will be a* **maximum** *when the total internal resistance is equal to the total external resistance.*

Examples.—*1. Determine the number of cells required to send a current of half an ampere through a body whose resistance is 30 ohms, if each cell has an E. M. F. of 2 25 volts and a resistance of 2 ohms.* (*C. U. 1925*)

If the cells are arranged in series, we have $C = \dfrac{nE}{nr+R}$ [See Art. 35(1)]

Here $C = \frac{1}{2}$ amp. ; $r = 2$ ohms. $R = 30$ ohms. ; $E = 1{\cdot}25$ volts.

$\therefore$ $\dfrac{1}{2} = \dfrac{1{\cdot}25n}{3n+30}$; *or* $3n + 30 = 2{\cdot}5n$; *or* $n = 60$ cells.

2. Two cells A and B, each having an e. m. f. of 1·5 volts and internal resistance of 0·8 ohms. are arranged in series. The positive and negative poles of this battery are connected with the positive and negative poles respectively of a third cell C exactly like A, the connecting wires having negligible resistance. What is the current in the circuit, and what is the potential difference between the poles of C and also those of A ? (*Pat. 1945*)

Here the cell C is arranged to work against the other two similar cells. So the current flowing in the circuit $I = \dfrac{(2 \times 1{\cdot}5) - 1{\cdot}5}{3 \times 0{\cdot}8 + 0} = \dfrac{5}{8}$ amp.

We have $E = IR$ (*i.e.* P. D. reqd.) $+ Ir$. (Art. 31.)

$\therefore$ P. D. between the poles of $A = 1{\cdot}5 - \left(\dfrac{5}{8} \times 0{\cdot}8\right) =$ volt, and P. D. between the poles of $C = 1{\cdot}5 + \left(\dfrac{5}{8} \times 0{\cdot}8\right) = 2$ volts [$\because$ in this case the current inside the cell is from positive to negative pole, (see Art. 31, eq. 1)] ;

3. A battery of 24 cells, each of internal resistance 2 ohms, and E. M. F. 1·4 volts is to be connected so as to send a maximum current through a wire of 12 ohms. Show how you will connect them ; amd find the strength of the current in each of the cells, also the potential difference at the ends of the external resistance. (*Pat. 1928*).

[See Art. 35(3)]. Let the cells be arranged in a mixed circuit. Then to get the maximum current, the internal resistance nr/m of the battery must be equal to the external resistance R ; where n is the number of cells in each row, m the number of rows, and r the internal resistance of each cell.

Then, we have, $mn=24$. $\therefore$ $n=24/m$..................(1)

and, $nr/m=R$; or $2n/m=12$; or $n=6m=6\times\frac{24}{n}$, from (1).

or, $n^2=144$; or, $n=12$; and so $m=2$.

Thus the cells should be arranged in 2 rows in parallel, 12 cells in series in each row.

Current in the external circuit $=\dfrac{12\times1{\cdot}4}{\frac{12\times2}{2}+12}=0{\cdot}7$ amp. This current is divided equally in the two rows. $\therefore$ The current in each row $=0{\cdot}35$ amp., which is also the current in each cell.

P. D. at the ends of the external resistance $=0{\cdot}7\times12=8{\cdot}4$ volts.

4. Five cells, each having an E.M.F. of 1·5 volts and internal resistance 3 ohms, are connected in series with an external resistance of 10 ohms. Find the current passing through the circuit. What will be the change in the current through the circuit, if the cells are formed in parallel ? (*Pat. 1941*)

When the cells are connected in series, the current, $C_1=\dfrac{5\times1{\cdot}5}{3\times5+10}=0{\cdot}3$ amp.

When the cells are connected in parallel, the current,

$$C_2=\frac{1{\cdot}5}{\frac{3}{5}+10}=0{\cdot}14 \text{ amp. (approx.)}$$

$\therefore$ The charge in the current $=C_1-C_2=0{\cdot}3-0{\cdot}14=0{\cdot}16$ amp. (approx).

36. Grouping of Resistances.—Resistances can be arranged in an electric circuit in two ways, (1) in **series**, and (2) in **parallel**.

(1) In series.—A number of conductors having resistances r_1, r_2, r_3, etc., are said to be joined in *series* when joined end to end in succession, *i.e.* when the same current passes through each of them in succession. The resistances r_1, r_2, r_3,...in series have a total P. D. of $Cr_1+Cr_2+Cr_3+\ldots$ or $C(r_1+r_2+r_3+\ldots)$ between the points A and D, C being the current passing

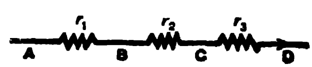


Fig. 45 —Resistances in Series

through each of them (Fig. 45). If R be the total resistance of the conductors, then $R = \dfrac{\text{Total P. D.}}{\text{Current}} = \dfrac{C(r_1 + r_2 + r_3 + \ldots)}{C}$;

i.e. $\mathbf{R = r_1 + r_2 + r_3 +}$.

Thus, **the total resistance of a number of conductors in series is equal to the sum of the resistances of the individual conductors.**

(2) **In Parallel** (or **Divided circuits**).—A number of conductors having resistances r_1, r_2, r_3, etc , are arranged in *parallel* when one end of each of them is joined at a common point A and the other end at another common point B in such a way that a current C entering at A is divided into parts, a part flowing in each of the branches, and again meeting at B to reconstitute the total current (see Fig. 46).

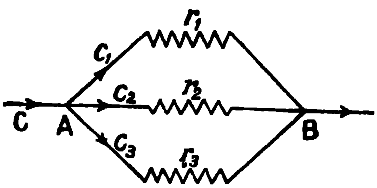


Fig. 46—Resistances in parallel

Let C be the total current, C_1, C_2, C_3, etc., be the currents in r_1, r_2, r_3, etc., respectively, and V_a, V_b, the potentials at A and B. Then, we have, $C = C_1 + C_2 + C_3 + \ldots\ldots\ldots\ldots\ldots$

By Ohm's Law, $C_1 = \dfrac{V_a - V_b}{r_1}$; $C_2 = \dfrac{V_a - V_b}{r_2}$; $C_3 = \dfrac{V_a - V_b}{r_3}$, etc.

Adding, $C_1 + C_2 + C_3 + \ldots\ldots = (V_a - V_b)\left(\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} + \ldots\ldots\ldots\right)$

or $$C = (V_a - V_b)\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots\ldots\ldots\right) \quad \ldots \quad \ldots \quad (1)$$

If R be the equivalent resistance of the combination, *i.e.* total resistance between the common points A and B, we have,

$$C = \frac{V_a - V_b}{R} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$\therefore$ From (1), $\dfrac{V_a - V_b}{R} = (V_a - V_b)\left(\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} + \ldots\ldots\ldots\right)$

or, $$\mathbf{\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots\ldots}$$

Thus, **the reciprocal of the equivalent resistance of a number of conductors in parallel is equal to the sum of the reciprocals of the resistances of the separate conductors.**

N. B. Notice that $\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$ is always greater than $\frac{1}{r_1}$ or $\frac{1}{r_2}$;

So. $\frac{1}{R}$ is greater than $\frac{1}{r_1}$ or $\frac{1}{r_2}$, and hence R is less than r_1 or r_2. So *the effective resistance of two or more conductors in parallel is smaller than that of either of them, when used alone.*

Note that in the case of several conductors joined in **series** the **P. D. is divided** among the conductors in the proportion of their *resistances*, but the **same current** passes through each of them; whereas all the conductors, when connected in **parallel**, have the **same P. D.**, but the total **current** is **divided** amongst them in proportion to their *conductances i. e.* reciprocals of resistance ($1/r$). The current in a circuit can be regulated by introducing a resistance or a combination of resistances in series or parallel with it.

Laws for Parallel Circuits are,—(*a*) The *voltage* across several resistances in parallel is the same for all the circuits. (*b*) The *total current* is the sum of the current through the different branches. (*c*) The *total resistance* is the voltage divided by the total current.

Electric lamps are invariably connected in parallel, for, if they were connected in series, then switching out one of them would cause the rest also to go out. Fig. 47 shows a lamp-board on which lamps are connected in parallel, and it is evident that any one of the lamps can be disconnected without affecting others. By disconnecting one lamp the current passing in others is not changed, because the current passing in each lamp is V/r, where V is the P. D. between the mains (which is also the P. D. for each lamp) and r the resistance of the lamp. The effect of disconnecting the lamp would be to change the current of the main circuit. If the lamps were connected in series then removal of one of the lamps would result in diminishing the total resistance and so increasing the current passing in the circuit, and thus the lamps would get more than their correct voltage with disastrous results. This is not the case when they are connected in parallel. Thus a *lamp-board* consisting of several lamps, which are alike, connected in parallel provides a very convenient form of adjustable resistance, which is often used in accumulator charging, etc.

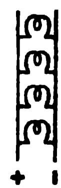


Fig. 47

Example.—*An electric current from a battery of 6 volts passes through a circuit having three lamps of resistances 2, 3, and 4 ohms joined in parallel. Calculate the current passing in the lamps when* (*a*) *all the lamps are in their sockets,* (*b*) *first lamp of resistance 2 ohms is removed.*

(a) If R be the total resistance, $\frac{1}{R}=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$; *or*, $R=\frac{12}{13}$ ohms

$\therefore$ By Ohm's Law, current in the main circuit, $C=6/\frac{12}{13}=6{\cdot}5$ amp.

If C_1, C_2, and C_3 be the currents passing in the lamps having resistances 2, 3 and 4 ohms respectively, we have, by Ohm's Law,

$C_1=\frac{6}{2}=$ amps. ; $C_2=\frac{6}{3}=2$ amps. ; $C_3=\frac{6}{4}=1{\cdot}5$, where 6 volts form the P. D. for each lamp.

Total resistance, $R=\frac{1}{\frac{1}{3}+\frac{1}{4}}=\frac{12}{7}$ ohms. $\therefore$ $C=\frac{6}{12/7}=3{\cdot}5$ amps.

And $C_2=\frac{6}{3}=2$ amps. ; and $C_3=\frac{6}{4}=1{\cdot}5$ amps.

That is, by taking out the first lamp, C_2 and C_3 are not changed.

37. **Shunts.**—When using a sensitive galvanometer it is not desirable to send a strong current through it. In such a case, the two terminals of the galvanometer are joined with a resistance of small value, called the **shunt**, which acts as a conductor parallel to the galvanometer (Fig. 48), so that most of the main current may pass out through the shunt. The main current is divided into the two branches according to the resistance of each branch.

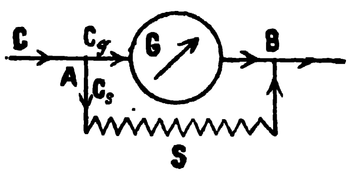


Fig. 48

Let G be the galvanometer resistance, S the resistance of the shunt, C_g the current in the galvanometer, and C_s the current in the shunt, then, if V_a and V_b be the potentials of A and B, we have by Ohm's Law,

$(V_a-V_b)=C_g\times G=C_s\times S$; *or* $\frac{C_s}{C_g}=\frac{G}{S}$; *i.e. the currents in the branched circuits are inversely proportional to the resistancss of the two branches.*

$\therefore$ $\frac{C_s+C_g}{C_g}=\frac{G+S}{S}$; *or* $\frac{C}{C_g}=\frac{G+S}{S}$ ($\because$ the main current, $C=C_g+C$).

(The factor $\frac{G+S}{S}$ by which the galvanometer current C_g is multiplied to give the total current C is called the **multiplying power** of the shunt, that is, the number of times the main current is stronger than the galvanometer current).

Now, $\frac{C_g}{C}=\frac{S}{G+S}$; *or*, the fraction of the total current through the galvanometer $=\frac{\textit{resistance of shunt}}{\textit{galv. resistance + shunt resistance}}$

Hence $\mathbf{C_g = C\frac{S}{G+S}}$ .... (1)

Thus, if $1/n$th of the total current is to be sent through the galvanometer we have, $\frac{C_g}{C}=\frac{1}{n}=\frac{S}{G+S}$; or $S(n-1)=G$;

or $S=\frac{1}{n-1}G$; *i.e.* if the value of S be $\frac{1}{n-1}$th of G, the current in the galvanometer will be $1/n$th of the total current.

By making S small in eq. (1), C_g can be made as small as possible. If it is necessary to send, (say), $\frac{1}{10}$ of the main current in the galvanometer, we have, from (1),

$$\frac{C_g}{C}=\frac{1}{10}=\frac{S}{G+S}\text{.......................}(2)$$

∴ From (2), $S=\frac{1}{9}G$, that is, the resistance of the shunt should be 1/9 of the resistance of the galvanometer in order that $\frac{1}{10}$ of the main current will pass through the shunt. Similarly, if $\frac{1}{100}$ or $\frac{1}{1000}$ of the main current is required to be passed in the galvanometer, the resistance of the shunt should be $\frac{1}{99}$ or $\frac{1}{999}$ of the resistance of the galvanometer. It is clear that the sensibility of the galvanometer is thus reduced to 10, 100 or 1000 times, as the deflection in each of the above cases is obtained by a very small fraction of the original current.

Example.—*The total resistance of a simple circuit is 80 ohms including the resistance of a tangent galvanometer which is 4 ohms. The galvanometer gives a deflection of 60°. It is then shunted with a 7·6 ohms coil. What is the new deflection?* (*Pat. 1944*)

Case I. We have, $C=\frac{E}{80}=K\tan 60°$.....................(I)

Case II. $\frac{1}{R}=\frac{1}{4}+\frac{1}{7\cdot6}$; $R=\frac{4\times7\cdot6}{11\cdot6}$.

Now, the main current in the circuit, $C_1 = \dfrac{E}{(80-4)+\frac{4\times7\cdot6}{11\cdot6}}$

and the current in the galvanometer, $C_g=C_1\times\frac{S}{S+G}$

$$= \frac{E}{76 + \frac{4 \times 7{\cdot}6}{11{\cdot}6}} \times \frac{7{\cdot}6}{(7{\cdot}6+4)} = 10K \tan \theta, \ldots\ldots\ldots\ldots \text{(II)}$$

where θ is the new deflection. From (I) and (II), $\frac{\tan \theta}{\tan 60} = \frac{2}{3}$.

$$\therefore \quad \tan \theta = \frac{2}{3} \times \tan 60^\circ = \frac{2 \times \sqrt{3}}{3} = \frac{2}{\sqrt{3}}; \quad \therefore \quad \theta = \tan^{-1} \frac{2}{\sqrt{3}}.$$

N. B. The resistance of the galvanometer is 4 ohms, but on being shunted the equivalent resistance between its terminals fell to $\frac{4 \times 7{\cdot}6}{11{\cdot}6} = 2{\cdot}62$ ohms approximately. Hence to keep the current in the main circuit constant, a resistance, $4 - 2{\cdot}62 = 1{\cdot}38$ ohms, has to be added in series.

(a) How to Increase the Range of Ammeters by Shunts.— By shunting a galvanometer, or an ammeter which is nothing but a low resistance galvanometer (see Art. 42), the range of the instrument can be increased considerably. Suppose, for example, an ammeter of resistance 9 ohms can read currents upto 5 amperes, *i.e.* it is calibrated to read 1 to 5 amperes. So a current stronger than 5 amperes should not be passed through the instrument.

Now, by connecting a wire of resistance equal to $\frac{1}{9}$ of the instrument (*i.e.* $\frac{1}{9} \times 9 = 1$ ohm) across it terminals, *i.e.* by using a shunt of 1 ohm resistance, we allow only $\frac{1}{10}$ of the total current to flow through the instrument. So the ammeter can now be used to read currents 10 times greater than the original current, *i.e.* it can now read up to (5×10) or 50 amperes. Thus the **range** of the instrument is **increased** 10 times. Similarly by using shunts of other values the range of the instrument can be conveniently changed. So a **shunt** can be defined as **a resistance put across the terminals of a galvanometer (or any instrument) in order to reduce its sensibility, and also to increase its range**.

Examples.—1 *An electric current of 5 amperes is divided into three branches, the length of the wires in the three branches being proportional to 1, 2, 3 : find the current in each. (The wires are of the same material and cross-section).*

(C U. 1912, '29)

Let V = P. D. between the terminals, C_1, C_2, C_3 = the respective currents in the three branches having resistances, say, r, $2r$, $3r$.

Then $V = C_1 r = C_2 \times 2r = C_3 \times 3r$. $\therefore$ $C_1 = V/2r$; $C_2 = V/r$. $C_3 = V/3r$. But the total current = 5 amperes. $\therefore 5 = C_1 + C_2 + C_3 = \frac{V}{r}(1 + \frac{1}{2} + \frac{1}{3}) = \frac{11}{6} C_1$.

Hence, $C_1 = \frac{30}{11}$amp. ; and $C_2 = \frac{C_1}{2} = \frac{15}{11}$amp. ; and $C_3 = \frac{C_1}{3} = \frac{10}{11}$ amp.

2. Two points A and B are maintained at a constant potential difference of 110 volts. A third point C is connected to A by two resistances of 100 and 200 ohms respectively in parallel, and to B by a single resistance of 300 ohms. Find the current in each resistance, and the potential differences between A and C, C and B. (*Pat. 1926*)

(Draw a diagram). If R be the combined resistance between the points A and C, we have, $\frac{1}{R}=\frac{1}{100}+\frac{1}{200}=\frac{3}{200}$; $\therefore$ $R=\frac{200}{3}$. $\therefore$ The total resistance between A and $B=\frac{200}{3}+300=\frac{1100}{3}$ ohms.

Hence, by Ohm's Law, the total current $C=100\Big/\frac{1100}{3}=\frac{3}{10}=0{\cdot}3$ amp.

This current flows in BC, but is divided into two branches having resistances of 100 and 200 ohms respectively.

The P. D. of C and B = current × resistance $=0{\cdot}3\times300=90$ volts.

$\therefore$ The P. D. of C and $A=110-90=20$ volts.

Hence, the current in the branch CA having 100 ohms resistance $=$ P.D. $\div$ resistance $=20/100=0{\cdot}2$ amp. ; and the current in the branch having 200 ohms resistance $=20/200=0{\cdot}1$ amp.

Thus, currents of $0{\cdot}1$, $0{\cdot}2$, and $0{\cdot}3$ amp. flow in the resistances 200, 100 and 300 ohms respectively.

Otherwise thus :—Let E be the P.D. between A and C, then the P.D. between C and $B=(110-E)$; and the currents through two parallel resistances are $E/100$ and $E/200$ respectively.

$\therefore$ The total current $=\frac{E}{100}+\frac{E}{200}=\frac{3E}{200}$ and this flows through CB.

Again, the current in $CB=\frac{110-E}{300}$. Hence $\frac{3E}{200}=\frac{110-E}{300}$; whence $E=20$.

The currents can be calculated as above.

3. Three wires of resistances 2, 6 and 12 ohms respectively are connected in parallel and are inserted in a circuit with a cell and a tangent galvanometer. The deflection is 60° ; the 2 ohms wire is removed, and the deflection becomes 45. Calculate the resistance of the galvanometer. (Neglect the resistance of the cell).

(*All. 1925*)

If R be the combined resistance of the three wires,

$$1/R=\tfrac{1}{2}+\tfrac{1}{6}+\tfrac{1}{12}=\tfrac{3}{4}\text{ ohm}\ ;\ or\ R=\tfrac{4}{3}\text{ ohms}.$$

If R_1 be the combined resistance of the two wires of resistance 6 and 12 ohms,

$$1/R_1=\tfrac{1}{6}+\tfrac{1}{12}=\tfrac{3}{12}=\tfrac{1}{4}\text{ ohm}\ ;\ \text{or } R_1=4\text{ ohms}.$$

If G be the galvanometer resistance and E the E. M. F. of the cell, the total resistance of the circuit in the first case $=G+\frac{4}{3}$ ohm, and that in the second case $G+4$ ohm. Now if C_1 be the current in the first case and C_2 that in the second case,

$C_1 = E/G + \frac{4}{3} = 10K \tan 60^\circ$ amperes, where K is the reduction factor of the galvanometer, and $C_2 = E/G + 4 = 10K \tan 45^\circ$ amperes.

Dividing one by the other, we get $\dfrac{3(G+4)}{3G+4} = \tan 60^\circ = 1{\cdot}732$;

or $2{\cdot}196\, G = 5{\cdot}072$; $\therefore$ $G = 2{\cdot}31$ ohms.

(4) *The poles of a Daniell cell of E.M.F. 1·1 volt and internal resistance 1 ohm are joined by two wires in series, a wire AB of 4 ohms and a wire BC of 6 ohms resistance. The positive pole of the cell is connected to the end A of the series. What will be the readings of a voltmeter connected between* (i) *A and B,* (ii) *B and C,* (iii) *A and C ?* *(C. U. 1938)*

Here the resistances are in series, so the total resistance $= 6+4+1 = 11$ ohms. $\therefore$ Current $C = \dfrac{E}{11} = \dfrac{1{\cdot}1}{11} = 0{\cdot}1$ amp.

Now, if V_1 be the P. D. between A and B, V_2 that between B and C ; and V_3 that between A and C, we have,

$V_1 = 0{\cdot}1 \times 4 = 0{\cdot}4$ volt ; $V_2 = 0{\cdot}1 \times 6 = 0{\cdot}6$ volt, and, $V_3 = 0{\cdot}1 \times 10 = 1$ volt.
[Note that V_3 = Terminal P. D. of the cell (Art. 31)].

5 *A cell has an E.M.F. of 1·5 volts and internal resistance 2 ohms. If the terminals of the cell are connected by two wires in parallel of 2 ohms and 4 ohms resistance respectively, what is the current in each wire and what is the potential difference between the terminals of the cell ?*

Calculate also the potential difference when (a) *the current is cut off in one of the wires, and* (b) *when it is cut off in the other.* *(Pat. 1941)*

Let V be the P. D. of the terminals of the cell, and C_1 the current passing through the wire of 2 ohms resistance and C_2 the current in the wire of 4 ohms resistance. Then, we have, the total current, $C = C_1 + C_2$.

If R be the combined resistance, $1/R = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$; $\therefore$ $C = V/\frac{4}{3} = \dfrac{3V}{4}$.

Again $C = \dfrac{1{\cdot}5}{2+\frac{4}{3}} = \dfrac{9}{20}$ amp. $\therefore$ $\dfrac{3V}{4} = \dfrac{9}{20}$; *or* $V = \dfrac{3}{5}$ volt.

We have, $C_1 = \dfrac{V}{2} = \dfrac{3}{5 \times 2} = 0{\cdot}3$ amp. ; $C_2 = \dfrac{V}{4} = \dfrac{3}{5 \times 4} = 0{\cdot}15$ amp.

Again, (*a*) when the current is cut off in the wire of 2 ohms resistance,

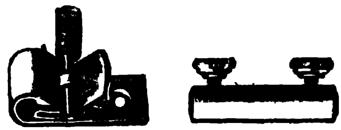
we have $C = \dfrac{1{\cdot}5}{2+4} = 0{\cdot}25$; $\therefore$ P.D. $= 0{\cdot}25 \times 4 = 1$ volt.

(*b*) When the current is cut off in the wire of 4 ohms resistance,

we have $C = \dfrac{1{\cdot}5}{2+2} = 0{\cdot}375$; $\therefore$ P.D. $= 0{\cdot}375 \times 2 = 0{\cdot}75$ volt.

38. Some Accessories for Electrical Experiments.—Some useful accessories often necessary for electrical experiments are given below.—

(a) Connectors and Binding Screws.—These are used for connecting two separate wires and also for other different purposes. These are of various shapes and types and are made of brass or copper (see Fig. 49).



Connector Fig. 49 Binding Screw

(b) Keys.—A key is used for opening or closing an electrical circuit. There are different types and shapes of keys of which two are generally used :—

(i) Plug Key.—Two thick metallic plates C, C are fixed on an ebonite base B being separated from each other by a small gap which can be bridged by a brass plug P with an ebonite handle T (Fig. 50), S, S being the two binding screws.

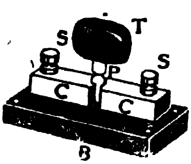


Fig. 50.—Plug Key

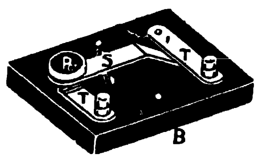


Fig. 51—Tapping Key

(ii) Tapping Key.—It consists of an ebonite base B on which two binding screws T, T are fixed, by means of which the key can be inserted in the circuit. One of the screws T is attached to a metal spring S having an ebonite tapping button P (Fig. 51) Below this there is a metal stud connected by means of a metal strip (or wire) with the other binding screw T. Contact is made only as long as P is pressed down on the stud. When the finger is withdrawn, the metal spring springs up and the circuit is broken.

Tapping key is used if the current is required for a *short time*, while the **plug key** is used if the current be required for a *longer time*.

(iii) Commutators.—A commutator is an arrangement for reversing the direction of a current passing through any portion of a circuit. There are different forms of commutators.

(1) Quadrant Commutator.—Fig. 52 represents a four-way plug which can be used as a commutator. Four thick metal plates 1, 2, 3, 4, are separated from one another by gaps. The plates are fixed on an ebonite base B and provided with binding screws A, C, B, D (Fig. 53). Contact is established between the plates by inserting metal plugs between the gaps as shown in the figure. Fig. 53 also explains the manner

in which the four-way plug key can be used as a commutator. The battery is joined to a pair of opposite terminals, *A* and *B*, and the two ends of the portion of the circuit in which the current should be rever-

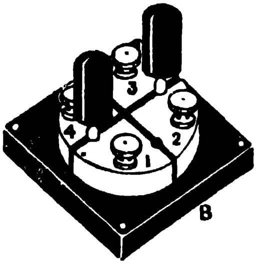


Fig. 52—Plug Commutator

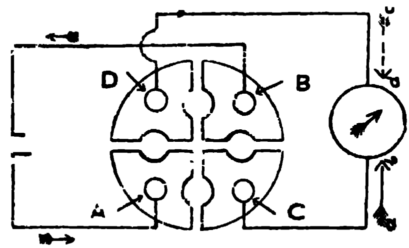


Fig. 53—Connection in Plug Commutator

sed (here the galvanometer circuit) are joined to the other pair of terminals, *C* and *D*. If one plug is inserted between *B* and *D*, and another between *A* and *C*, the current reaches the galvanometer through *C* (as shown by the arrow *ab*) and comes back to the battery through *B*. If, again, the plugs are inserted between *A*, *D*, and *B*, *C*, the direction of current in the galvanometer is changed (as shown by the dotted arrow *cd*). The direction of the battery current is the same in both the cases.

(2) Pohl's Commutator.—Fig. 54 represents another commutator known as the *Pohl's Commutator*. This is generally used for rapidly reversing the direction of a current. It consists of six mercury cups

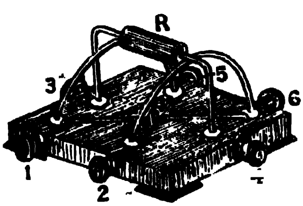


Fig. 54—Pohl's Commutator

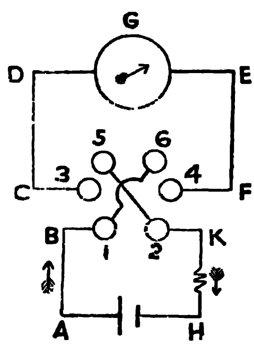


Fig. 55—Connections in a Pohl's Commutator

on an insulating base, each provided with its own binding screw. One diametrically opposite pair of cups is connected by a piece of thick copper wire, and another such pair is also connected by another wire

which is bent in order to avoid contact with the first. Connections are made as shown in Fig. 55. The two middle legs of the wire-frame rocker R remain always in contact with the mercury cups 3 and 4. The rocker rocks in such a way that when the two front legs dip in the cups 1 and 2, the two back legs do not touch the mercury cups 5 and 6. Again, when the two back legs touch 5 and 6, the front legs do not touch 1 and 2.

Now suppose in Fig. 55 the front legs of the rocker touch the cups 1 and 2 The current from the battery goes to the cup 3 from the cup 1 through the wire frame and then passes through the galvanometer G in the direction $ABCDEF$. Then from the cup 4 it reaches the cup 2 through the wire frame, and returns to the battery in the direction KH.

(iv) **Resistance Coils and Resistance Boxes.**—Coils of insulated wire having resistances from fractions of an ohm to hundreds or thousands of ohms are made by instrument-makers and are usually

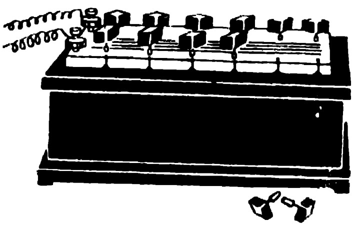


Fig. 56

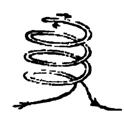


Fig. 56(a)—Double Wire (Non-inductive Winding)

arranged in a box, called **a resistance box** (Fig. 56). Coils having a standard resistance are also separately sold and used, each silk-covered wire in the coil being doubled over itself [see Fig. 56(*a*)] to eliminate the effects of self-induction (see Chapter VII). The free ends of the wire are soldered to adjacent bars A, A (see Fig. 57). When a plug P is out, the coil is in the circuit, *i.e.* the current passes from one brass bar A to the other (A) through the wire, and when the plug is in, the current passes straight across, and no resistance is included. Thus by changing the plugs in the box any required combination of resistances may be used.

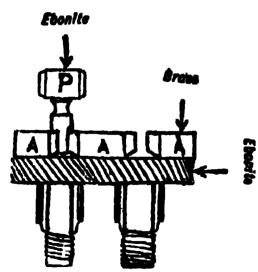


Fig. 57—Resistance coil in a Resistance Box

(v) **Rheostat.**—The resistance in a circuit, and hence a current,

can be changed by including in the circuit a resistance box, but when the value of the included resistance need not be known, an adjustable resistance is sometimes used. Such a variable resistance is called a **rheostat**. Fig. 58 illustrates a sliding rheostat. This usually consists of a coil of uncovered wire of *high specific resistance* wound on a slate cylinder in such a way that the neighbouring turns do not touch one another. The two ends of the wire are connected to the two front binding screws *T. T.* The instrument is inserted in a circuit by means of the binding screw *A* at the top and one of the front ones *T*, *T*. A sliding piece on the top bar *R* makes contact with the wire of the cylinder, and resistance can be altered by moving the sliding piece along the bar.

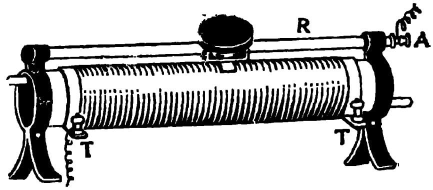


Fig. 58—Rheostat.

39. Measurement of Resistance.—Electric resistance can be measured in two ways :—(*a*) *Method of Substitution* ; (*b*) *Method of Comparison.*

(a) Method of Substitution.—The unknown resistance is joined in series with a battery and a galvanometer (say the low resistance coil of a tangent galvanometer), and the deflection is noted. After this, a suitable resistance box is inserted in the place of the unknown resistance and the resistances in the box are adjusted until the galvanometer gives the original deflection. The total value of the resistances used in the box gives the value of the unknown one. This is only an approximate method and is not ordinarily used.

(b) Method of Comparison.—The usual method of measuring a resistance is by the *principle of Wheatstone's Bridge*, which embodies a method of comparison.

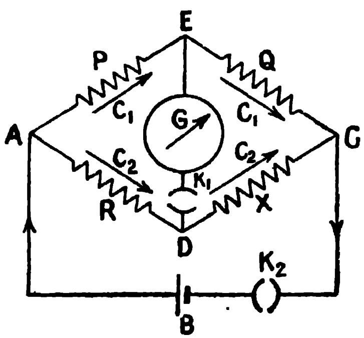


Fig. 59—Wheatstone's Bridge

Wheatstone's Bridge.—It consists of four conductors *AE*, *EC*, *CD*, *DA* having resistances *P*, *Q*, *X*, *R* respectively (Fig. 59). The two junction points *E* and *D* are connected by a galvanometer *G* through the key K_1 and a battery *B* is included between the other two junction points *A* and *C* through the key K_2. The resistances of the different branches are so adjusted that when the two

keys K_1 and K_2 are closed and a current passes through the system, there is no deflection in the galvanometer. *This happens when the potentials of E and D are the same, and so no current flows through the branch ED and so through the galvanometer.* Any experimental method by which this condition of 'no deflection' is obtained is called a **null method**. The current coming from the battery is divided at A into two portions C_1 and C_2 flowing in the two branches AEC and ADC respectively, which again unite at C and flow back to the battery. Let V_1 and V_2 be the potentials at A and C, and let the potentials at the points E and D, which are supposed to be the same, be V. As there is no flow of current between the points E and D, the same current C_1 flows through P and Q while C_2 flows through R and X. By Ohm's Law,

we have $C_1 = \dfrac{V_1 - V}{P} = \dfrac{V - V_2}{Q}$; again $C_2 = \dfrac{V_1 - V}{R} = \dfrac{V - V_2}{X}$

Hence, $\dfrac{V_1 - V}{V - V_2} = \dfrac{P}{Q} = \dfrac{R}{X}$; $\therefore$ $\dfrac{\mathrm{P}}{\mathrm{Q}} = \dfrac{\mathrm{R}}{\mathrm{X}}$; or, $\mathrm{X} = \dfrac{\mathrm{Q}}{\mathrm{P}}\mathrm{R}$.

Or, knowing any three of these resistances, the fourth can be calculated.

Two forms of Wheatstone's Bridge, which are generally used for the measurement of resistance, are (*i*) the *Metre Bridge*, and (*ii*) the *Post Office Box*.

40. The Metre Bridge.—This consists of a fine *bare* wire AC of *uniform* cross-section, one metre long, stretched along a metre scale which is fixed upon a wooden board (Fig. 60). The two ends of the wire are soldered at the points A and C of the series of copper plates AT, TL, MN, JF, and FC of negligible resistance, binding screws being provided at the points marked by letters, A, T, L, M, D, N, J, F and C.

There is a **slider** E, also called the **jockey**, by pressing which contact can be made at any point of the wire. The bridge has two gaps LM and NJ for the insertion of resistance coils.

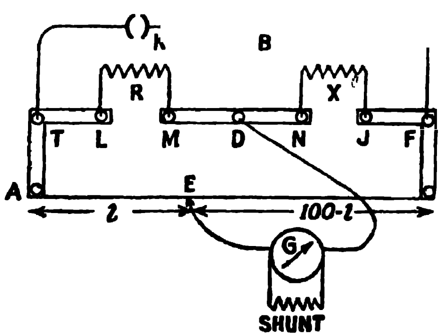


Fig. 60—The Metre Bridge

To measure a resistance X, it is placed in the gap between N and

J. A known resistance R is placed in the gap between L and M. The two terminals of a galvanometer are joined to the binding screws at D and E, E being on the slider. Two terminals of a battery are joined at T and F through a key K. Now, closing the battery circuit, move the slider along the wire and press it at different points until a position is found for which there is *no deflection* of the galvanometer. *At this position the potentials of D and E are the same.* Let l be the length of the portion AE of the wire and $100-l$ that of EC, and let P and Q be their respective resistances.

Then, we have, $\frac{R}{X}=\frac{P}{Q}$, from Wheatstone's Principle.

But, since the wire is uniform, the resistance of any portion of it is proportional to its length : for if ρ be the resistance per unit length of AC, the resistance P of the length $l=\rho l$, and that (Q) of the length $(100-l)=\rho(100-l)$.

$$\frac{P}{Q}=\frac{l}{100-l} \quad \text{Hence, } \frac{R}{X}=\frac{P}{Q}=\frac{l}{100-l}\ ; \text{ or } X=\frac{R\times(100-l)}{l}$$

[**Note.**—It may be noted that the bridge will be most sensitive when the position of the balance point is near the middle of the wire. For this reason R should be as nearly as possible equal to X].

41. The Post-Office Box.—This is a compact form of the Wheatston's Bridge (Fig. 61). It is so named because this instrument was originally intended for service in the British Post Offices for measuring resistances of telegraph wires, etc. On examining the box it will be found that in the first line AC (Fig. 61) there are two sections, AE and EC, each containing a set of three coils of resistances 10, 100, 1000 ohms respectively. These sections form P and Q arms of Fig. 59, which are called the **ratio arms** of a Wheatstone Bridge. The resistances of the remaining coils vary from 1 ohm to 5000 ohms or may be any other suitable series. This section forms the arm R of Fig. 59, called the **rheostat arm**, which should be adjusted so that no current will pass in the galvanometer. In order to insert a resistance in the circuit, the plug in the corresponding coil should be taken out. The unkown resistance (X) is put in the fourth arm between D and C. The

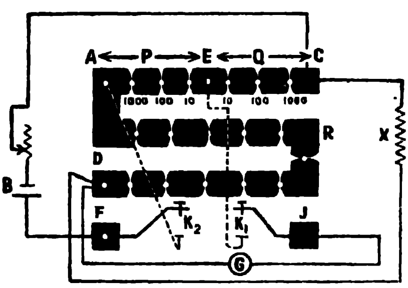


Fig. 61—The Post Office Box Connection

galvanometer (G) is inserted between E and D through the tapping key (K_1), while the battery B is connected between A and C through the other key (K_2).

Expt.—For the measurement of an unknown resistance X, first draw a diagram of the Wheatstone's Bridge, and make the connections as in Fig. 61. In making the measurement, first take out the two plugs corresponding to resistances 10 ohms from each of the *ratio arms* AE, EC, and adjust the resistance R in the *rheostat* arm AD, till the galvanometer gives no deflection.

In this position, we have, $P/Q = R/X$

or $$X = R\frac{Q}{P} = R\frac{10}{10} = R.$$

Or, the resistance in the AD arm gives directly the value of the unknown resistance. If, however, the exact balance point is not obtained, the correct value can be calculated by the principle of proportional parts. By changing the values of the ratio arms, and correspondingly changing the value of AD arm, the resistance up to the second place of decimal can be determined.

For example, say the value of an unknown resistance is 2·31 ohms. Changing the ratio of $P : Q$ from 10 : 10 to 100 : 10 and finally to 1000 : 10, the corresponding resistances in the AD arm should be changed from 2 to 23, and finally to 231 in order that there may be no deflection in the galvanometer.

Then we have, $\dfrac{1000}{10} = \dfrac{231}{X}$. Therefore $X = 2{\cdot}31$ ohms.

Note.—Remember always to close the battery circuit before the galvanometer circuit; for, if the galvanometer circuit is closed before the battery circuit, a temporary current may flow through the galvanometer in the *opposite direction* to the main current due to self-induction (see Chapter VI). Follow the reverse order when breaking the circuit, *i.e.* break the galvanometer circuit before the battery circuit.

41(a). Uses of Metre-Bridge and P. O. Box.—

(*i*) Metre bridge is suitable for measuring small resistances whilst the Post Office Box is suitable for measuring fairly high resistances.

(*ii*) The **specific resistance** ρ of the material of a wire whose resistance X is determined with either of them may be calculated by

very *small* in order that it may not appreciably change the strength of the current in the circuit which is to be measured.

A Voltmeter—a frequent type of which resembles an ammeter in appearance—is an instrument for measuring the difference of potential between any two points of a circuit. The *difference* between an ammeter and a voltmeter is that the moving coil in the voltmeter itself is of very high resistance, or it is connected in *series* with a high resistance, while the resistance of the coil in an ammeter is low. So a *voltmeter* is essentially a **high resistance** *moving-coil galvanometer* of the type described above. *It is always placed* **in parallel** *with the main circuit*, the two terminals of the voltmeter being connected with the two points of the circuit, the potential difference of which is to be measured, and as it is placed in parallel connection (see Fig 63), the current passing through the instrument is very small and so the current in the main circuit is practically unaltered. The P.D. under measurement, therefore, is also not affected.

Theory of Voltmeter.—

If C be the strength of the current flowing between two points in a circuit before completing the voltmeter circuit, the P. D. between the points is Cr by Ohm's Law, r being the resistance between the two points. Now, when the voltmeter circuit is joined across the portion of the main circuit having resistance r, it taps off a part of the main current, though a very small part, say C_1. Then the current flowing through the main circuit is $(C-C_1)$, and the P. D. between the points is $(C-C_1)r$. This shows that any current C_1 passing through the voltmeter circuit causes a diminution (C_1r) of the P. D. originally existing between the two points, and thus the P. D. measured on the voltmeter will be smaller by C_1r than the actual P. D. (Cr) which existed between the points before the voltmeter was connected up. Now, if the resistance of the voltmeter be very great in comparison with r, the current C_1, flowing through the voltmeter, will be so small that it may be neglected in comparison with C (which may thus be taken to be unchanged), and so the P. D. between the points becomes practically equal to Cr.

The deflection of the voltmeter is proportional to the small current flowing through it, and this is, by Ohm's Law, proportional to the P. D. between the points with which the terminals are in contact. Hence, if the voltmeter be already calibrated to measure P. D. directly in volts, the position of the pointer over the graduated scale of the instrument gives the P. D. between the points.

The scale of ampere-meters may be graduated to read directly in amperes, milli-amperes ($\frac{1}{1000}$ of an ampere), or micro-amperes

($1/10^6$ of an ampere)—the instruments being called *ammeters*, *milli-ammeters*, and *micro-ammeters* respectively.

Similarly there are *voltmeters*, *milli-voltmeters*, and *micro-voltmeters*.

43. How to use a Suspended-coil Galvanometer as an Ammeter or a Voltmeter.—In order *to use a sensitive moving-coil* **galvanometer as an ammeter**, a short thick wire should be joined across its terminals to allow only a small fraction of the current through the coil of the galvanometer, the major portion of the current passing through the thick wire (see Art. 37). The galvanometer should then be calibrated in comparison with a standard instrument In fact, all ammeters are fitted with a low resistance shunt placed in parallel with the coil, firstly, *to protect the moving coil from excessive currents by using only a small fraction of the current under measurement*, *and*, secondly, *to reduce the equivalent resistance to a very small magnitude so that the shunted instrument can be safely put* **in series** *with a circuit without appreciably changing the strength of the current.*

By having a suitable shunt the ammeter can be used for a very **wide range of current strengths** (see Art. 37).

For example, let us suppose that the maximum current allowed to pass through a given ammeter is 0·1 amp. and that the instrument is to be used up to 10 amp., *i.e.* the range is to be increased 100 times. For this a shunt S is to be joined across the terminals of the ammeter so as to allow $\frac{99}{100}$ of the main current to pass along S, and $\frac{1}{100}$ to pass through the instrument. So the resistance of the shunt should be $\frac{1}{99}$ of the resistance of the ammeter, as explained in Art 37.

Again, *to use* a **galvanometer as a voltmeter** a high resistance should be added externally to the voltmeter circuit in series, and the calibration should be effected by comparison with a standard voltmeter.

A **voltmeter** may be modified so as to measure voltages higher than its maximum scale-reading by connecting an additional resistance in series to its circuit, because the voltage across any part of a simple circuit is proportional to the resistance of the other. Thus a voltmeter designed to read up to 5 volts may be modified to read upto 10 volts by adding a resistance in series equal to the resistance of the instrument, for $V/V' = 5/10 = R/(R+R')$, where V is the original and V' the final difference of potential, and R the resistance of the instrument and R' the new resistance to be added in series, where $R' = R$. Similarly, if the same instrument be required to read up to 50 volts, $R' = 9R$, *i.e.* nine times the resistance of the instrument should be added.

44. Method of connecting Ammeters and Voltmeters.—It

should be remembered that an **ammeter** should always be put **in series** in the circuit, and the **voltmeter** should be put **in parallel** between two points whose difference of potential is required. This will be clear from Fig. 63, where the ammeter A reads the current in amperes flowing through the circuit, and the voltmeter V measures the potential difference between B and C, the battery circuit being completed through a key.

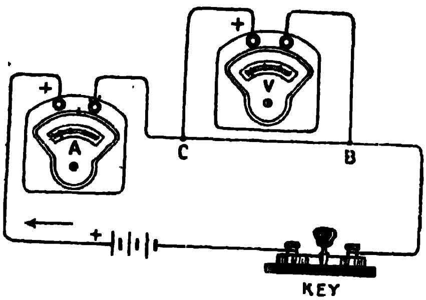


Fig. 63

Remember.—When introducing ammeters and voltmeters in a circuit, particular care should be taken to see that the binding-sbrew marked (+) should always be joined with the higher potential terminal of the battery so that the current enters the instrument at the terminal marked (+).

45. **Resistance of an Incandescent Lamp by means of Voltmeter and Ammeter**.—The resistance of the filament of an electric lamp can be determined by a P. O. box when the filament is cold, but this method is not suitable when the filament is hot, in which case the following method is adopted. In Fig. 63 put the lamp between B and C and send sufficient current through the lamp so that the filament becomes incandescent and gives out light Now take reading of the voltmeter and ammeter. If the voltmeter reading is V volts and the ammeter reading is A amperes, then, by Ohm's Law, the resistance R of the filament, when incandescent, will be given by, $R = V/A$ ohms.

Examples.—*1. A galvanometer gives a full-scale deflection with a current of 0·01 ampere, and has got a resistance of 10 ohms. How would you use this galvanometer,*

(i) as an ammeter reading upto 10 amperes ?
(ii) as a voltmeter reading upto 10 volts ?

(*i*) The maximum current allowed through the galvanometer is 0·01 ampere ; so in order to use it as an ammeter reading up to 10 amperes we must use a shunt of low resistance which will cary $(10-0{\cdot}01)=9{\cdot}99$ amperes. If S be the resistance of the shunt, we have

$$0{\cdot}01 = 10\frac{S}{10+S} \text{ (see Art. 37) ; or, } 0{\cdot}01\,(10+S) = 10S \qquad S = 0{\cdot}01 \text{ ohm.}$$

So, by using a shunt of 0·01 ohm with the galvanometer it can be used as an ammeter reading up to 10 amperes.

(*ii*) In order to use the galvanometer as a voltmeter (Art. 43) reading up to 10 volts, a resistance R' should be joined *in series* with the galvanometer of resistance 10 ohms; so the voltage across the total resistance of $(R'+10)$ ohms will be 10 volts. The current passing through the galvanometer and also through R' is the same, *i.e.* 0·01 ampere. Therefore, by Ohm's Law, we have

$(R'+10)\times 0{\cdot}01=10$; or, $0{\cdot}01R'+0{\cdot}1=10.$

$\therefore\quad R'=\dfrac{9{\cdot}9}{0{\cdot}01}=990$ ohms. Thus the required resistance is 990 ohms.

2. *A certain current measuring instrument has got 50 scale divisions and reads 0·005 amp. per scale division. The instrument which has a resistance of 25 ohms is to be used for measuring volts. Calculate the resistance which must be used in series with the instrument in order that ranges of 1 to 50 and 2 to 100 can be read.*

(*i*) 1 scale division indicates 0·005 ampere, but it should indicate 1 volt: so we have $C=E/R$; or, $0{\cdot}005=1/R$; $\therefore\ R=200$ ohms.

$\therefore$ Resistance to be joined in series $=200-25=175$ ohms.

(*ii*) For the second range, we have, $0{\cdot}005=2/R$; or $R=400$.

$\therefore$ Required resistance $=400-25=375$ ohms.

Questions

Art. 28.

1. Define volt, ohm and ampere. (C. U. 1929).

1(a). What are the practical units for (*a*) E.M.F., (*b*) current, (*c*) resistance, and (*d*) charge? (U. P. B. 1947).

2. Distinguish clearly between an electromotive force and a potential difference as applied to a cell. (Pat. 1931, '32; Mad. 1932).

3. Define the absolute units of current, potential difference and resistance, and obtain the practical units from them. Name the instrument by which each is measured and describe any of them. (C. U. 1934).

Art. 29

4. How does the current delivered by a cell differ from that delivered by a static machine? (C. U. 1936).

5. Define electro-static unit quantity of electricity and name some experiments that go to show that electricity obtained by friction is similar in nature to the electricity obtained from a Voltaic cell. (Pat. 1939).

Art. 30.

6. One gm. of copper wire is drawn into a wire (a) 1 cm. thick, (b) 2 cms. thick. Compare their resistances. (All. 1926).

[*Ans*: 16 : 1.]

Arts. 31 & 32.

7. Explain what you mean by the term 'Resistances of an electric circuit.' How would you compare two resistances in the laboratory? The terminals of

a battery of E.M.F. 4 volts and resistance 3 ohms are connected by a wire of resistance 9 ohms. Find the current strength. (C. U. 1926).

(For comparison of two resistances see Art. 40). [*Ans* : $\frac{1}{3}$ amp.]

7(a) Three wires, each of 60 ohms resistance, are arranged in parallel and connected to a battery of 10 Leclanche cells in series, each cell having an E.M.F. of 1·4 volts and a resistance of 2·2 ohms. Calculate the P. D. across the battery terminals and the current in each wire. (Utkal. 1948).

[*Ans* : 20/3 volts ; 1/9 amp.]

8. Two cells having a resistance of 2 ohms and E. M. F. of 1·5 volts are connected in series to the binding screws of a galvanometer having a resistance of 6 ohms. Find the current through the galvanometer. (C. U. 1920)

[*Ans* : 0·3 amp.]

8(a) A uniform wire 4 metres long and of resistance 6 ohms per metre is bent into the form of a square *ABCD*. The adjacent corners *A* and *B* are connected to a battery of E. M. F. 3 volts and internal resistance 4 ohms. Find the current along *AB*. (Utkal. 1947).

[*Ans* : 9/34 amps.]

9. An electric circuit contains a tangent galvanometer which gives a deflection of 45°. When, however, an additional resistance of 5 ohms is put in the circuit, this deflection is reduced to 30°. Calculate the total resistance of the circuit in the first instance. (Pat. 1926)

[*Ans* : 6·82 ohms.]

10. State Ohm's Law, and show how it provides the definition of electrical resistance. (Pat. 1926).

11. A cell, which has an E.M.F. of 1·5 volts on open circuit and the internal resistance of which is 2 ohms, has its terminals joined by two wires of 4 ohms and 10 ohms placed in parallel. Find the current in each wire. (Pat. 1937).

[*Ans* : $C_1 = 0·22$; $C_2 = 0·08$.]

11(a). Find the resistance of a battery which on open circuit gives an E. M. F. of 6 volts and which, when producing a current of 2 amp., has a P. D. of 4 votes between its poles. (Pat. 1946).
[*Ans* : 1 ohm.]

12. State Ohm's Law. Show how you will verify it ?
(C.U. 1910, '11, '18, '20, '25, '36, '39 ; Pat. 1918, '19, '20, '25, '27, '28, '35, '43 ; Dac. 1932 ; All. 1923, '25, '26, '28. '32, '46 ; Utkal 1947).

12(a). The E.M.F. of a battery is 18 volts and its resistance is 3 ohms. The P.D. between its poles when they are joined by a wire *A* is 15 volts, and it falls to 12 volts when *A* is replaced by another wire *B*. Find the resistances of *A* and *B*. (Pat. 1948).
[*Ans* : 15 ohms and 6 ohms]

12(b). Two wires of resistances 8 and 2 ohms respectively are connected in parallel. The combination is then connected in series with a third wire

of resistance 4 ohms. When the circuit is completed with a battery, the main current is 0·5 Amp. Calculate the terminal voltage of the battery as well as its E. M. F. The internal resistance of the battery is 0·8 ohm.

[*Ans* : E. M. F. = 3 volts ; terminal voltage = 2·6 volts.] (U. P. B. 1948)

13. State Ohm's Law and explain its meaning as carefully as you can. Apply the law to prove that the conductance of a number of coils in parallel is equal to the sum of the conductances of the coils separately, and explain how the relation can be verified experimentally. (Pat. 1930, cf. '46).

[**Hints.**—See Arts. 31 and 36. This can be verified by making experiments with a P. O. Box. [(See Art. 41.)]

Art. 33.

14. Describe and explain the principle of action of 'Resistance thermometer'. (Pat. 1947).

Art. 34.

15. State and explain the difference between a large and small Daniell cell in respect of (i) E.M.F., (ii) resistance. (Pat. 1938)

16. State, in general terms, on what the E.M.F. of a cell depends. (C. U. 1919)

Art. 35.

17. How would you arrange 30 cells, in each of which the resistance is 5 ohms, so as to send the most powerful current through an external circuit of 6 ohms resistance ? (Pat. 1915)

[*Ans* : 5 rows, each containing 6 cells.]

18. A battery of 10 cells joined in series yields a current of 1 ampere when the external resistance is 10 ohms, and a current of 0·6 ampere when the external resistance is 20 ohms. Find the E.M.F. and the internal resistance of the cells (these being the same for all.) (C. U. 1911)

[*Ans* : E.M.F. = 1·5 volts. ; $r = 0·5$ ohms.]

19. Explain the terms 'in series' and 'in parallel' as applied to a voltaic battery giving diagrams. (C. U. 1930)

20. A battery of 5 similar cells is connected with a rheostat and a tangent galvanometer of resistance 20 ohms, all in series. When 100 and 190 ohms are successively used in the rheostat, the galvanometer records deflections of 60° and 45° respectively. Calculate from the data the internal resistance of a single cell. (Pat. 1939)

[*Ans* : 0·59 ohm].

21. A number of cells are given to you ; how would you arrange them in order to supply the largest amount of current in a circuit of known external resistance ? (C. U. 1926)

Art. 36.

22. A cell having an E. M. F. of 2 volts and a resistance of 0·5 ohm is connected up with three lengths of wire having resistances of 1, 2, and 3 ohms

respectively, the wires being in series. Find the difference of potential between the ends of the middle wire. (C. U. 1930 : Dac. 1934)

[*Ans* : 0·615 volt.]

23. State Ohm's law, and apply it to find the equivalent resistance of a system of conductors joined (a) in series, (b) in parallel. (Pat. 1941)

24. Prove the law of parallel resistances. How would you verify this experimentally. (Pat. 1947)

Ten cells, each of E. M. F. 1·5 volts and internal resistance 0·3 ohm, are joined in series to form a battery. The terminals of the battery are joined by resistances of 20 ohms and 30 ohms in parallel. Find the current passing through each resistance.

[*Ans* : $C_{20} = 0·6$ amp. ; $C_{30} = 0·4$ amp.]

Art. 35.

25. A galvanometer of 40 ohms resistance is shunted by a shunt of 5 ohms. Find the combined resistance of the shunt and the galvanometer. Find also the current which flows through the galvanometer when a battery of 20 volts and an external resistance of 10 ohms are connected in series.

[*Ans* : $R = \frac{40}{9}$ ohms ; $C_g = \frac{2}{13}$ amp.] (Dac. 1933)

26. A circuit contains a shunted galvanometer. If C be the current in the main circuit, G the resistance of the galvanometer, and S that of the shunt, prove that the current through the galvanometer, $C_g = \dfrac{S}{S+G}C$.

A certain galvanometer of resistance 10 ohms is shunted with a resistance of 2 ohms and the current through the galvanometer is 0·1 amp. What additional shunt must be applied to the instrument so that the current through it may be 0·01 amp., the main current remaining constant. (Pat. 1937)

[*Ans* : 0·185 ohm.]

27. If a shunt is to be applied to a galvanometer of resistance 20 ohms so that only 1% of the total current passes through the galvanometer, what must be the resistance of the shunt ? (Pat. 1930)

[*Ans* : $\frac{20}{99}$ ohm.]

28. Explain the use of shunts. What is the resistance of a shunt which when joined to a galvanometer of resistance 'g' will cause $1/n$ of the total current to flow through the galvanometer ? (Pat. 1944)

[*Ans* : $g/(n-1)$ ohm.]

Art. 39.

29. Give the theory of the Wheatstone's bridge method of measuring resistances. (C. U. 1909, '12, '26 ; Pat. 1918, '20, '34 ; All. '29 ;U. P. B. 1948)

Explain how will you use it to find the specific resistance of a wire. (See Art. 41). (All. 1929, '45 ; U. P. B. 1948 ; Cf. '22, '26 ; Pat. '40)

30. If the resistance of a wire of length 120 cms. and diameter 0·4 mm. is found to be 2·5 ohms, what is the specific resistance of the material ?

[*Ans :* $\rho = 26{\cdot}2 \times 10^{-6}$ ohm-cm. (approx.)] (Pat. 1920)

31. Describe one form of Wheatstone's bridge and prove the formula employed when using the bridge to measure a resistance. (Cf. Pat. 1945, '46)

Art. 40.

32. Discuss the theory and describe the practical arrangement of comparing two resistances by a Metre-bridge.

(C. U. 1927, '45 ; All. '21, '25 : Dac. 1922)

Art. 41.

33. Describe the construction of the Post Office Box and explain the principle upon which its working depends. Draw a diagram, showing the connections in an experiment with it. (Pat. 1929 ; cf. All. 1916 ; C. U. 1938)

Art. 42.

34. What is an ammeter ? How is it used ? How does it differ from a voltmeter ? (All. 1924 ; Cf. '29, '44)

35. Give a brief account of (a) a voltmeter, (b) an ammeter.

(C. U. 1932 ; Pat. '44)

Art. 43.

36. How do you convert a 100-ohm galvanometer, reading up to 120 micro-amperes, into a voltmeter reading up to 2·4 volts ? (L. U.)

[*Ans :* Adding resistance = 19,9000 ohms].

CHAPTER V

Thermal Effects of Electric Currents :

Thermo-Electricity

46. **Work done by a current : Joule's Law.**—When a current flows through a wire of some appreciable resistance, the wire is heated. The heating is due to work done in overcoming the resistance to the flow of electricity. So the heat is produced at the expense of electrical energy. It has already been stated (Art. 28) that unit work is done in carrying unit quantity of electricity between two points differing in potential by unity. Suppose a wire of resistance r

is to be traversed by a current of strength C. Let E be the potential difference between the two ends of the wire and Q units of electricity conveyed through it in time t seconds (both being measured in C.G.S. electro-magnetic units) ; then the work done,

$W = EQ$ ergs $= ECt$ ergs ($\because$ $Q = Ct$) $= C^2rt$ ergs ($\because$ $E = Cr$, by Ohm's Law).

Here, C and r are expressed in C. G. S. units. If these are expressed in practrical units, then C amperes $= C \times 10^{-1}$ C. G. S. units, and r ohms $= r \times 10^9$ C. G. S. units.

$$\therefore \quad W = (C \times 10^{-1})^2 \times r \times 10^9 \times t = C^2rt \times 10^7 \text{ ergs.}$$

If H be the amount of heat generated, and J the mechanical equivalent of heat,

$$\mathbf{W = JH}$$

$\therefore \quad H = \dfrac{W}{J} = \dfrac{C^2rt \times 10^7}{J} = \dfrac{C^2rt \times 10^7}{4{\cdot}2 \times 10^7}$ calories ($\because$ $J = 4{\cdot}2 \times 10^7$ ergs per calorie) $= \dfrac{\mathbf{C^2rt}}{\mathbf{4{\cdot}2}}$ calories $= 0{\cdot}24\ C^2rt$ calories.

From the above expression the following law, known *Joule's Law*, can be derived. The law was first established by Dr. Joule, of Manchester, in 1841.

Joule's Law of Generation of Heat.—The law states that,

(*i*) The amount of heat generated in a conductor in a given interval of time is proportional to the square of the current to which it is due,

i.e. $H \propto C^2$, when r and t are constant.

(*ii*) The amount of heat generated by a given current in an interval of time is proportional to the resistance of the conductor,

i.e. $H \propto r$, when C and t are constant.

(*iii*) The amount of heat generated in a given conductor by a given current is proportional to the time for which the current passes,

i.e. $H \propto t$, when C and r are constant.

[**Note.**—The heating effect of electric current is used for lighting purposes in incandescent-filament lamps, arc lamps, etc. (see Art. 72)].

47. Determination of J.—The value of the mechanical equivalent J can be determined by electrical method in the following way :—

Expt.—A calorimeter with a stirrer (S) is taken (Fig. 64). The calorimeter contains a quantity of some liquid in which is dipped a coil of fine manganin wire W (say, of resistance r), the two ends of which are connected to the two terminal binding screws B and C, which are fixed on an insulating lid placed on the calorimeter. The circuit contains a battery D, a rheostat R, an ammeter A, and a plug key K in series with the heating coil W, and the connections are made as in Fig. 64. A sensitive thermometer T, the bulb of which should be immersed in the liquid is introduced in the calorimeter. The voltmeter V reads the potential difference between the ends of the coil W.

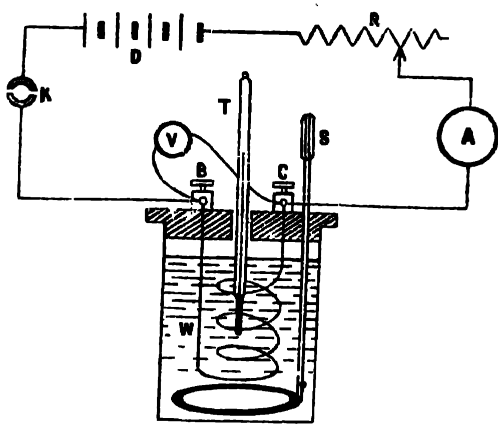


Fig. 64

The initial temperature of the liquid is read. Then the current is started, the strength of which is adjusted by means of the rheostat, and its value C is read from the ammeter. The current is allowed to pass through the circuit for a known interval of time t. The liquid is stirred and the final temperature of the calorimeter and liquid noted.

Calculation.—Let m = mass of liquid of sp.-heat S; θ_1 = initial temperature of water; θ_2 = final temperature of water; w = water equivalent of the calorimeter and stirrer; then,

The amount of heat developed $H = (mS \times w)(\theta_2 - \theta_1)$

We have, $JH = C^2rt \times 10^7$; $\therefore$ $J(ms + w)(\theta_2 - \theta_1) = C^2rt \times 10^7$;

$$or, \quad J = \frac{C^2rt \times 10^7}{(ms + w)(\theta_2 - \theta_1)}.$$

48. Experimental Verification of Joule's Law.—(*i*) *The amount of heat generated in a conductor* (resistance r) *in a given time is directly proportional to the square of the strength of the current flowing in it, i.e.* $H \propto C^2$, *when* r *and* t *are constants.*—Take the same apparatus as shown in Fig. 64. Put a known quantity of some oil in the calorimeter; note its temperature, and pass a current strring the oil all the time. Take the rise in temperature θ_1°$C.$, after passing the current for, say, 10 minutes, and read the ammeter to know the strength of the current.

Let it be C_1. Now increase the strength of the current to C_2 by suitable adjustment of the resistance R, and note the rise in temperature $\theta_2{}^\circ C.$, after an equal interval of time as before (*i.e.* 10 minutes). It will be found that $\frac{\theta_1}{\theta_2}=\frac{C_1{}^2}{C_2{}^2}$.

But as the mass and the water-equivalent of the calorimeter and its contents remain the same, the amount of heat produced is proportional to the rise in temperature, which again will be proportional to the square of the current strength as shown above. For example, if the strength of current is doubled (*i.e.* $C_2=2C_1$), the rise in temperature, and hence the amount of heat produced, will be 4 times the previous values, (*i e.* $\theta_2=4\theta_1$) Thus it is verified that the amount of heat produced, $\mathbf{H \propto C^2}$.

(*ii*) *The amount of heat generated by a given current in a given interval of time is directly proportional to the resistance of the conductor, i.e.* $H \propto r$ *where* C *and* t *are constants.*—

Take two coils having different resistances r_1 and r_2. First perform the above experiment with one of the coils having resistance r_1 in the liquid of the calorimeter, and note the rise in temperature. Now replace the coil by the second coil of resistance r_2 and proceed as before keeping the strength of the current constant by means of the rheostat R. If θ_1 and θ_2 be the respective rise in temperature of water in the above cases, it will be found that $\frac{\theta_1}{\theta_2}=\frac{r_1}{r_2}$. But $H \propto \theta$. So, $\mathbf{H \propto r}$.

(*iii*) *The amount of heat generated by a given current in a given conductor is directly proportional to the time during which the current flows ; i.e.* $H \propto t$, *where* C *and* r *are constants.*

Take the temperature of the oil at equal intervals of time, say every minute, in experiment (*i*) above. It will be seen that the temperature of the water rises uniformly, *i.e* by equal amounts in equal intervals of time. This verifies that, $\mathbf{H \propto t}$.

49. Effect of Current on the Resistance of a Conductor—The effect of current on the resistance may be better explained by taking an illustration. Suppose, we have a chain made of alternate links of silver and platinum wires of the same diameter. When a current from a battery is sent through the chain, the platinum wires will glow brightly while the silver wires will remain dull. The same current passes through each wire, but the specific resistance of platinum being six times that of silver, the heat developed in the platinum wires is greater than that in the silver. The fact that the thermal capacity of platinum is less than that of silver also increases the effect.

Again, if a current from a battery is sent through a long piece of platinum wire so that the wire glows feebly, and if a part of the wire is dipped in a beaker of cold water, then the rest of the wire will glow brightly. This is due to the fact that the resistance of a metal wire decreases when cold, so the cold part of the wire reduces the total resistance of the circuit, which again increases the current and thus produces more heat.

The thinner the wire the hotter it becomes when a given current passes through it, because its resistance is greater, and the surface from which heat can be radiated is smaller in this case in comparison with a thicker wire.

The fact that, when a current passes through a resistance, the *heat* produced (and the loss of energy) is proportional to the *square* of the strength of the current, has to be carefully considered when transmitting electric power. Thick, and therefore costly, conductors are needed in order to avoid fusing of the wire due to the heat, which represents a wastage of energy. So it is much cheaper to transmit a small current at a high voltage than a large current at a low voltage (*vide* Ch. VIII).

Examples.—*1. The ends of a coil of 50 ohms resistance are connected to an voltmeter reading 0·5 volt to a division. This coil is immersed in an oil of specific heat 0·2. The temperature of the oil rises through 20° when a current passes through it for 15 minutes. Calculate the mass of the oil. Voltmeter reads 12 while the current passes. The vessel in which the oil is contained is of negligible heat capacity.* *(Pat. 1927)*

We know, $H \propto C^2rt$ (C and r are expressed in C.G.S. units).

or $JH = C^2rt$, where J = Joule's equivalent, and is equal to $4{\cdot}2 \times 10^7$ ergs.

If C and r are expressed in practical units,

$$H = \frac{C^2 \times 10^{-2} \times r \times 10^9 \times t}{4{\cdot}2 \times 10^7} = \frac{C^2rt}{4{\cdot}2} \text{ cal.}$$

If m be the mass of the oil, the amount of heat produced,

$$H = m \times 0{\cdot}2 \times 20 \text{ cal.}$$

The potential difference of the two ends of the wire is $12 \times 0{\cdot}5 = 6$ volts.

$\therefore$ Current $= \frac{6}{50} = \frac{3}{25}$ amp. Hence $m \times 0{\cdot}2 \times 20 = \frac{(\frac{3}{25})^2 \times 50 \times 15 \times 60}{4{\cdot}2}$;

whence $m = 38{\cdot}57$ grams.

2. An electric copper kettle holding 1800 gm. of water weighs 1000 gm. If a current of 5 amps. at 200 volts pass through the kettle, calculate the time taken by water to reach boiling point from 20°C. (Sp. ht. of copper is 0·1; $J = 12 \times 40^7$ ergs.).

We have $JH = ECt$ ergs. (Art. 46)

$$\therefore \quad H = \frac{ECt}{J} = \frac{E \times 10^8 \times C \times 10^{-1} \times t}{4{\cdot}2 \times 10^7} \text{ cals.} = \frac{ECt}{4{\cdot}2} \text{ cals.}$$

Here $H = (1000 \times 0{\cdot}1 + 1800)\ (100 - 20) = 1900 \times 80 = 152{,}000$ cals.

$$\therefore \quad t = \frac{4{\cdot}2 \times 152{,}000}{EC} = \frac{4{\cdot}2 \times 152{,}000}{200 \times 5} = 10 \text{ min. } 38{\cdot}4 \text{ seconds.}$$

3. *ABCD is a network of resistances 5, 8, 7 and 16 ohms taken in order. A battery of 29·9 volts is connected between A and C. Find simple numbers which will represent the heat generated in AB, BC, AD, and DC.* (*Pat. 1932*)

Draw the diagram. The resistances AB, BC, CD and DA are 5, 8, 7 and 16 respectively. When the battery is joined between A and C, the resistance of the branch $ADC = (16 + 7)$ or 23 ohms, and that of the parallel branch $ABC = (5 + 8)$ or 13 ohms. If C_1 be the current flowing in the branch ABC, $C_1 = 29{\cdot}9/13$ ($\because$ the P. D. of A and $C = 29{\cdot}9$ volts) $= 2{\cdot}3$ amp. and if C_2 be the current flowing in the branch ADC; $C_2 = 29{\cdot}9/23 = 1{\cdot}3$ amp.

If H be the amount of heat generated by a current C flowing in any conductor of resistance r for time t secs., $H = 0{\cdot}24 \times C^2rt$ calories (Art. 46).

$\therefore$ If H_1, H_2, H_3, and H_4 be the number of heat units generated per sec. in AB, BC, AD, and DC respectively, we have,

$H_1 = 0{\cdot}24 \times (2{\cdot}3)^2 \times 5 = 6{\cdot}348$ calories. $H_2 = 0{\cdot}24 \times (2{\cdot}3)^2 \times 8 = 10{\cdot}1568$ calories. $H_3 = 0{\cdot}24 \times (1{\cdot}3)^2 \times 7 = 2{\cdot}8392$ calories. $H_4 = 0{\cdot}24 \times (1{\cdot}3)^2 \times 16 = 6{\cdot}4896$ calories.

50. Electrical Work : Energy : Power.—

Electrical Work.—Electrical work is said to be done when a charge flows under a potential difference. When unit charge flows under a P.D. of E, the work done is E. Therefore, when Q charge flows under a P.D. of E, the work $W = Q \times E$ (C.G.S. units) $=$ charge $\times$ pot. diff.

The practical unit of work or energy is the **Joule**, *which is the work done when* **1 coulomb** *is moved through a potential difference of* **1 volt**.

$\therefore$ 1 Joule $= 10^{-1} \times 10^8 = 10^7$ C.G.S. units of work (or ergs).

$\therefore$ $J = 4{\cdot}2$ joules per calorie.

Power.—*Power* is the **rate** at which an agent does work, *i.e.* it is the work done by an agent per unit time.

That is, the power $P = \dfrac{Q \times E}{t} = CE$; where $C (= Q/t)$ is the current strength, and E the potential difference. The practical unit of power is known as the **watt**, *which is the power expended when one* **ampere** *of current flows under a P.D. of one* **volt**. Hence,

Power (in watts) = Volts × amperes

1 watt = 1 amp. × 1 volt per sec. $= 10^{-1} \times 10^8$
$= 10^7$ **ergs. per sec. = 1 joule per sec.**

1 Kilowatt = 1000 watts.

The energy expended in an electric circuit at the rate of one kilowatt for one hour is known as one **kilowatt-hour (K. W. H.)** of energy. It is the Board of Trade Unit (**B. T. U.**) by which the consumption of electrical energy is measured and charged by the Electric supply companies.

1 K W.H. = 1000 watts × 60 × 60 secs. = **3,600,000 joules** = **36 × 10^5 joules**

51. Relation between Horse-Power and Watts.—The Engineering unit for power is called one Horse-power.

1 H. P. = 33,000 ft.-lbs. per minute = 550 ft.-lbs. per second.
= 550 × 30·48 × 453·6 × 981 ergs per sec.
= 746 × 10^7 ergs. per sec. = 746 joules per. sec.
= **746 watts** ($\because$ 1 watt = 1 joule per sec.) = 0·746 kilowatt.

Examples.—*1. Calculate the time required to boil a litre of water which is at 25·4°C., the available energy being at the rate of 1 H. P.*

Heat required to raise the temperature of 1 litre (1000 c.c.) of water through (100 − 25·4)°, or 74·6°C. = (1000 × 74·6) = 74600 calories.

But 1 calorie is equivalent to 4·2 × 10^7 ergs *or* 4·2 joules.

$\therefore$ The electrical energy equivalent to 74600 calories is 746000 × 4·2 joules.

The rate of available energy = 1 H. P. = 746 joules per sec.

$$\therefore \quad \text{Time required} = \frac{74600 \times 4{\cdot}2}{746} \text{ seconds} = 7 \text{ minutes.}$$

2. A generator produces current at 200 volts, but owing to the resistance in the cable leads, the available voltage in a station is only 210 volts, where a motor is working on a current of 20 amperes. Find (a) the power of the generator in (i) K. W., (ii) H. P.; (b) the power taken by the motor in K. W.; (c) the energy wasted in the leads; (d) the resistance of the leads.

(*a*) Power in watts = volts × amperes.

$\therefore$ Power of the generator = 220 × 20 watts = (220 × 20)/1000 K. W.
= 4·4 K. W.; and (220 × 20)/746 H. P. = 5·9 H. P. (approx.)

(*b*) Power taken by motor = (220 × 20)/1000 = 4·2 K. W.

(*c*) Energy wasted in leads = (220 − 210) volts × 20 amp. = 200 watts.

$$(d) \quad \text{Resistance of leads} = \frac{\text{P.D. between ends}}{\text{current}} = \frac{220-210}{20} = \frac{10}{20} = 0{\cdot}5 \text{ ohm.}$$

3. The lighting of a village requires 40 amps at 200 volts. This is supplied by a dynamo in a distant town, developing a P D. of 220 volts. Find, (i) the resistance of the mains from town to village; (ii) the number of B.T. Units consumed in 10 hours in the village: (iii) the number of B. T. Units wasted in the mains in the same time. (*U. P. B. 1948*)

(*i*) Fall of P.D. in the mains = 220 − 200 = 20 volts = resistance of the mains × current in the mains = resistance × 40. $\therefore$ Resistance of the mains = 20/40 = 0·5 ohm.

(*ii*) B. T. units consumed in 10 hours $= \frac{200 \times 40 \times 10}{1000} = 80$ K.W.H.

(*iii*) B. T. units wasted in the mains $= \frac{(220-200) \times 40 \times 10}{1000} = 8$ K.W.H.

(For applications of Heating Effect of Electric Current see Ch. VIII.).

Thermo-Electricity

52. Seebeck Effect : Thermo-couple.—In 1821 Seebeck observed that if two wires of different metals were soldered together at their ends so as to form a closed circuit, then, on heating one of the junctions, a current flowed round the circuit. This phenomenon is known as the **Seebeck effect.** The current is called **thermo-current**, and the pair of metals combined in this way is called a **thermo-couple.** Experimenting with various metals Seebeck arranged several metals in a series, called the *thermo-electric series* or **Seebeck series** (given below), such that when any pair of them is used, the current flowed through the cold junction from the metal which occurs earlier in the series to that which occurs later in the series. Thus, in an iron-copper couple (see Fig. 65), the current flows from iron to copper through the cold junction, the position of copper in the Seebeck-series being earlier than that of iron.

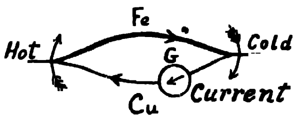
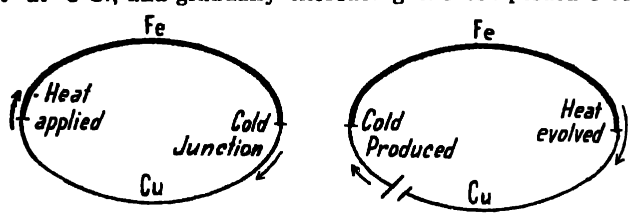


Fig. 65—Thermo-couple

Thermo-Electric Series

(1) Antimony ;	(3) Silver ;	(5) Copper
(2) Iron ;	(4) Gold ;	(6) Bismuth.

It has been found experimentally that the *E. M. F.* produced in the circuit increases as the difference of temperature between the two junctions is increased. Keeping the temperature of the cold junction constant at 0°C., and gradually increasing the temperature of the hot



(a) Fig. 66 (b)

junction, the *E. M. F.* attains a ***maximum value*** at a certain temperature after which the *E. M. F.* diminishes and gradually becomes zero,

although the temperature continues to rise. With further rise of temperature the *E. M. F.* is *reversed*, and the current begins to flow in the opposite direction. *The temperature of the hot junction at which the E. M. F. becomes a maximum is called the* **Neutral temperature.** It is changed if the pair forming the couple is changed.

Peltier Effect—In 1834 Peltier found that when an electric current is made to pass through the junction of two dissimilar metals, then the junction is either heated or cooled, according to the direction in which the current flows. This is known as the **Peltier effect**, which is the converse of the Seebeck effect.

The rule about evolution and absorption of heat is that the junction, which on applying heat sends the thermo-electric current in a certain direction, absorbs heat when a current is sent through it in the same direction from a cell. Sending the current in the opposite direction evolves heat at the junction. Fig. 66 (a) illustrates an *Cu-Fe* couple in which, on application of heat at a junction, current flows from *Cu* to *Fe* across that junction. Fig. 66 (b) shows that in the same couple, on sending a current from a battery from *Cu* to *Fe*, heat is absorbed at that junction resulting in a fall of temperature.

This phenomenon has been explained by supposing that there exists an *E.M.F.* (called the Peltier *E.M.F.*) at the junction of two dissimilar metals. The magnitude of this *E.M.F.* is proportional to the temperature of the junction, and its direction for the two metals is fixed. Thus in a copper-iron couple the *E. M. F.* is directed from copper to iron. That is why, when a current is passed through this couple, cold is produced at the junction where the current passes from copper to iron, *i.e.* in the direction of the Peltier *E.M.F.* and heat is evolved at the other junction when the current passes in opposition to the Peltier *E.M.F.*

Peltier Effect and Joule's Effect Compared.—The heat produced due to the Peltier effect should not be confused with the ordinary heating effect (Joule's effect) of a current. The ordinary effect (Joule's effect) depends upon the resistance of the metal and is independent of the direction of the current, whereas the Peltier effect depends upon the direction of the current. The Joule's effect is proportional to the square of the current (C^2), whereas the Peltier effect is proportional to the current (C). Thus, the **Peltier effect** is an illustration of a **reversible** process between heat and electrical energy, whereas the **Joule's effect** is an illustration of an **irreversible** process.

Thomson Effect.—When a current is passed from a battery through a thermo-couple, heat is evolved or absorbed not only at the

two junctions (Peltier effect) but also along the two elements forming the couple. On reversing the direction of the current, the effects are also reversed. This remarkable effect was discovered by Sir William Thomson (Lord Kelvin), and known as the **Thomson effect.**

This has been explained by supposing that there exists an *E. M. F.* in a *conductor* whose ends are at different temperatures. In some metals the direction of the *E. M. F.* is from the hot to the cold end, and in some other metals the direction is the opposite. Thus, in iron there is an *E. M. F.* acting from the parts of higher to those of lower temperature, while in copper the *E. M. F.* acts from the parts of lower to those of higher temperature. Due to the existence of this *E. M. F.*, heat will be evolved or absorbed in the conductor depending on the direction of the current.

53. **Thermo-pile.**—The instrument originally constructed by *Nobile* comprises a number of small rods of two dissimilar rods, usually *Antimony* and *Bismuth* alternately joined in series in a zigzag way as shown in Fig. 67. Each of the junctions is well soldered while the other parts of the rods are carefully insulated. The two free ends of the *series* is connected to two binding screws between which a sensitive galvanometer is placed.

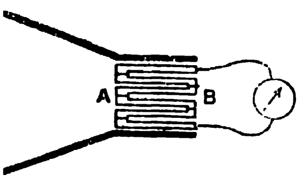


Fig. 67—Thermo-pile

The combination, therefore, is a number of thermo-couples joined in series in order to increase the total *E.M.F.* like that of a series battery of cells Moreover, the combination is usually made in the form of a cube where the alternate junctions of the series are on the opposite faces *A* and *B* of the cube. The front face *A* which is turned towards a source of heat is coated with *lamp-black* to absorb radiant heat as readily as possible. The cube, ordinarily called a **thermo-pile,** is enclosed in a brass casing having a conical mouth-piece around the front face *A* by which *radiant heat* is collected. Antimony and Bismuth are used in order that for a given difference of temperature between the two faces of the pile a comparatively large *E. M F.* may be obtained.

The instrument is very sensitive to even a small difference of temperature between its two faces. It is used to detect *radiant heat.* It is employed for the measurement of solar radiation, and the comparison of the distribution of heat energy in a spectrum. It can be calibrated by *check observations* to record directly the difference of temperatures between the two faces of the pile.

Questions

Art. 46.

1. A conductor carrying a current divides into two branches whose resistances are in the ratio of 4 to 5. Compare the amounts of heat generated in the branches. (C. U. 1931)

[*Ans :* $H_1 : H_2 : 5 : 4$]

2. How does the rate at which heat is developed by an electric current depend on the current strength and the resistance of the conductor ? (Pat. 1929)

3. Obtain an expression for the quantity of heat generated in a conductor of resistance 'R' when a current 'C' passes through it for 't' seconds. (Pat. 1932, '39, '42 ; Cf. C. U. '42, '44 ; Utkal 1948)

3(a). Deduce an expression, in practical units, for the development of heat per second in a conductor carrying a current. (U. P. B. 1947)

4. In what respects does a wire carrying a current differ from a wire which carries no current ?

How can the effects so produced be utilised for the measurement of current ? (All. 1917)

[**Hints.**—Differs in two respects :—when a current passes through a wire (*i*) a magnetic field is set up round the wire (see Art. 11) ; (*ii*) heat is developed in the wire (see Art. 46).

For the measurement of current, magnetic effects are utilised in instruments known as ammeters, and the heating effects are utilised as noted in Art. 46, where C is calculated from the relation $JH = C^2 rt \times 10^7$].

5. Compare the amounts of heat developed in the four arms of a balanced Wheatstone's bridge when the resistances of the arms are 100,10,300,30.

[*Ans :* 30 : 3 : 10 : 1] (All. 1931)

6. A coil of wire of resistance 2 ohms is soldered to two thick copper rods immersed in 1000 grams of oil (specific heat 0·6). A current of strength 3 amperes is passed for 30 minutes. Neglecting the water equivalent of the calorimeter, loss of heat by radiation, etc., find the rise in temperature of the oil. A current of 1 ampere passing through a resistance of 1 ohm for 1 second generates 0·2387 calories. (C. U. 1909)

[*Ans :* 12·89 approx.]

7. A 220-volts, 500-watt aluminium electric kettle weighing 1,200 gm. is used to boil water from a 220-volt electric supply. How long will it take to bring 1 kgm. of water from 30°C. to the boiling point, assuming all the heat generated to be utilised without loss due to radiation ? (Sp. ht. of Al. = 0·2)

[*Ans :* 12 min. approx.] (Pat. 1937)

8. A constant current of one ampere flows in a platinum wire of resistance 5 ohms, stretched along the axis of a cylindrical tube, through which a steady stream of water passes at the rate of 15 c.c. per minute. The steady

difference between the temperatures of the water entering and leaving is 4·8°C. Neglecting losses of heat, calculate a value of the mechanical equivalent of heat. (C. U. 1937)

[*Ans :* $4{\cdot}16 \times 10^7$ ergs.

9. An electric kettle whose water equivalent is 100 gm. contains 1000 gm. of water at 15°C. If the kettle takes 4 amp. at 230 volts (*a*) find the rate of generation of heat in it, and (*b*) the time required to boil the water assuming that 10 per cent. of the heat generated is lost.

[*Ans* : (*a*) 220 calories per sec. ; (*b*) 7 min. 52 sec.]

10. A current of 5 amperes flows through a wire of resistance 10 ohms for 2 minutes. If the heat produced is exclusively supplied to 100 gm. of water, through how many degrees will the temperature be raised ? (C. U. 1941)

[*Ans* : 70°C.]

Art 47.

10(a). Describe the electric method of determining "J". (All. '1946)

Art. 48.

11. State Joule's Law regarding the development of heat in an electrical circuit. Describe an experiment to verify it. (C. U. 1927, '30, '45 ; Pat. '28, '40, '43, '48 ; All. '31, '44, '46 ; Dac. 1934 ; Cf. U. P. B. 1947).

Art. 49.

12. Two equally long copper and silver wires are suspended from two supports and are so connected that a current travelling along the copper wire returns along the silver. It is found that the copper wire sags. The current is now caused to flow along both the wires in parallel, and the silver wire sags. Explain fully the above effects. (Pat. 19?9)

Hints :—$H \propto C^2 rt.$; The specific resistance of copper being greater than that of silver, the resistance of the copper is greater than that of the same length of the silver wire. In the first case, C is the same in both the wires. ∴ $H \propto r$, hence the heat produced in the copper wire is greater, and so it sags. In the next case, the current is distributed inversely as the resistance, so current in the silver wire is greater, and because $H \propto C^2$, the heat produced is greater in the silver wire. Therefore it sags].

13. A copper wire and an iron wire are connected to an accumulator first in parallel and then in series. In the first case the copper wire gets red hot and in the second case the iron wire. Explain the facts and show how to compare the resistances from the rates at which the heat is developed in each case. (Pat. 1926, '38 ; All. '24)

Hints.—(*a*) See answer to *Q.* 12. Here the specific resistance of iron is greater than that of copper.

(*b*) If r_1 and r_2 be the resistances of the iron and the copper wires respectively, their total resistance R_1. when in parallel, $= r_1r_2/(r_1+r_2)$ and R_2, when in series, $= r_1 + r_2$. ∴ $\dfrac{R_1}{R_2} = \dfrac{r_1 r_2}{(r_1+r_2)^2}$. Then if C_1 and C_2 be the

currents (neglecting the battery resistance) when the wires are joined in parallel and series respectively, $C_1R_1 = C_2R_2 =$ P.D. of the cell.

$\therefore \dfrac{C_1}{C_2} = \dfrac{R_2}{R_1} = \dfrac{(r_1+r_2)^2}{r_1r_2}$. Hence if H_1 and H_2 be the rates at which heat is developed, $\dfrac{H_1}{H_2} = \dfrac{C_1{}^2R_1}{C_2{}^2R_2} = \dfrac{(r_1+r_2)^4}{(r_1r_2)^2} \times \dfrac{r_1r_2}{(r_1+r_2)^2} = \dfrac{(r_1+r_2)^2}{r_1r_2} = \dfrac{R_2}{R_1}$. Thus the rates at which the heat is developed is inversely proportional to the combined resistances of the two wires.]

Art. 50.

14. An incandescent lamp with carbon filament works at 2·5 watts under a voltage of 20 volts. What is the resistance of the lamp ? (C. U. 1927).

[*Ans* : 160 ohms.]

14(a). State Joule's law on the production of heat in an electric circuit.

An electric iron, which when hot has a resistance of 80 ohms, is used on a 200 volt circuit. What will be the cost of using it for 2 hours if energy costs 3 as per K. W. H ? (C. U. 1947).

[*Ans* : 3 as.]

15. Explain the terms Watt, Board of Trade Unit, Efficiency of a lamp. (All. 1932 ; U. P. B. 1948.)

Calculate the amount of heat produced in 5 min. in a 20 watt lamp. (All. 1928.)

[**Hints.**—$H = (20 \times 5 \times 60/4{\cdot}2$ cal.]

16. Define and compare (*a*) coulombs and amperes, (*b*) joules and watts. State how joule is related to the volt and ampere.

Describe an experiment which illustrates the statement that one calorie equals 4·2 joules. (See also Art. 47.) (C. U. 1946)

Art. 52.

17. What is a thermo-electric current ? Describe how you would demonstrate its presence. How do you explain the existence of such a current from the principle of conservation of energy ? (C. U. 1934)

[**Hints.**—As the current is obtained at the expense of heat energy, the existence of thermo-current follows the principle of conservation of energy.]

18. What is a thermo-couple ? How will you demonstrate currents generated in such a couple ? Give a diagram of the arrangement to be used by you. (All. 1928 ; C. U. '45.)

Art. 53.

18. Give a brief account of a thermo-pile. (C. U. 1932 ; All. '25).

CHAPTER VI

Chemical Effects of Electric Currents

54. **Electrolysis.**—When an electric current flows along a wire, molecules or atoms of the wire do not move from their places, but when a current flows through a solution of a salt or an acid, it decomposes it, and the two decomposed portions tend to move in *opposite* directions. This process of decomposition is known as **electrolysis**, and the liquid which undergoes decomposition is known as an **electrolyte**. The terminals by which the current enters and leaves the electrolyte are called **electrodes**,—that by which the current enters the electrolyte is called the **anode**—(from *ana* = up, and *hodos* = a way), and that by which the current leaves is called the **kathode** (from *kata* = down, *hodos* = a way), also written as *cathode*. The liberated elements, or groups of elements, which move in opposite directions and are given off at the electrodes, are called **ions** (or *wanderers*), those liberated at the anode are called **anions**, and those at the kathode are called **kations**. The vessel which contains the electrolyte is called an *electrolytic cell* or a **voltameter**.

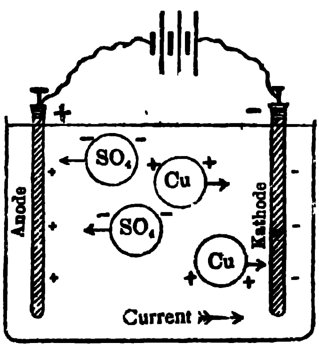


Fig. 68 —Migration of Ions

Charge carried by an Ion :—Electrolysis is explained by supposing according to Arrhenius, that the molecules of the electrolyte break up, by the very act of solution, into two parts, called the *ions*. These ions are oppositely charged with equal electric charges attached to them. When a potential difference is set up between the electrodes by means of an external source of *E M.F*. the positively charged ions (kations) move towards the negative electrode (or kathode) and are **electro-positive** in character. Similarly, negatively charged ions (canions) move towards the anode, and so the anions are **electro-negative** (Fig. 68). The movement of the ions in opposite directions between the two electrodes constitutes the current.

It has been found that hydrogen and the metals are electro-positive in character. The electric charge carried by the ion of a divalent element, such as copper, is twice as great as the charge carried by the ion of a monovalent element, such as hydrogen or silver, and are represented thus (Cu^{++}), (H^+), etc.

All solutions of salts, acids, and bases, in water are decomposed by the passage of an electric current. Liquids, such as turpentine and some oils, which do not allow electricity to pass through them, cannot be decomposed ; while there are certain other liquids (including *mercury*) which conduct electricity, but are *not decomposed* by it.

55. (a) **Copper Voltameter.**—When copper sulphate is dissolved in water, some of the molecules are broken up or ionised into Cu^{++} and SO_4^{--} ions. So when an *E. M. F.* is applied between two electrodes set up in the solution, the postiively charged copper ions move to the kathode and are deposited on the plate, while the negatively charged (SO_4) ions go to the anode where acting on the copper of the anode form copper sulphate which passes into the solution to replace that which has been used up. Here the copper taken from the anode is equal in amount to the copper deposited on the kathode, and the rate of deposition is a measure of the strength of the current. Hence a copper voltameter is often used as a current-measurer (see Art. 59.)

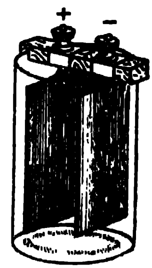


Fig. 69—Copper Voltameter

Such an arrangement is called a *copper voltameter.* The measurement of any given current requires considerable time, and this voltameter is, therefore, chiefly used in the standardisation and calibration of ammeters and tangent galvanometers.

A **copper voltameter** (Fig. 69) consists of a glass vessel containing a 15 per cent. solution of copper sulphate (crystals), to each litre of which 5 c.c. of concentrated sulphuric acid have been added, in which two copper plates connected to two separate binding screws are dipped from a non-conducting support placed on the top of the vessel. In an improved form of the copper voltameter, the anode consists of two plates of copper, electrically connected together, and the kathode is placed between the two anode plates, but not touching them as shownin Fig 69(a), where the anode is in the form of two thin identical copper plates *A*, *A* hanging one on either side of a similar copper plate *K* which serves as the kathode. A bent brass or copper rod *DEFG*, on which the plates *A*, *A* are supported electrically, connects them, while another brass or copper rod *JL*, which is electrically insulated from the bent rod *DEFG*, supports the kathode. With this arrangement the deposit occurs on both sides of the kathode and thus the maximum area is utilised. For

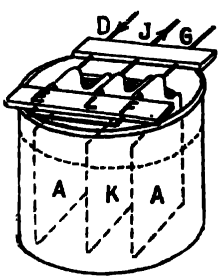


Fig. 69(a)

good deposition the area of the kathode dipped in the solution should be such as to allow **50 sq. cm. of surface for each ampere of current**.

In the case of electrolysis of copper sulphate solution, the kathode will always be coated with the copper, but what happens at the anode depends upon the material of which the electrodes are made. If the anode is of some *inert material* like carbon or platinum, the SO_4 ions will give up their charge at the anode and will act upon the water of the solution, forming sulphuric acid and liberating oxygen : thus,

$$2H_2O + 2SO_4 = 2H_2SO_4 + O_2.$$

This oxygen is liberated at the anode. This action is similar to the case of the electrolysis of water acidulated with sulphuric acid as described under "Water Voltameter" in (c).

(b) **Silver Voltameter.**—Similarly, in a voltameter with Ag-electrodes and $AgNO_3$ solution, a certain amount of silver is deposited on the kathode, and (NO_3) ions attack the anode, forming $AgNO_3$. The kathode, in this case, very often consists of a silver, or a platinum cup, in which the solution is kept. Such a voltameter is used whenever more accurate determination of current, than is possible with other equipments, is necessary. The maximum limit of a current through it is about 0·3 ampere for every 50 sq. cms. surface of the kathode.

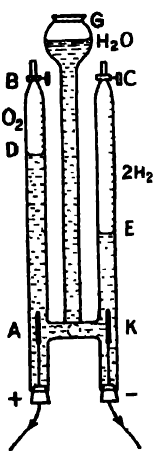


Fig. 69 (b)

(c) **Water Voltameter : Electrolysis of Water or Dilute Sulphuric Acid.**—Pure water is not a good conductor of electricity and so for electrolysis of water a dilute solution of sulphuric acid (5 c.c. of strong acid in 1000 c.c. of water) is used.

The cell in which the acidulated water is taken is called a **water voltameter**, and it consists of two vertical graduated tubes AB and KC, united at their bases by the short piece AK; the gases formed on the electrodes may be caught in the tubes and measured. It will be found that the gas evolved at the kathode K has twice the volume of that evolved at the anode in the same time. On opening the top C the gas in CE escapes, and if a light be applied, the escaping gas, burns showing it to be hydrogen. The gas in BD as it escapes will ignite a glowing taper or splinter of wood showing that it is oxygen. *Thus hydrogen*

is evolved at the kathode, oxygen at the anode, and the two are in the proportion in which they combine to form water.

Theory.—The diluted acid dissociates as follows: $H_2SO_4 = 2H^+ + SO_4^{-}$. The hydrogen ion, on application of the electric field between the two electrodes, migrates to the kathode and giving up its charge to it escapes as neutral hydrogen. The suphion radical moves to the anode and delivers its charge to it. It, being unstable, then undergoes a **secondary reaction** at the anode as follows: $SO_4 + H_2O = H_2SO_4 + O$. Thus the sulphuric acid molecule returns to the original condition to be again dissociated as before. The sulphuric acid molecules are not wasted, and it is the **electrolysis of water** that really takes place in the voltameter according to the following equation, $H_2SO_4 + H_2O = H_2SO_4 + H_2 + O$. So the final products at the kathode and the anode are hydrogen and oxygen in the ratio 2 : 1 by volume. In practice, however. the two gases cannot be obtained in the above ratio of volumes due to (*i*) the unequal absorption of the gases in the solution, (*ii*) occlusion of hydrogen in the kathode, and (*iii*) transformation of a small quantity of oxygen into ozone.

The same explanation and the same equations apply to the case of other acid solutions in water.

A continuous electrolysis of water yields a residue rich in **heavy hydrogen** or **deuterium**, which is an *isotope* of ordinary hydrogen having an atomic mass twice as much as that of a proton but carrying the same charge (vide Chapter IX) as that of the proton. Urey used this fractional electrolysis of water to isolate heavy hydrogen.

Some Terms

The **chemical equivalent** *of an element is numerically equal to the* **atomic weight** *divided by its* **valency**, *and it is this weight which will combine with, or replace, one part by weight of hydrogen.*

In electrolysis, it is also found that *metals and hydrogen*, which always travel *with the current* and go to the kathode, are called **electro-positive**, while the acid particles and non-metals, which travel *against the current* and go to the anode, are called **electro-negative.** Pure water does not conduct electricity, but it is difficult to get water without traces of some salt or acid dissolved in it ; so ordinary water is generally a conductor.

The **valency** of an element is given by the number of hydrogen atoms which will combine with, or are replaced by, one atom of the element.

A **gram-equivalent** of a substance is the quantity in grams equal to its chemical equivalent.

The atomic weight of silver is 108 and its valency is 1, so the chemical equivalent of silver is 108, and the gram-equivalent of silver is 108 grams.

56. Faraday's Laws of Electrolysis.—The facts of electrolysis are given by Faraday in the following laws :—

(*i*) *The mass (W) of an ion liberated at an electrode is proportional to the quantity of electricity Q*, which is passed, *i.e* to the product of the strength of the current C and the time t during which it flows. That is $W \propto Q \propto (C \times t)$.

(*ii*) *If the same quantity of electricity passes through several electrolytes, the masses of ions liberated are proportional to their chemical equivalents* (m). That is, $W \propto m$.

Verification.—The laws can be verified as follows.—

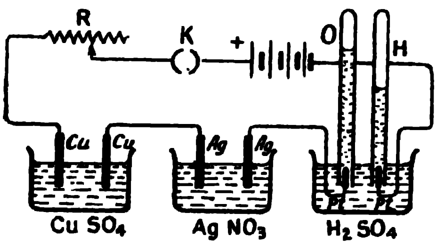


Fig. 70—Electrolysis

A current from a battery is passed through three electrolytic cells in series one containing a solution of *copper sulphate with Cu electrodes*, the second a solution of *silver nitrate* ($AgNO_3$) *with silver electrodes*, and the third containing *acidulated water with platinum electrodes* (Fig. 70).

1. (a) Let the current pass through the electrolyte for a period t_1 ; then determine the weight W_1 of copper deposited on the kathode of the copper volta meter, which was previously weighed. Repeat the experiment by passing the same current for another interval t_2 and determine theweight W_2 of copper deposited, then it will be found that,

$$\frac{W_1}{W_2} = \frac{t_1}{t_2} \text{ ; } i.e.\ W \propto t, \text{ when } C \text{ is constant} \quad \dots \quad \dots \quad (1)$$

(b) Next repeat the experiment by keeping the time interval the same, but alter the strength of the current (C) by means of the rheostat. Determine the amount of deposit on the kathode for each value of the current and let them be W_3 and W_4, the corresponding currents being C_1 and C_2 respectively, then it will be found that,

$$\frac{W_3}{W_4} = \frac{C_1}{C_2} \text{ ; } i.e.\ W \propto C, \text{ when } t \text{ is constant} \quad \dots \quad \dots \quad (2)$$

Then from (1) and (2), we have, $W \propto Ct$.

2. All the kathode plates of the different cells are first cleaned, dried and then weighed. To collect hydrogen, a graduated tube filled with water is inverted over the negative terminal of the water-voltameter. Pass a suitable current for some time and as usual find out the masses deposited on the plates. To find the mass of H_2 evolved, the volume of hydrogen collected is reduced to normal temperature and pressure and then multiplied by the density at N.T.P. It will be found that the masses liberated (W_1, W_2 and W_3) will be related to the chemical equivalents (m_1, m_2 and m_3) of the elements as follows.—

$$W_1 : W_2 : W_3 :: m_1 : m_2 : m_3.$$

That is, $W \propto m$, when C and t are constant.

57. Electro-Chemical Equivalent.—From Faraday's laws of electrolysis, $W \propto Ct$; or $W = ZCt$, where Z is a constant for an element. This constant is called the Electro-Chemical-Equivalent ($E.C.E$) of the element. From this it follows that, "*The electro-chemical equivalent* ($E.\ C.\ E.$) *of an element is the weight of it in grams which is deposited by the passage of one coulomb of electricity, i.e. by one ampere for one second.*"

Relation between E. C. E.'s of Elements.

If W_a and W_b be the masses of two elements A and B, of $E.C.E.$ given by Z_a, Z_b, liberated by the same current in the same time,

$$W_a = Z_a . C.t. \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$W_b = Z_b . C.t. \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

From (1) and (2), $\dfrac{W_a}{W_b} = \dfrac{Z_a}{Z_b}$. But from Faraday's second law of electrolysis,

$$\frac{W_a}{W_b} = \frac{\text{Chemical Eq. of } A, (m_a)}{\text{Chemical Eq. of } B, (m_b)}.$$

$$\therefore \frac{Z_a}{Z_b} = \frac{m_a}{m_b} = \frac{\text{Chemical Eq. of } A}{\text{Chemical Eq. of } B}.$$

The E.C.E. of silver is 0·001118 gm. per coulomb : of hydrogen, 0·00001044 gm. per coulomb. Knowing the value of E.C.E. of one element, the E. C. E. of the other elements can be obtained from the knowledge of the chemical equivalents of the elements. Thus, the chemical equivalent of silver being 108, its E.C.E. $= 108 \times 0{\cdot}00001044$. $= 0{\cdot}00112$ gm. per coulomb, since the chemical equivalent of H_2 is unity.

Since 0·001118 gram of silver is deposited by 1 coulomb of electricity, 108/0·001118, *i.e.* 96600 coulombs are required to deposit 108 gms. of silver. *This same quantity will also deposit one gram-equivalent of any other monovalent substance*

A table is given below of the E. C. E. of some common elements :—

Element	Atomic weight	Valency	E. C. E. in grams per coulomb
Electro-positive			
Copper	63·5	2	0·000329
Hydrogen	1·008	1	0·00001044
Silver	108·0	1	0·001118
Electro-negative			
Oxygen	16·0	2	0·0000829
Chlorine	35·5	1	0·00036

58. Weight of Ion deposited.—From the first law we have,

$$W \propto Ct,$$

where W = weight of an ion deposited in grams ;

C = current in amperes ; t = time in seconds.

or $W = ZCt$, where Z is a constant. By putting $C=1$, and $t=1$, Z becomes numerically equal to W—*the weight of ion deposited by 1 coulomb of electricity* and in this case the weight of the ion (Z) is called the **electro-chemical equivalent** of that particular element.

(a) Charge on an Ion.—According to the **modern theory**, also supported by conclusive experiments, the atoms themselves are the carriers of electricity, and the charge carried by a positive ion of each monovalent substance like hydrogen, silver, sodium, etc., is of exactly the same magnitude as the free negative charge, called "electron", which is the natural unit of electricity.

It has been already stated that the passage of 96600 coulombs will liberate one gram-equivalent of any monovalent substance. Then, if n be the number of atoms in one gram-equivalent of a monovalent substance and e the charge carried by each atom (*ion*) in electrolysis, we have $ne = 96600$ coulombs.

But the number n is constant for all substances and its mean value is $6{\cdot}16 \times 10^{23}$. Hence the charge carried by each monovalent

ion $= \dfrac{96600}{6{\cdot}16 \times 10^{23}} = 1{\cdot}57 \times 10^{-19}$ coulombs $= 4{\cdot}7 \times 10^{-10}$ E. S. U. So this charge is associated with an **electron**.

Since the gram-equivalent of a diavalent substance contains ***half*** *the number of atoms stated above, the charge carried by each* ***divalent ion*** *is twice that carried by each* ***monovalent ion.***

The two terms **ion** and **electron** should be distinguished by remembering that the *ion* is the *material atom carrying the charge* and **the electron** is the **charge itself without the atom.**

Example.—*A current of 5 amperes is passed for 5 hours through three voltameters arranged in series containing solutions of copper sulphate, silver nitrate, and sulphuric acid. Calculate the masses of silver, copper, and hydrogen obtained, given that E. C. E. of silver = 0·001118 gm /coulomb ; at. wt. of silver = 107·88 ; at. wt. of copper = 63·57, and at. wt. of hydrogen = 1·008.* (*Pat.* **1944**)

The total quantity of electricity passed in 5 hours, or $(5 \times 60 \times 60)$ seconds $= 5 \times 5 \times 60 \times 60 = 90{,}000$ coulombs.

(1 coulomb is the quantity conveyed by 1 amp. in 1 sec.)

1 coulomb liberates 0·001118 gm. of silver.

∴ 90000 columbus will liberate $(0{\cdot}001118 \times 90000)$ gm. $= 100{\cdot}62$ gm. of silver.

In this case, by the second law, the masses of liberated ions are proportional to their chemical equivalents.

∴ Mass of hydrogen $= (100{\cdot}62 \times 1{\cdot}008)/107{\cdot}88 = 0{\cdot}94$ gm.

Mass of copper $= (100{\cdot}62 \times 63{\cdot}57)/(2 \times 107{\cdot}88)$ (copper being divalent) $= 29{\cdot}65$ gms.

59. Measurement of Current by Voltameters.—From the relation, $W = ZCt$, the strength of a current C can be calculated by knowing W, Z, and t. For the purpose of measuring current strength, a copper voltameter is conveniently used. Before dipping the kathode in the solution it should be carefully cleaned, dried, and then accurately weighed in a good balance. Current is then passed for a known time, after which the kathode is taken out, carefully washed with distilled water, and dried in air. Now, by weighing the kathode the weight of copper deposited will be known ; so the current strength can be calculated by knowing t, and Z of copper.

59(a). Reduction Factor of Tangent Galvanometer.—

Expt.—A circuit is made of a copper voltameter V, a battery B, an adjustable resistance R, and a tangent galvanometer (T. G.) with a commutator C, all connected in series (Fig. 71). By adjusting the resistance R the needle of the galvanometer is brought as near to 45° as possible. The kathode is then taken out, cleaned, dried, and then

weighed accurately in a good balance. It is then put in its position in the voltameter and a current is passed for a known time t seconds. For a good deposit, the maximum limit to the current strength is *1 ampere for every 50 sq. cms. of the kathode surface.* With larger currents the deposits fall down being feathery. The current (through the galvanometer only) should be reversed, when one half of the time occupied by the experiment has passed, and the mean of the four readings (readings for both ends of the pointer each time) is taken as deflection θ. The kathode is then removed, washed, dried, cooled and finally weighed in a delicate balance and thus the mass of copper deposited is found. Then from the relation C (amp.) = $10K \tan\theta$, the reduction factor K is calculated. The value of C is calculated from the relation $W = ZCt$, where W is the mass of copper deposited on the kathode of the voltameter. Thus,

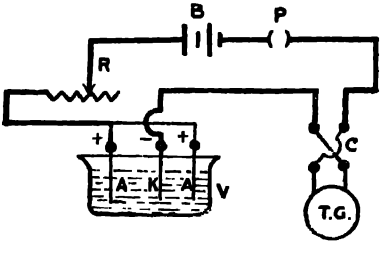


Fig. 71

$$C = \frac{W}{Zt} \text{ (ampere) ; or } 10\,K \tan\theta = \frac{W}{Zt} \text{ ; or } K = \frac{W}{10\,Zt \tan\theta}.$$

N. B. The above method may be used to determine the **E.C.E.** of **Copper**. Here $Z = \dfrac{W}{10\,Kt.\ \tan\theta}$. Knowing the reduction factor K, W, t, and θ being found from the experiment, Z for copper may be calculated.

Examples.—*1. In order to test the readings of an ammeter it was connected in series with a silver voltameter, and a steady current was passed through the two for one hour. The ammeter indicated 0·26 ampere, and 1·0062 gm. of silver was deposited. Was the ammeter reading correct ? If not, what was the error ? [E. C. E. of silver = 0·001118]* *(Pat. 1920)*

From the relation, $W = ZCt$ (Art. 58) ; $C = W/Zt$ (ampere).

$\therefore\ C = 1{\cdot}0062/(0{\cdot}001118 \times 3600) = 0{\cdot}25$ ampere.

Hence the reading indicated by the ammeter is wrong, and the error is $0{\cdot}26 - 0{\cdot}25 = 0{\cdot}01$ ampere. [This example illustrates a method of calibrating an ammeter (see Art. 42)].

2. A circuit consists of a solution of silver salt and a coil of wire of resistance 20 ohms immersed in an oil bath in series. Constant current flows for 10 seconds and deposits 0·0279 grm. of silver. Calculate how much heat energy is developed in the oil both. (E. C. E. of silver = 0·0011183 gr./coulomb.) *(Pat. 1922)*

From the relation $W = ZCt$, we have $C = \dfrac{0{\cdot}0279}{{\cdot}0011183 \times 10} = 2{\cdot}49$ ampere.

The amount of energy developed $= C^2rt$ ergs (when C and r are expressed in C. G. S. units) $= C^2rt \times 10^7$ ergs, when C and r are expressed in practical units

$$= (2{\cdot}49)^2 \times 20 \times 10 \times 10^7 = 1240 \times 10^7 \text{ ergs.}$$

$\therefore$ The amount of heat energy in calories $= \dfrac{C^2rt \times 10^7}{J} = \dfrac{1240 \times 10^7}{4{\cdot}2 \times 10^7} = 295{\cdot}2.$

3. A current is passed for 30 minutes through a silver voltameter in series with a tangent galvanometer of one turn and diameter 40 cms. The deflection of the galvanometer is 45°, and 2·312 gms. of silver are deposited. What value does this give for the E C. E. of silver ? (H = 0·36). **(Pat. 1924)**

From $C = \dfrac{rH}{2\pi n}\tan\theta$ (Art. 23), $C = \dfrac{20 \times 0{\cdot}36}{2 \times 3{\cdot}1416 \times 1} \times \tan 45° = \dfrac{0{\cdot}3}{0{\cdot}2618}$ C.G.S. units.

Again, we have, $W = CZt$; or $2{\cdot}312 = \dfrac{0{\cdot}3 \times 10}{0{\cdot}2618} \times Z \times (30 \times 60)$; whence

$Z = 0{\cdot}00112$ gm. per coulomb (for here C should be taken in amperes).

4. Calculate the electro-chemical equivalent of hydrogen, given that 1 ampere deposits 0·65 grm. of copper from a solution of copper sulphate in 33 min. **(Atomic wt.** *of copper = 63 ; valency = 2).* **(Pat. 1938).**

As above, $0{\cdot}65 = Z \times 1 \times (33 \times 60)$; $\therefore Z = \dfrac{0{\cdot}65}{33 \times 60}$

Chemical eq. of copper $= \frac{63}{2}$; But E. C. E. of copper $= \frac{63}{2} \times$ E. C. E. of hydrogen. Or $\dfrac{0{\cdot}65}{33 \times 60} = \dfrac{63}{2} \times$ E. C. E. of H.

From this, E. C. E. of $H = 0{\cdot}00001042$ per coulomb.

5 If 96,500 coulombs of electricity liberate one gramme equivalent of any substance, how long will it take for a current of 0·15 ampere to deposit 20 milligrammes of copper from a solution of copper sulphate ? [Atomic weight of copper = 64] **(Pat. 1926)**

The gram equivalent of a substance is the quantity in grams equal to its chemical equivalent, and chemical equivalent = atomic wt. ÷ valency

The atomic weight of copper is 64 and its valency in copper sulphate is 2.

$\therefore$ Chemical equivalent of copper $= \frac{64}{2} = 32$, and its gram equivalent $= 32$ gms. Again, 96500 coulombs liberated 32 gms. of copper ;

$\therefore$ 20 mgms. or 0·02 gm. will be liberated by $\dfrac{96500}{32} \times 0{\cdot}02$ coulombs.

Now if t seconds be the time required by 0·15 ampere to deposit 0·02 gm., we have, 0·15 ampere flowing for t secs., *i.e.* $(0{\cdot}15 \times t)$ coulombs

$$= \frac{96500}{32} \times 0{\cdot}02 \text{ coluombs ; whence } t = 6 \text{ min. } 42 \text{ secs.}$$

6. Two copper plates are immersed in a solution of copper sulphate, and a current is passed through them and a tangent galvanometer. The deflection of the galvanometer is 45°, and after an hour it is found that 216 milligrams

of copper have been deposited on one plate. Having given that a current of 1 ampere deposits 19·8 milligrams of copper per minute, deduce the reduction factor of the galvanometer. (*Pat. 1919*)

1 ampere deposits 19·8 mg. of copper per minute ;

i.e. $\frac{19·8}{1000 \times 60}$ gm. of copper per sec. ;

$\therefore$ E. C. E. of copper $= \frac{19·8}{100 \times 60} = 0·00033$ gm. per coulomb.

We know, that $W = ZCt$.........(1). Again, if θ be the galvanometer deflection and K the reduction factor of the galvanometer, C (in amp.) $= 10\ K \tan\theta$

or, $K = \frac{C}{10 \tan\theta} = \frac{W}{Zt \times 10 \tan\theta}$.........from (1)

$= \frac{216/100}{0·00033 \times 60 \times 60 \times 10 \times 1} = 0·018$ in C. G. S. units.

60. Storage Cells (or Accumulators).—Due to the back E. M. F. of polarisation in a cell, reverse current can be obtained and this is utilised in storage cells ; such cells may be considered as voltameters in which chemical electrolysis is first produced by the passage of current from a battery, and this gives rise to a P. D. between the two electrodes of the voltameter, which is afterwards used as a cell after disconnecting the battery.

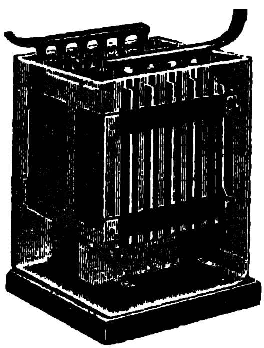


Fig. 72—Accumulator.

It two lead plates are immersed in dilute* sulphuric acid and an electric current from a battery is passed through the acid, hydrogen bubbles are deposited on the plate at which the current leaves the solution (kathode), and oxygen on the plate where the current enters (anode) forming *lead peroxide* (PbO_2). After this, the current in the solution decreases due to the polarisation effect. If the battery is now removed, and the two lead plates are joined together by a wire, a current will flow in the *opposite* direction [see Art. 4(ii)]. The original current from the battery turned one of the plates into a plate of lead peroxide ; so after stopping the current of the charging battery, when the plate of lead and the newly formed plate of lead peroxide, dipped in dilute sulphuric acid, are joined by a wire, we get a current, which is really the *polarisation current*. The arrangement after charging is like that of a

*Note—It should be noted that sulphuric acid without water is not an electrolyte. The electrolysis takes place only in a very dilute solution.

voltaic cell of two dissimilar metals dipped in an acid. The cell works until the peroxide is used up, when the cell is said to be **discharged** ; after which a fresh **charge** should be given to get the original state. The cell is called a **secondary** cell, as it is the secondary or the reverse current which is really taken from the cell. It is also called a **storage cell** or **accumulator**. It must not be thought that there is really an accumulation or storage of electricity. **It is the chemical energy which is stored, and not electricity.**

In the *modern form* of secondary cells, the lead plates are replaced by lead grids, the interstices [Fig. 72(*a*)] of which are filled up with *litharge*, *i.e.* lead monoxide (PbO). In some cells *red lead* (Pb_3O_4) is packed in the grids of the positive plate and litharge in those of the negative. After charging, the interstices in the positive plate are oxidised to lead peroxide (PbO_2), and in the negative reduced to spongy lead. Before charging, litharge and the acid react forming $PbSO_4$ in both the plates as follows :—

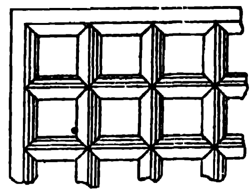


Fig. 72(a)

$$PbO + H_2SO_4 = PbSO_4 + H_2O.$$

And during **charging**, the following chemical action takes place—

At the **positive** plate : $PbSO_4 + O + H_2O = PbO_2 + H_2SO_4$.

At the **negative** plate : $PbSO_4 + H_2 = Pb + H_2SO_4$.

So on charging, the specific gravity of the acid increases owing to the formation of acid molecules.

During the **discharge**, *i.e.* when the cell is giving a current, hydrogen is set free at the *positive* peroxide (PbO_2) *plate* and the following chemical action takes place—

$$PbO_2 + H_2 + H_2SO_4 = PbSO_4 + 2H_2O.$$

At the *negative* lead *plate*, oxygen is set free and the following action takes place—

$$Pb + O + H_2SO_4 = PbSO_4 + H_2O.$$

Thus during the discharge, water is set free and the specific gravity of the acid is lowered, the E. M. F. of the cell falling at the same time.

The internal resistance of the cell is considerably reduced by the use of large plates placed close together and using a series of plates parallel to each other connected alternately to two electrodes which end in two binding screws. Insulating separators are kept between the plates so that the plates may be placed close together without touching. The cell was originally due to Plante but subsequently modified by Faure.

A Few Facts about Accumulators.—The specific gravity of the acid should be 1·3 when fully charged. It falls during discharge, but it should not be allowed to fall below 1·18.

The voltage of a fully-charged accumulator is 2·2, which should be steady at 2 volts during discharge The voltage should not be allowed to fall below 1·8, after which the cell should be recharged.

Resistance.—The internal resistance of the accumulators is very small—from $\frac{1}{50}$th to $\frac{1}{100}$th of an ohm. For this reason when the cell is short-circuited, *i e.* the two poles are joined together without any external resistance, enormous current will flow inside the cell (see Art. 31) and the heat generated by current will damage the cell.

Thus, suppose that the internal resistance of a cell is 0·01 ohm and the terminals are joined by a piece of wire whose resistance is negligible, then the current becomes $1/0·01 = 200$ amperes. The current being very high, so much heat will be generated that the plates will expand and break away from the grids.

Caution.—The two terminals of an accumulator must never be *short-circuited* by connecting them by a good conductor, say a connecting wire. Distilled water must be added from time to time to make up the water lost by evaporation during action of the cell.

The *capacity* of an accumulator is expressed in **ampere-hours.** Thus a cell of capacity 30 amp.-hr. with a maximum discharging rate of 0·5 amp. can give a current of 0·5 amp. for 60 hours, or 0·25 amp. for 120 hours and so on.

Alkali-Cell (Edision Cell or Ni-Fe Cell.—This cell consists of a *Ni*-plated iron container filled with *KOH*-solution. The positive plate is formed by *Ni*-plated perforated iron-tubes filled with *Ni*-flakes and hydrated nickel-oxide while the negative plate consists of nickel-plated perforated iron pockets filled with iron-oxide During charging, current from an external source is passed from the nickel oxide plate to the iron-oxide plate, when the positive plate is covered with Ni_2O_3. During discharge, Ni_2O_3 is reduced to NiO and Fe is oxidised to FeO. Zn or Cd plate in place of Fe makes the cell more efficient. The maximum voltage of the cell is 1·25 volts and a large current may be drawn from it.

Lead and Alkali Cells Compared.—Both of them after discharge may be recharged over and over again.

An alkali cell is only half as heavy as a lead cell of the same amp-hour capacity, but its efficiency is somewhat less. The lead cell may be easily damaged but the life of an Edison Cell is much larger. It is mechanically stronger and cannot be easily injured by rapid charging

or discharging. When discharged too much, the lead cell *sulphates* and may be permanently out of use, but the alkali cell, even when completely discharged may be recharged.

Uses of Storage Cells.—They are extensively used for laboratory purposes, including high and low tension work in wireless, in automobiles to supply current to, the horn, the self-starter, the light, and the ignition system. They are used in train-lighting systems. In power stations, they are used to act as stand-by, as also supplementary to dynamos. They are used in telephone and telegraph systems, in calling bells, burglar alarms, electric-clock systems, etc. In some countries, they are also used by the farmers in non-electrified areas for lighting purposes, the charging of the cells being carried by small dynamos run by gasolene engines.

61. **Applications of Electrolysis.**—There are several industrial applications of electrolysis of which a few cases are given below :—

(*a*) **Electro-plating**—The process of depositing a layer of a metal, such a gold, silver, nickel, on any object by electrolytic method, is known as **electro-plating**. It takes place in a vessel containing a solution of a salt of the substance (say, silver or nickel) which is to be deposited, and in which the anode consists of a rod of the metal (say, silver or nickel), and kathode forms the object which should be electro-plated.

In the case of **nickel-plating**, the electrolyte used is a solution of nickel sulphate in ammonium sulphate, and, in the case of **silver-plating**, a solution of silver nitrate or silver cyanide in potassium cyanide is used.

As soon as a current passes, the solution is decomposed, and the metallic ion is deposited on the object forming the kathode. The metal rod at the anode is dissolved by the current to keep up the concentration of the solution. Before introducing in the bath, the object should be carefully cleaned in order to be perfectly free from dirt and grease.

In order that the deposition may be hard, durable, and of the usual colour, the current used in electro-plating must be small.

In **Electro-guilding**, gold is deposited on the kathode by using ordinarily a solution of gold cyanide in potassium cyanide.

Galvanizing Iron—Zinc is deposited on iron sheets by electrolytic process in order to prevent it from rusting.

(*b*) **Electro-typing.**—It is a process of securing exact copies of types, medals, coins, etc. For this purpose a mould of the object is first obtained in wax or plaster of Paris, the surface of which is treated with graphite in order to render it conducting. It is then

placed as the kathode in an electrolytic bath of copper sulphate solution to receive a sufficiently thick coating of copper by passing a current through the solution. The mould is then taken out of the bath. The surface of the deposited sheet facing the mould gives an exact reproduction of the object.

(*c*) **Production of Pure Metals.**—Electrolytic methods are adopted for the purification of metals, such as *copper*, *silver*, and *gold*. This is done by making the impure copper the anode in a bath of copper sulphate, and the pure copper is deposited electrically on the kathode. Similar process is adopted for other metals.

The cheapness of aluminium articles is solely due to the electrolytic process of refining by dissolving aluminium oxide (Al_2O_3) in a bath of molten cryolite ($AlF_3.3NaF$), found largely in Greenland.

Example.—*A spoon having an area of 20 sq. mm. is to be coated with silver to a thickness of 0·01 mm. If a current of 0·15 amp. is used, calculate the time for which it must flow. (E.C.E. of silver is 0·001118 gm. per coulomb; density of silver is 10·5 gm. per c.c.* (*Pat. 1940*)

$$\text{Volume of silver deposited} = \frac{20}{100} \times \frac{1}{100 \times 10} \text{ c.c.}$$

$$\therefore \quad \text{Wt. of silver} = \frac{20}{100} \times \frac{1}{1000} \times 10{\cdot}5 = \frac{21}{10000} \text{ gm.}$$

$$\text{From the relation } W = ZCt, \quad \frac{21}{10000} = 0{\cdot}001118 \times 0{\cdot}15 \times t\,;$$

$$\text{whence } t = 12{\cdot}52 \text{ seconds.}$$

62. Uses of Electrolysis.—From what has been considered above electrolysis is seen to be valuable for (*a*) ascertaining the constituents of chemical compounds in liquid forms, for which the principle applied in Art. 54 is applied ; (*b*) measuring the strength of a current (see Art. 59), and determining E.C.E. of elements, and calibrating an ammeter ; (*c*) electro-plating (Art. 61) ; (*d*) electro-typing (Art. 61) ; (*e*) purification of metals (Art. 61).

Questions

Arts. 54, 55 & 56.

1. Explain clearly how you would find experimentally the ratio of the electro-chemical equivalents of hydrogen and copper.

(C. U. 1930 ; All. '30 ; Pat. '42)

[**Hints.**—Pass the same current for the same time through a copper voltameter and a water voltameter. Observe increase in weight of the copper kathode and also determine the weight of hydrogen liberated at the kathode by measuring its volume and reducing it to N. T. P. (1 litre of H at N. T. P.

weighs 0·089 gm.) Then by Faraday's second law, the ratio of the weights of *Cu* and *H* is as that of their electro-chemical equivalents.]

2. Describe a water voltameter and explain how you would use it for the verification of the laws of electrolysis. (C. U. 1916)

3. State Faraday's laws of electrolysis and explain how you can verify them with the help of a copper voltameter. Discuss the steps in the process and state what precautions are to be taken to ensure good results. (C. U. 1944)

4. Two plates of zinc are immersed in a solution of zinc sulphate and connected to the terminals of a voltaic battery. Describe and explain briefly the effects observed on the two plates. (C. U. 1917)

5. Explain the terms—electrolyte, electrodes, kathode, anode, ions.

A current is passed through three electrolytic cells, the first containing dilute sulphuric acid with platinum electrodes, the other two containing a saturated solution of copper sulphate with platinum electrodes in one cell and copper electrodes in the other. State what occurs at each electrode. (C. U. 1924, '28.)

6. Describe an arrangement for obtaining oxygen by the decomposition of water. Point out the most important difference between electrical conduction in metals and in solutions. (C. U. 1920)

[Hints —For the first part see Art. 55. Decomposition takes place in a solution due to electric conduction, while in a metal no decomposition takes place.]

7. How would you determine which is the positive and which is the negative terminal of a voltaic battery by (*a*) the magnetic effects, and (*b*) the electro-chemical effects of currents. (All. 1925)

[(*a*) Art. 9 ; (*b*) Art. 55].

8. Without measuring currents by means of their magnetic force Faraday was able to prove experimentally that the amount of chemical action occurring in an etectrolytic cell is proportional to the quantity of electricity passing through it. Show how this can be done. (Pat. 1926)

9. State the laws of electrolysis and describe experiments to verify them. (C.U. 1917, '40, '44 ; Pat. 1936, '38, '42, '44, '49 ; Dac. 1927 ; All. '27, '45)

Art. 57.

10. What do you understand by the electro-chemical equivalent of an element ? (Bom. 1930 ; Pat. 1929, '30. '33, '38, '42 ; Cal. '49)

Arts. 58 & 59.

11. State the laws of electrolysis and show how they may be applied to measure the strength of an electric current. (Pat. 1937, '47 ; C.U. 1937)

11(a). State and explain Farady's laws of electrolysis. How may the strength of an electric current be measured by means of a copper voltameter ? (Utkal 1947).

12. What is the amount of charge carried by a monovalent atom in the process of electrolysis. (C. U. 1944)

[*See* Art. 58(*a*)].

13. A metal plate, having a total surface area of 300 sq. cm., is to be nickel-plated. If a current of 1·5 amp. is used for 3 hours, find the thickness of nickel deposited on the plate. [Density of nickel = 8·8 gm. per c.c. E. C. E. of nickel = 0·000304 gm. per coulomb.] (Pat. 1942)

[*Ans* : $1·87 \times 10^{-3}$ cm.]

13(*a*). A copper voltameter is connected in series with a battery and a standard 2 ohm coil. The current is passed through 30 minutes and the increase in weight of the cathode is 1·476 gm. The mean reading of a voltmeter connected across the 2 ohm coil is 5 volts. Determine the E. C. E. of copper. (C. U. 1949)

[*Ans* : 0·000328.]

14. Three copper voltameters in parallel are connected to the ends of a battery with resistance. If after 30 minutes the deposits are 0·763, 0·742, and 0·785 gms. respectively, find the strength of the current drawn from the battery (E. C. E. of copper is 0·0003993 gm./coulomb.) (Pat. 1930)

[*Ans* : 3·862 amp.]

15. A tangent galvanometer has a current passed through it which produces a deflection of 45°. The same current passes through a copper voltameter, where it deposits 0·3 gm. of copper in 30 minutes. If the E.C.E. of copper be 0·00033 gm./amp. sec. find the value of the current, and show how to determine the current for any other reading of the galvanometer.

[*Ans* : 0·505 amp.] (Pat. 1931)

[**Hints.**—When θ is 45°, current $C = K$, so K is known for the galvanometer and thus current for any other deflection will be known].

15(a). A battery sends a current through a tangent galvanometer which it deflects through 2°, a voltameter in which it evolves 10 c.c. of hydrogen in a certain time and a coil in a calorimeter in which the thermometer rises 0·3°C., all the instruments being in series. If the current be made 5 times as great, describe and explain the effect of the increased current on the three instruments.

16. The coil of a Tangent Galvanometer having 10 turns and radius 5 cms. is placed in series with a copper voltameter. If the deflection is 60° and $H = 0·36$, calculate the weight of copper deposited in 30 minutes. [Electro-chemical equivalent of Hydrogen = 0·00001045 and atomic weight of copper (divalent) = 63·57]. (Pat. 1934)

[*Ans* : 0·296 gm.]

17. Describe an experiment to determine the electro-chemical equivalent of copper. (Pat. 1940)

18. Calculate the strength of the current that is to be sent for an hour through an electrolytic cell containing silver nitrate solution for depositing 0·805 gm. of silver on the cathode. (E.C.E. of silver = 0·00118 gm. per coulomb.

[*Ans :* 0·2 ampere.] (C. U. 1948)

19. Describe a copper voltameter and explain how with the help of such a voltameter, the reduction factor of a Tangent Galvanometer can be determined. (Pat. 1944)

20. In an experiment the weight of silver deposited was 1·372 gram in 45 minutes. The deflection of the galvanometer needle was 30°. Draw a diagram showing the necessary connections, and find the reduction factor (E. C. E. of silver = 0·00112 gm. per coulomb)

$$[\textit{Ans :}\quad K = \frac{1{\cdot}372}{0{\cdot}00112 \times (45 \times 60) \times 10 \times 1/\sqrt{3}} = 0{\cdot}07]$$

21. Explain carefully how you would use a tangent galvanometer to measure an electric current. Deduce any expression you use. (Pat. 1949)

Art. 60.

22. Describe the action and working of a lead accumulator.
(Pat. 1926 ; Cf. All. '29 ; C. U. '33, '45)

23. Describe and explain the principle of action of a storage cell.
(Pat. 1947 ; U. P. B. 1948)

Art. 61.

24. Briefly explain the process of electro-plating. (All. 1920)

25. Describe, with a neat diagram, the arrangement for having a uniform coat of silver on brass spoons. (Pat. 1932)

26. How will you proceed to deposit silver on a copper vessel. Find the strength of the current which will deposit 2 gm. of silver in 2 min.

[*Ans :* 1·49 amp.] (All. 1931)

27. A piece of metal weighing 2000 gms. is to be electro-plated with $2\frac{1}{2}$ per cent. of its weight in gold. If the current strength is 1 ampere and the electro-chemical equivalent of gold is 0·0006808 grammes per coulomb, how long will it take to deposit the required weight of gold ? (Pat. 1925 ; All. '27)

[*Ans :* 2 h. 2 m. 24 s.]

Art. 62.

28. State and explain Faraday's laws of electrolysis and point out three applications of the phenomenon of electrolysis. (Utkal. 1948)

CHAPTER VII

Electro-magnetic Induction

63. Induced Currents : Faraday's Experiments.—It has already been found that a magnetic field is produced in the space surrounding a wire carrying a current. In 1831 **Faraday** showed that a momentary electric current can also be set up in a closed coil of wire by only moving it near a magnet or in any other magnetic field ; and a current can also be produced if a magnet be moved near the coil. *No current* will, however, be produced so long as there is *no relative motion* between the two, namely the magnetic field and a conductor. The momentary current thus produced is called **induced current** aud the phenomenon is called **electro-magnetic induction**.

(1) **Currents induced by Currents**—Two cylindrical coils of insulated wire are taken, one of which having a large number of turns is connected with a sensitive galvanometer G (Fig. 73) ; and the other, which can easily be introduced into the hollow of the first, is connected with a battery, a variable resistance R and a key. The former, which is connected to the galvanometer, is called the **secondary circuit** and the latter which contains the battery the **primary circuit**.

First connect a cell with a suitable high resistance in the secondary circuit and notice the direction of deflection of the galvanometer. Trace the direction of the current in the coil knowing the polarities of the cell. Thus the direction of the current is indentified with the deflection in the galvanometer.

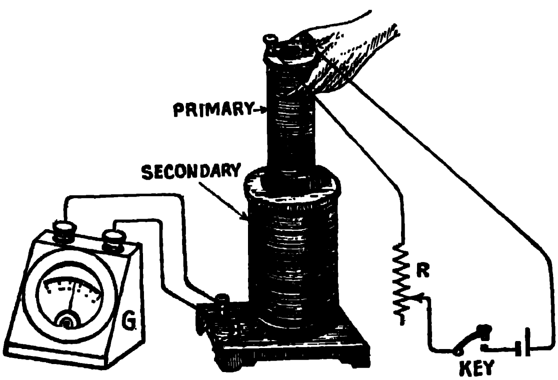


Fig. 73

(*a*) After putting the primary coil inside the se condary, if the key in the primary is pressed down, a *sudden deflection* is observed in the galvanometer. It is found that the *direction* of this *instantaneous* current is *opposite* to that passing in the primary. When the primary circuit is broken by disconnecting the key, the galvanometer is again deflected, but in the *opposite direction*. This

time the *direction* of the instantaneous current through the secondary is the *same* as that passing in the primary. The induced current is only momentary and exists only while the change in the primary is made.

(*b*) Similarly, when the current strength of the primary circuit is **increased**, by decreasing the resistance, or the primary coil is thrust more and more into the secondary, an instantaneous **inverse** (*i.e.* in the opposite direction to that in the primary) current is produced in the secondary ; and when the current strength of the primary is **diminished**, or the primary coil is removed to a greater distance, an instantaneous **direct** (*i.e.* in the same direction as that in the primary) current is produced in the secondary.

In the above experiments the strength of the induced E. M. F. (and hence the current) is greatly increased by keeping a bundle of soft iron wires inside the primary coil. In this case, increased deflections will be obtained on making or breaking the primary circuit. This is due to the high permeability of iron owing to which the change in the number of lines of force is increased by the introduction of the soft iron wires.

Explanation of Induced Current from Lines of force.—Fig. 74 gives a vivid picture of the simple facts of electro-magnetic induction stated above. The 'primary' and the 'secondary' circuits are represented as two parallel straight wires, *AB* and *CD*, where *AB* is the 'primary' and *CD* the 'secondary'. As soon as the primary circuit is closed, current passes in the direction from *A* to *B*, and a number of circular magnetic lines of force concentric with *AB* is established, whose direction is clock-wise, when viewed from the end *A*. During establishment some of these lines cut through the secondary *CD* and thus start up a momentary current in the secondary in a direction so as to oppose the sudden appearance of these magnetic lines, by setting up lines of force in *CD* in the anti-clockwise direction when looked at from the end *C*. So the direction of the current in *CD* must be from *D* to *C*. If the current in the primary is increased, an exactly similar effect is produced, but the effect is due to the additional lines of force due to increased current.

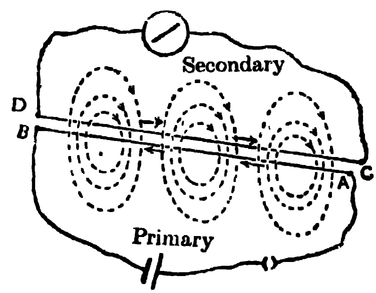


Fig. 74

On 'breaking' the circuit in AB, the lines of force shrink up in both AB and CD ; in CD an induced current is set up which tends to maintain the original field during the time of disappearance of the lines of force ; thus the direction of the induced current this time in CD must be from C to D in order that the lines of force may be in the clockwise direction so as to maintain the original field. If the current in the primary is decreased, or the primary circuit removed to a greater distance from the secondary, a similar effect is produced, but the effect is due to the decrement of lines of force linked up with the secondary.

(2) **Current Induced by Magnets.**—Similar effects as in "currents induced by currents" are observed by using a **bar-magnet,** instead of the primary coil (vide Fig. 75). The direction of the current induced in the coil is determined as in Art. 63 (1).

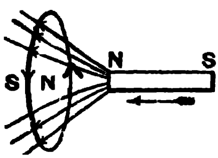


Fig. 75 (*a*)

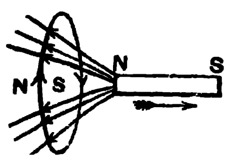


Fig. 75 (*b*)

(*a*) When the north pole of the magnet is brought near the secondary, *i.e.* the coil of wire in this case, a temporary current is induced in the coil which flows in such a direction (anti-clockwise) that the near end of the coil acquires also north polarity [Fig. 75 (*a*)].

When the north pole of the magnet approaches the coil, more and more lines of force of the magnet will embrace the coil and the induced current developed in the coil will be such as to oppose this increase or, in other words, the front face of the circuit will acquire similar polarity, *i. e.* north polarity. Hence the induced current is anti-clockwise in direction.

(*b*) When the magnet is withdrawn, the current flows in the opposite direction (clockwise), and so the *polarity* of the near end is also *opposite*, *i.e.* south [Fig. 75 (*b*)].

Here the number of lines of force linked with the secondary gradually diminishes, so the direction of the induced current will be such as to oppose this diminution, *i.e.* there will be a tendency to maintain the original number of lines of force. So the front face will acquire south polarity and so the direction of the current will be clockwise.

(*c*) When the movement of the magnet becomes quicker, the induced current becomes stronger.

Here it should be carefully noted why the induced current in the coil becomes stronger with the quicker movement of the magnet. When the magnets move quickly, *more lines of magnetic force are cut per second by the coil* and so the induced E.M.F. (and hence the induced current) becomes greater.

(*d*) No current is produced when the magnet does not move, *i.e.* when the number of lines of magnetic force through the coil does not change.

(*e*) The reverse effect is produced when the south pole of the magnet is used.

Thus **the change of magnetic lines of force** *may be regarded as the reason for inducing in the coil an electro-motive force which gives rise to a momentary current.*

N.B. It should be noted that in these experiments [(*a*) to (*e*)] we obtain *electrical energy in a coil by the motion of the magnet near it.*

The above results are tabulated below :

By	Primary	Secondary
Current	(a) at make (b) at approach (c) when strength increasing	Instantaneous, and inverse
Magnet	When approaching	
Current	(a) at break (b) when receding (c) when strength decreasing	Instantaneous, and direct
Magnet	when receding	

64. Laws of Electro-magnetic Induction.—As a result of his celebrated experiments on electro-magnetic induction Faraday in 1831 stated the following laws :—(1) *When the number of magnetic lines of force (which is often termed the* **magnetic flux**) *passing through a circuit is altering, an induced E.M.F. is set up, the magnitude of which is pro-*

portional (a) to the rate at which the change of flux is taking place, and (b) to the number of turns in the circuit.

In the above experiments of Art. 63, when the primary current is made, or increased, or it is brought nearer the secondary coil, the number of magnetic lines of force passing through the secondary is *increased*; and when the primary circuit is broken, or removed to a distance, or the north pole of the bar-magnet is taken away, the number of magnetic lines of force in the secondary *decreases*.

(2) *An* **increase** *in the number of lines of force linked up with the circuit induces an* **inverse** *current, while the* **decrease** *in the number of lines of force induces a* **direct** *current in the circuit ; and the current continues only while the change in the number of lines of force is actually taking place.*

Mathematical form of Faraday's Law.—If N be the number of lines of force passing through a circuit at any instant, and N' be the lines after a short interval t, two cases may arise :

(i) If $N'>N$, $(N'-N)$ is positive, and the induced E. M. F. (e)

$$=K\frac{N'-N}{t}=\frac{N'-N}{t};$$

since in the electro-magnetic system, the constant of proportionality, $K=1$. But the induced E. M. F. (e) is inverse ; so with its proper sign, we have, $-e=\frac{N'-N}{t}$; or $e=-\frac{N'-N}{t}$.

(ii) If $N'<N$, $(N'-N)$ is negative and in this case, there is a decrease in the number of lines of force and so the induced E M. F. (e) is direct. So, with proper sign, we have, $e=\frac{N-N'}{t}=-\frac{N'-N}{t}$.

Thus in both cases we find *that the induced E. M. F. (e) is the rate of change of magnetic flux with the sign changed.*.

If the secondary consists of n turns of wire and so the total flux is $n(N'-N)$, the value of (e) becomes,

$$e=-\frac{n(N'-N)}{t}=\frac{\text{Total no. of lines of force cut}}{\text{Time in seconds}}\ (\text{E.M.U.})$$

or $$e=\frac{\text{Total change in flux}}{\text{Time in seconds}}\ (\text{E.M.U.})=\frac{\text{Total change in flux}}{10^8\times\text{Time in seconds}}\ (\text{volts}).$$

65. Magnitude of the Induced E. M. F.—The magnitude of the induced E. M. F. can be increased in two ways.—*(i) by increasing the number of turns of wire of the secondary coil.* **By this the actual**

number, by which each line of force is cut by the turns of wire, is increased, and so the induced E. M. F. is also increased ; (*ii*) *by winding the primary on a bundle of soft iron wires*, which increases the number of lines of force in the primary due to the high permeability of iron ; (*iii*) *by increasing the rate of change of the number of lines of force, i.e.* by increasing the rate at which the primary circuit is made or broken, or by any other means.

The first part is very important as it means that an extremely high E. M. F. can be produced by taking a secondary coil of a very large number of turns, in which case the wire must be thin and so the resistance of the secondary coil will be great. Thus the current produced will be small, but the pressure (or voltage) will be high.

66. Lenz's Law.—In the experiments under Art. 63(2), it has been found that if the north pole of a bar-magnet approaches a coil, the direction of the induced current in the coil, as seen from the magnet, is anti-clockwise, that is, it is such that the face opposite to the north pole of the magnet acquires north polarity, which, therefore, tends to oppose the motion of the magnet. Similarly, if the magnet recedes from the coil, the same face of the coil will acquire south polarity, which, therefore, tends to attract the magnet. Hence there is always mutual opposition between them. This is true in every case of induced current. So *to obtain the direction of the induced current* there is a law, known as **Lenz's Law**, which runs as follows :—

In all cases of induced currents, the induced current is in such a direction that its reaction tends to stop or oppose the very cause to which the induced current is due.

Lenz's Law and the Principle of Conservation of Energy.—In the experiments already mentioned it is seen that when N-pole of a magnet moves relatively to a closed circuit, a current is induced in the circuit which sets up magnetism to oppose the motion of the magnet. Similarly when a primary coil approaches the secondary, an unlike current being induced in the secondary, there is a force of repulsion between the two. So in these experiments an amount of mechanical work is done in overcoming the opposition while producing induced currents, and it is this *mechanical energy which is transformed into the energy of the induced currents.*

In the case where the primary and secondary coils are fixed and the induced current is produced by making or breaking the primary current, or by changing the strength of the current in the primary, the flux through the secondary changes due to which there is a change of *electrical potential energy* of the secondary coil, which *is converted into electrical kinetic energy*, *i.e.* the energy of the induced current.

Lenz's Law follows from the principle of **Conservation of energy,** as all laws ultimately should, for if the direction of the induced current were such that it helped the motion of the approaching coil or magnet, instead of opposing it, the motion of the approaching coil or magnet would be increased. So in a magnetic field it would only be necessary to give a slight movement to a conductor, when its velocity, and hence its kinetic energy, according to the above condition, would go on increasing without receiving energy from any other source ; that is, electrical energy will be continuously generated without an expenditure of equivalent energy of some other form or forms which is against the principle of conservation of energy.

67. Fleming's Right-hand Rule :—A simple rule known as Fleming's right-hand rule gives a convenient method of deducing the direction of the induced current (or E. M. F.)

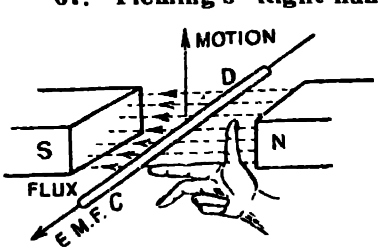


Fig. 76

Hold the thumb, first finger, and middle finger, of the right hand mutually at right angles to each other. If the first finger points in the direction of the magnetic lines of force and the thumb in the direction of motion, then the middle finger will point in the direction of the induced current or E. M. F. (Cf. Fleming's Left-hand Rule in Art. 17).

In Fig. 76 the rule has been illustrated. A straight horizontal conductor *CD* is made to move vertically upwards at right angles to a uniform field. The fore-finger of the right hand points in the direction of the magnetic field and the thumb in the direction of motion of the conductor. The middle finger, which is directed at right angles to both the motion and field, gives the direction of the E. M. F. induced in the conductor.

68. Mutual-induction and Self-induction.—(*i*) *The phenomenon of production of induced current in a circuit by changing the magnetic field due to a current in another circuit is known as* **Mutual induction**. The experiments, described in Art. 63(1), are all instances of mutual induction. The circuit which carries the source of E.M.F., and in which the varying magnetic field is produced is called the *primary circuit*, and the other circuit, which is usually a bigger coil of many turns of wire wound on the primary, or near it, in which the induced E.M.F. is produced, is called the *secondary*. To increase the inductive effect the coils are often wound on a soft iron core.

The co-efficient of mutual induction (or simply, **mutual inductance**) of two circuits may be defined as the magnetic flux linked up with the secondary when a unit current (E.M.U.) flows in the primary, or as the E M F. acting round the secondary due to a unit rate of change of current in the primary.

(*ii*) *Induced current may also be developed in a circuit due to its own movement in a magnetic field, or by change of flux through itself due to variations of its own current strength. This phenomenon is called* **Self-induction.**

Thus when a current is started, or suddenly increased in a coil of many turns, there is an increase in the magnetic lines of force passing through the coil, and so an inverse induced current is set up, *which* **opposes** the growth of current in the circuit *and thus weakens temporarily* **the current** *in the circuit.* Again, when the current through the coil is suddenly stopped, there is a decrease in the number of the magnetic lines of force in the coil, and so a direct induced current (*i.e.* in the same direction as the original current) is set up, *which tends to* **prolong the steady current** *for a short time*, and which frequently gives rise to a spark at the point where the circuit is opened. The induced current, thus produced at *break*, is sometimes called the "**extra current.**" This is due to **self-induction.**

The co-efficient of self-induction (or **self-inductance** or simply **inductance**) of a circuit is defined as the magnetic flux associated with the circuit due to unit current (E.M.F.) flowing in the circuit or as the extra E.M.F. produced in the circuit due to a unit rate of change of current in the circuit.

The practical unit of inductance (mutual or self) is called the **Henry.** It is the inductance which causes an E.M.F. of **1** volt to act round a circuit for a change of current of 1 amp. per sec.

That is, $1\ henry = \dfrac{1 \text{ volt}}{1 \text{ amp.}} = \dfrac{10^8}{10^{-1}} = 10^9$ E.M.U.'s of inductance.

Milli-henry (10^{-3} henry) or *Micro-henry* (10^{-6} henry) are found convenient units for expressing inductances which are ordinarily used in the laboratory.

Note.—It should be noticed that in order to avoid the effects of self-induction in the coils of resistance boxes, each silk-covered wire is doubled over itself and wound in a coil. In this case, the direction of the current in one half of the wire is opposite to that in the other half, and so the effect due to one is neutralised by the other. This is called **non-inductive winding** [(see Fig. 56(a)].

Demonstration of Self-induction in a Coil.—Faraday observed the effect of self-induction as follows.—The coil CD is connected to the battery B and the galvanometer G as in Fig. 76(a). When the key K is pressed and the current is established, suppose the needle is deflected to the position $a'b'$ where it is retained by means of a stop. Then on breaking and again closing the circuit immediately, the needle is deflected momentarily beyond $a'b'$. This is due to self-inductance of CD delaying the establishment of the current through it, as a result of which the fraction of the total current of the cell which traverses G increases. If the needle is held in its normal position ab, a deflection in the opposite direction takes place on breaking the circuit ; in this case, the self-induced current in CD is in the direction of the principal current, *i.e.* it passes *through* the galvanometer G from right to left.

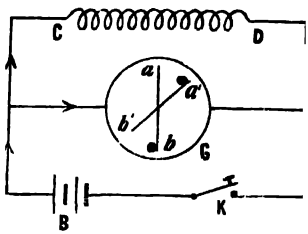


Fig. 76(a)

69. **Ruhmkorff's Coil.**—This is also simply called the **Induction coil**. It is a practical application of the principle of *mutual induction.* By this instrument an induced E.M.F. of very high voltage is produced between the ends of a secondary coil by rapidly making and breaking a primary current of low voltage.

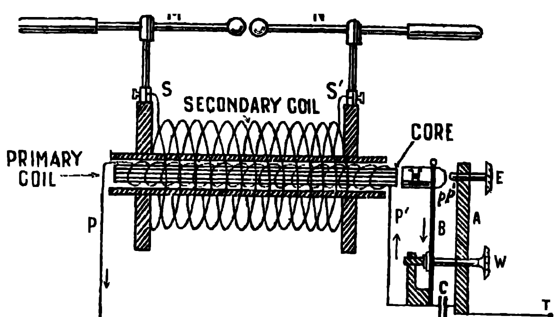


Fig. 77—Ruhmkorff's Induction Coil

Parts.—This instrument essentially consists of the following parts.

(*i*) **A Primary coil.**—It is a coil PP' (Fig. 77) of a few turns of insulated thick copper wire wound usually on a hollow tube of some good insulating material such as vulcanite, etc. ; in the hollow of the tube is placed co-axially a core of a bundle of soft iron wires (or a laminated core of soft iron) which increases the magnetic flux produced by the primary coil and also acts as an electromagnet.

(*ii*) **A Secondary coil.**—It is a coil SS' of a very large number of turns of very fine insulated copper wire wound on a wider outer

tube, also made of a good insulating material such as vulcanite. Sometimes shellac is used to insulate each turn from the other. In order that there may not be any sparking between neighbouring turns, which are at high potential difference when the instrument is in action, insulating separators are also often times used at regular intervals between the two ends S, S'. The terminals S, S' are connected to two adjustable conductors, M and N, which end in two knobs and which may be set apart leaving any suitable air path in between.

(*iii*) **An Iron Hammer.**—One end P' of the primary coil is connected to a metallic spring B which supports at its top an **iron hammer** H provided with a platinum contact point p at its back.

(*iv*) **A Contact-Breaker.**—This consists of an *adjustable metallic screw* E with a platinum point p' in the line with p, fixed to a vertical metallic pillar A. The primary circuit is closed when the screw E is adjusted so that its contact point p' touches the hammer contact point p. By adjusting the base screw W (made of an insulator), the spring can be set at any desired position, *i.e.* its stiffness can be regulated.

(*v*) **A Condenser.**—This is a fixed condenser of a very large value. The condenser C is inserted with one of its two sets of plates being connected to the hammer and the other to the adjustable screw, so as to be in parallel with the contact breaker, as shown in the figure.

(*vi*) **A Commutator.**—By this the primary current can be started, stopped, or reversed (not shown in the figure). It is connected at T, T', which are the two leads from the primary.

Action.—To start the action of the instrument, a battery of low voltage (2 to 6 volts) is connected through the commutator to the two terminals T, T' of the primary coil, and the contact screw E is worked till the two platinum points p, p' touch each other. Thus the primary circuit is closed.

Action in the Primary.—As soon as the primary circuit is closed the current passing through the primary coil magnetises the soft-iron core, which attracts towards it the soft-iron hammer H which is in front. As the hammer is drawn towards the core, the circuit is broken between p, p', reducing the primary current to zero and thereby the core is demagnetised. The spring B brings the hammer back to its original position by virtue of its elasticity and again the circuit is made as before. By this automatic arrangement the primary current is alternately made and broken at a rapid rate.

Action in the Secondary.—When the primary current is made, a strong magnetic flux increasing from zero-value to a maximum is produced by the iron-cored primary coil and this causes a large induced E.M.F. depending on the number of turns used, to act in the secondary coil in the reverse direction, as the primary current grows. Next when the primary circuit is broken, the same flux linked up with the secondary will decrease from the maximum to zero value causing an induced E.M.F. to act in the secondary in the same direction as that of the primary, as long as the current decays. So, for a complete cycle comprising a *make* and the next *break*, it might appear that two electro-motive forces of the same magnitude will act in the secondary in opposite directions. That is, an alternating E. M. F. will be produced at the ends of the secondary terminals. But actually an intermittent undirectional E.M.F. is obtained because the induced E. M. F. at *break* is much greater than that at *make*, which may be explained as follows.—

(*i*) When current in the primary is made, due to the self-induction of the primary coil, an inverse induced E. M. F. acts in the primary, which opposes the growth of the current. So the flux generated increases less rapidly than its rise when there would have been no self-induction. So, due to the effect of self-induction, the inverse induced E.M.F. in the secondary is also correspondingly decreased.

(*ii*) When the primary circuit is broken, a very high resistance is introduced at the air-gap between the contact points p, p' and so the primary current tends to fall to a zero-value instantaneously. Within the secondary the flux is therefore withdrawn at a much greater rapidity than its rate of growth 'at *make*'. So the direct induced E M.F. 'at *break*' should be much larger than the inverse E M.F. 'at *make*'. This is, however, not the only aspect of the action.

The self-induction of the primary creates a direct induced E. M. F. in the primary when the circuit is broken, on account of which the primary current tends to persist, or, in other words, it retards the cessation of the primary current. So the flux produced by the primary current is withdrawn at a lesser rate than it should be when there is no self-induction. Thus, due to the effect of self-induction of the primary, the secondary E M.F. 'at *break*' should be correspondingly diminished, but, nevertheless, the direct E.M.F. produced 'at *break*' will be much larger than the inverse E.M.F. produced 'at *make*' *in the secondary coil* The direct E.M.F. produced in the *primary* 'at *break*' is large enough to cause a spark at the air-gap between the contact-pieces, where most of the resistance of the circuit is localised. The contact points, though made of platinum in order that they may not

wear out easily, are subjected to these sparks which tend to burn them away.

Use of the Condenser.—The condenser C, which should be of large capacity, placed parallel to the air-gap between p, p' reduces the occurrence of such sparks and also increases the efficiency of the instrument, which may be explained as follows. The 'self-induced E.M.F. 'at *break*' in the primary is imparted to the air-gap, *i.e.* to the coats of the condenser. As the capacity of the condenser is large, the plates are charged to a. P. D. too small to lead a spark across the air-gap. So the sparking is diminished. The tendency of the primary current to persist being thus reduced, the rate of withdrawal of the flux during '*break*' is considerably increased, which helps to develope a larger E. M. F. in the secondary 'at *break*'. There is yet another aspect of the action of the condenser. During '*break*', as soon as the condenser is charged, it discharges itself immediately owing to the low resistance of the primary circuit; that is, the charge in the condenser *rebounds* and traverses the primary coil in the opposite direction. This is equivalent to removal of the flux and their re-insertion in the opposite direction. The change of the flux is, thereby, almost doubled; so the secondary induced E. M. F. produced during '*break*' of the primary current is also made almost double. In a complete cycle comprising 'a *make* and a *break*', the direct induced E.M.F. 'at *break*' will therefore greatly predominate over the inverse induced E. M. F. 'at *make*' resulting in a unidirectional secondary discharge, but of the intermittent type, taking place only during '*breaks*'.

It should be noted that the **current** induced in the secondary *depends*, besides the rate of change of the number of lines of force, *upon the resistance of the circuit* which is fairly high, whereas the induced **E. M. F.** *depends* on (a) the *number of turns of the secondary*, and (b) the *rate of change of the number of lines of force*, and *not on the resistance of circuit*; so, though the **voltage** obtained in an induction coil is **very high**, the **actual current** (amperage) of the secondary circuit is **very small**

N. B. For a spark of 1 inch length, a pressure of about 20,000 volts is required. The P.D. produced by an induction coil of the medium size may vary from 20,000 to 30,000 volts.

It will be noticed that in power-stations, where very strong currents are used, switches of special design are used so that they can be pulled away very rapidly in order to prevent damage to the contacts where the circuit is broken. In houses also switches fitted similarly with a spring are used to serve the same purpose.

By an induction coil a greater quantity of electricity at high voltage is obtained than by a Wimshurst machine.

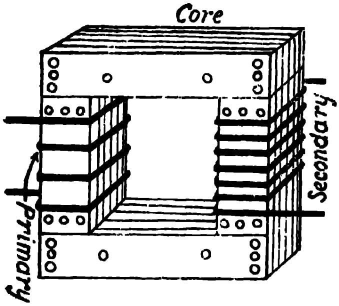


Fig. 78.—Transformer

70. Transformer.—This is an arrangement in which a larger current at low voltage is transformed into a smaller current at higher voltage, or a smaller current at higher voltage is transformed into a larger current at lower voltage. The former is known as a **"step up" transformer**, and the latter as a **"step-down" transformer**. An *induction coil* is an example of the *step-up* transformer. The only difference between a step-up transformer and an induction coil is that in the former the iron forms a closed magnetic path while in the latter it does not

A transformer essentially consists of two coils of insulated wire to one of which—called the *primary* coil—the current to be transformed is supplied, and the current is delivered by the other, called the *secondary* coil. The coils containing different numbers of turns are wound separately on a continuous soft iron core (Fig. 78). When a current flows round the primary, the iron core becomes magnetised, and lines of force from one coil will almost all remain in the core, and so pass through the other coil. In a transformer we have,

$$\frac{\text{E. M. F. in the secondary}}{\text{E. M. F. in the primary}} = \frac{\text{No. of turns in secondary}}{\text{No. of turns in primary}}.$$

So in a **step-up** transformer the secondary contains more turns than the primary, and in a **step-down** transformer, the secondary contains fewer turns than the primary.

A transformer is not used on a direct current circuit, but with only an *alternating current*, *i.e.* a current which reverses its direction of flow at regular intervals of time (see Art. 77) and thus reverses the magnetic flux which causes an induced E.M.F. in a secondary linked up with it.

Questions

Art. 63.

1. What is an induced current ? Describe typical experiments whereby the production of induced currents may be illustrated. **(C. U. 1909, '13, '15, '16, '18, '19, '26, ; Pat. 1928, '21, '28, '42 ; All. '20, '23, '29)**

2. Given (*a*) a coil of wire whose ends are connected to a sensitive galvanometer, (*b*) a coil of wire whose ends are connected to a battery, (*c*) a bar-magnet; devise experiments from which the laws of electromagnetic induction can be deduced. (C. U. 1949)

3. A coil is connected to a sensitive galvanometer. Another coil carrying a current is then (*a*) quickly introduced into the first, (*b*) while still there, the current in the second coil is reversed, and (*c*) finally withdrawn quickly. Explain the effect on the galvanometer. Can you produce a similar effect by some other means? If so how? (Pat. 1928)

[**Hints**.—Similar effect can be produced by a bar-magnet.]

3. (*a*). It is known that a current can be induced in a coil by moving a magnet near it. What conditions determine (*a*) the direction, (*b*) the duration, (*c*) the magnitude of the induced current? Give experimental evidence in support of your answer. (C. U. 1946)

4. Describe some experiments to show that electric current may be produced even without batteries. (Dac. 1932)

(See also Arts. 77 & 78).

5. Give a brief account of the principal phenomena of electro-magnetic induction. (C. U. 1931, '39 ; Pat. '43)

6. What do you understand by 'induced current'? How may it be produced? Upon what factors and in what manner, do (*i*) the strength, and (*ii*) the direction, of the induced current depend? (Pat. 1940, '48)

[For (ii) see Art. 65.]

• **Art. 64.**

7. State the laws of electro-magnetic induction and describe suitable experiments illustrating each of them. (C. U. 1926 ; Bom. 1932 ; Pat. 1949)

(See also Art. 63).

Art. 66.

8. State Lenz's law and apply it to explain the production of electrical currents by induction. Show that it follows from the principle of conservation of energy. (C. U. 1933, '36, cf. '47 ; Cf. Pat. '41, '48)

8. (*a*). State and explain Lenz's law. (Utkal, 1948)

Art. 69.

9. Describe an induction coil, and state the reasons for making the primary coil consist of a few turns of thick wire and the secondary of a very large number of turns of thin wire. What is the part played by the condenser? What is the function of the soft iron core? (C. U. 1936 ; Pat. 1938, '39, '45)

10. Sketch the parts of a simple Ruhmkorff's coil, and explain its action. Is there any difference between the high potential differences obtained with a Ruhmkorff's coil and with an electro-static machine? If, so, account for it. (Pat. 1920, '30)

11. Describe a sectional diagram of a Ruhmkorff's coil, with an index of parts, and explain its action. (C. U. 1922, '25, '27, '36, '42, '44 ; Pat. 1929, '32, '42, '43 ; All. 1924, '30, '45 ; Dac. 1934.)

12. Explain the construction and working of an induction coil, pointing out the ways in which the different parts of it contribute to increase the lengths of the spark between the terminals of the secondary. Mention some uses to which it has been put in a laboratory. (Pat. 1946)

CHAPTER VIII

Generation and Technical Applications of Electricity

71. Applications of Electricity.—Most of the applications of an electric current depend upon one or other of the following three properties :—

(1) *An electric current produces heat* in a conductor when it flows through it. This heating effect has been utilised in electric lamps, furnaces, etc., some of which are given below.

(2) *An electric current produces chemical changes* when it passes through certain solutions. This effect is known as electrolysis, which has been utilised in electro-plating, purification of metals, etc. (*vide* Chapter VI).

(3) *An electric current produces magnetic fields* surrounding it. Some of the applications of this effect have been given before and some are given below.

72. **Electric Lamps and their Progress.**—(*Arc lamps*).—Sir Humphry Davy made the earliest attempt (1810) at lighting by produ-

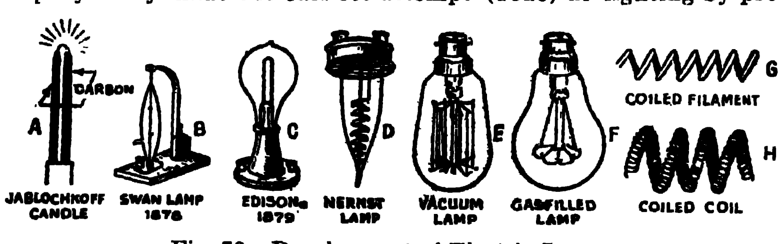


Fig. 79—Development of Electric Lamps

cing an *arc* between two carbon rods [vide Art. 72(2)]. It could not

be universally accepted for domestic uses owing to its large power consumption and difficulty of even and continuous operation, Jablochkoff devised a sort of carbon arc, called "arc candle" (1877), operated by alternating current. The two electrodes, being alternately positive and negative, wore out equally and the space between them could thus be kept constant, and so this arc ran more evenly than the Davy's arc. It had therefore a greater measure of success; but this as well had to be discarded owing to large comsumption of power and shortness of life.

Filament lamps.—Grove and Moleyns (1840) first used a metallic wire (*platinum*) for lighting by heating it to incandescence by electric current. Such wires became brittle due to *oxidation in air*. Starr and King suggested (1845) metal or carbon filaments, *in vacuum*, to prevent oxidation. Springel introduced an efficient Vacuum pump in 1875 and this enabled Edison in America and Swan in England, during 1878-79, to make the first successful carbon filament lamps. Edison's carbonised bamboo filament lamps (1880) gave about 3 *lumens per watt.**

Carbon filament lamps have, however, many defects. Resistance of carbon decreases with rise of temperature. So more current is allowed through the filament when it is hot than when cold. This produces over-heating and much wastage of energy. Carbon disintegrates slowly causing a gradual blackening of the inside of the bulb. At about 1800°C. it volatilizes, whereas a higher temperature is necessary for a strong illumination. Again, carbon filament lamps consume relatively larger power than other lamps. So, now-a-days such lamps are mainly used as lamp resistances. Nernst devised a lamp where the filament was made of *rare earths*, which became conducting at incandescence, no vacuum in this case being necessary to prolong life The filament used in vacuum lamps was up to this time in the form of a long open grid. Welsbach (1898-1902) used *osmium* wire which consumed less power but proved to be brittle. Von Bolton and Feuerlein (1905) used *tantalum* which grew soft when hot and required too many supports along its length, but was found suitable to be drawn into fine wires. Huntington (1884) first used *tungsten* as the material for the filament, but it came to be accepted as the *standard material* for the filament since Coolidge (1909) found a process for improving its ductility and enabling it to be drawn into fine wires.

Introduction of gas-filled lamps.—In vacuum lamps the filament can be raised to high temperatures no doubt (to increase efficiency), for

*One C. P. = 4π lumens.

little heat is lost by the filament through conduction, but the difficulty at such high temperatures is that the filament evaporates and blackens the bulb. It has been, however, recently discovered that this evaporation would stop if the bulb were filled with an **inert gas**, such as argon, nitrogen, or a mixture of argon and nitrogen, etc., after it is pumped clear of air. But high temperatures could not be reached in such bulbs owing to heat losses through conduction having increased due to the gas used. This difficulty was overcome by Langmuir in 1913 by having the **filament made in the form of a coil** whereby heat losses have been greatly reduced. Further improvement in this direction has been made recently (1934) by coiling the coiled filament between two metal electrodes. Such lamps are known as "**coiled coil lamps**". Gas-filled lamps (also known as **half-watt lamps**) available in the market are quite satisfactory from the standpoint of illumination, cost, and length of life. Such lamps are ordinarily nitrogen filled and they consume about 0·5 watt (*i.e. half watt*) per candle when running at the specified voltage. Bulbs filled with argon emit a bluish light which is somewhat soothing to the eye. Extreme precision and rigid control in the art of filament-making are the trends towards improvement at the present times. A small change in the diameter of the filament may cause a great difference in the length of life of a bulb.

Luminous Discharge Tubes.—In recent times luminous discharge tubes have begun to be also used for lighting purposes. Such tubes fall into two main categories.

Cold Cathode lamps, containing a gas at low pressure between two metallic electrodes connected to a high voltage supply, produce a defuse glow filling the tube, which may be bent to any shape (vide Art. 84, Chapter IX). The colour of the light depends on the gas used ; thus :—neon, red ; argon, blue ; mercury vapour in brown glass, green ; helium in yellow glass, yellow, etc Such lamps are used frequently for advertisement purposes. **Hot Cathode lamps**, comparatively more recent, use cathodes in the form of cylinders (thoriated tungsten) with heating wires inside them. The cathode emits electrons on being heated and the discharge forms an arc. In the mercury discharge type of this lamp, the discharge takes place inside an inner vessel of quartz glass. Discharge tubes, coated with *luminescent powders*, have recorded efficiencies between 5 to 60 lumens per watt due to the energy conversion effected by them. Tubes, run at high pressures, have been noticed since 1935-36 having efficiencies near 80 lumens per watt.

Informations on Electric Lamps.—The **Illuminating Power** (or **brightness**) of a lamp is expressed in candle-power (c. p.). Thus

"16 c. p." marked on a lamp indicates that the illuminating power of the lamp is equal to that of 16 standard candles.

The rate of consumption, or the power absorbed, by an electric lamp is expressed in watts (watts = volts × amperes, see Art. 50).

The power (in watts) absorbed by a lamp for each candle power of illumination is called the **efficiency** of the lamp. Thus

$$\textbf{Efficiency} = \frac{\text{Watts absorbed by lamp}}{\text{Candle-power of lamp}} = \textbf{Watts per candle-power.}$$

The P. D. which should be applied to a lamp is stated on it by the maker. If a higher P. D. is applied, the current through the filament becomes too great and its temperature higher than it should be. Though in this case the lamp gives more light, its '*life*' will be *short*. On the other hand, if the applied P. D. is too low, the temperature will be lower, and the *life* of the lamp in this case may be *long*, but its efficiency will be lower.

If a 16 candle-power carbon lamp, when run on a circuit which maintains a potential difference of 220 volts, consumes 0·25 ampere, the efficiency (the rate of consumption per candle-power) of the lamp $= \frac{220 \times 0{\cdot}25}{15} = 3{\cdot}4$ watts per c. p.

(**Efficiency.**—Earlier type of tungsten filament lamps had an efficiency of 1·4 – 1·7 watts per c. p. The efficiency of the modern gas-filled lamps is about 1·1 – 1·4 watts per c. p. for small lamps, while for large lamps consuming more than 100 watts, the efficiency is 0·7 – 1·0 watt per c.p.)

[**We pay for Energy.**—The bulb of a motor car light is often found to be marked "6V – 24 W.", *i.e.* 6 volts 24 watts. So this lamp requires a current of 4 (= 24/6) amperes, whereas a common lamp used for lighting a room, which is often marked "220V – 60W," will require three-elevenths of an ampere. But the second lamp will give much more light than the small motor lamp. At first sight the house-lamp appears to be the cheaper source of light, but in fact it is not, because the number of accumulators required to be used for the motor lamp is only three, while the house lamp will require no fewer than one hundred and ten. So it is evident that the cheapness does not depend only on the current used in the lamp but also in the voltage at which the current is drawn, *i.e.* on the total energy or power (current × volt). What the electric supply company charges for is not the current, but the *energy* consumed.

A consumer usually pays for his light by the "kilowatt-hour", *i.e.* 1000 watts for 1 hour. The energy consumed by the above lamp is 220 × 0·25 = 55 watts or 0·055 kilowatts. One such lamp running for 20 hours will consume 0·055 × 20 = 1·1 kilowatt-hours.]

(2) **Arc-Lamp.**—If two carbon pencils, which are at a difference of potential of about 60 to 70 volts, are brought into contact and then slightly separated, current continues to pass forming a luminous "arc" between the points. The discharge is characterised by intense light and heat. This light is called the **arc light.**

At the point of contact of the pencils, the resistance is very high; so, when current passes, the temperature becomes very high and corbon volatilises. When they are separated, the hot carbon particles pass from the positive to the negative end. Hence the positive carbon becomes hollowed out and it develops a crater at its mouth, and the negative one becomes pointed (Fig. 80). The positive carbon is consumed twice as fast as the negative one. If the distance between the carbons becomes too great, the arc goes out. To obtain steady illumination, the carbon pencils, when worn away, are made to approach each other by some automatic arrangement. The approximate temperature between the points is 3500°C.

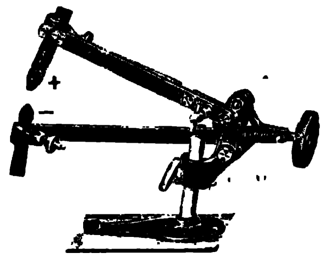
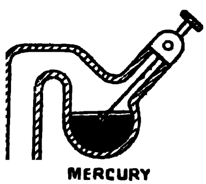


Fig. 80—Arc Lamp

(3) **Mercury Arc Lamp.**– The usual form of this lamp [Fig. 80(a)] is a bent tube of quartz having two terminal bulbs containing mercury. An electrode sealed to each bulb is placed in communication with the mercury. The electrodes may be connected to the supply mains through an adjustable resistance. As the mercury in the two bulbs are not in touch, in order to start the arc, the tube has to be slightly tilted when mercury from one bulb runs into the other producing the necessary metallic connection. The large current that passes developes sufficient heat to turn some mercury into vapour which afterwards forms the conducting link between the electrodes. As the current passes, the inside of the tube glows with a brilliant bluish light. This radiation, which is rich in ultra-violet light, is widely used in ultra-violet spectroscopy and medical therapy. As ultra-violet light is injurious to the eye, a worker should wear coloured glass spectacles, as a protective device, which absorbs these ultra-violet rays.



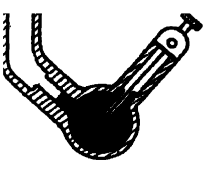


Fig. 80(a)—Mercury Arc Lamp

(4) **Electric Furnaces.**—In small electric furnaces spirals of high resistance wire, say nicrome (nickel 60, iron 25 and chromium 15 parts) or molybdenum, are wound round tubes of fireclay, and heat is developed when a strong current is passed.

(5) **Electric Stoves, Kettles, Irons, etc.**—Those are other applications of the "Joule Effect", and all of these are constructed on the same principle. In each of these contrivances high temperature is developed by passing a strong current through coils of wire of high resistance.

(6) **Electric Welding.**—It is a process by which two pieces of metal, such as tram-rails, are joined together.

73. **House Wiring.**—Two wires from the street-supply mains between which a constant P.D is maintained by the Electric supply company are taken to a house, and are first of all connected in series with a machine, called an *watt hour-meter*, in which the units of electricity consumed in the house in any period are recorded. Then the two wires (marked + and −) are taken to the different rooms of the house where the lamps and fans are all connected in parallel, as shown in Fig. 81. Fig. 81 represents the wiring of a building. The wiring is always carried out with the lamps placed in **parallel** for the following advantages,—(*i*) the remainder of the circuit will not be affected if a lamp is fused or taken out ; (*ii*) each lamp of the circuit is run with a constant P.D. supplied by the generator ; (*iii*) addition of more lamps in the circuit does not affect the current strength in any particular lamp (Art. 36). The *street supply mains* may be *two wire* or *three wire* systems. Two leads from the mains are taken to a house.

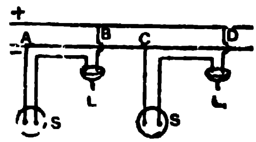


Fig. 81

Examples.—(*1*) *An electric lamp bears the mark "220V – 60 W." Explain this fully.*

The lamp is meant for 220 volts supply. A 60-watt lamp expends energy at the rate of 60 joules per second, *i.e.* 60×10^7 ergs per second.

The current taken by the lamp will be $\frac{60}{220} = 0{\cdot}27$ amp.

The hot resistance of the lamp is $\dfrac{200}{0{\cdot}27} = 814{\cdot}8$ ohms.

(*2*) *How many joules of energy are consumed when a 50-watt lamp burns for 10 minutes ?* (*C. U. 1933*).

(See Art. 50.) 1 watt = 1 joule per sec. So a 40-watt lamp consumes 40 joules of energy per sec. ∴ In 10 minutes or 600 seconds, the energy consumed = 40 × 600 = 24,000 joules.

(3) *A railway carriage is lit up by thirteen lamps, each taking 1·2 amperes at 15 volts. Find the resistance of a lamp, and also the total power used in lighting the compartment.* *(C. U. 1929)*

Here, resitance of the lamp is $\frac{E}{R} = \frac{15}{1\cdot5} = 12\cdot5$ ohms.

As there are 13 lamps, the total power used

$= 13 \times \{$E. M. F. (in volts) $\times$ current (in amperes)$\} = 13 \times 15 \times 1\cdot2 = 234$ watts.

(4) *A university hostel has 360 lamps installed. The lamps consume 50 watts each and are lighted for 6 hours daily for 9 months. The voltage of the supply is 220 and the current costs 6 annas per killowatt-hour. Find the cost of the current as well as the maximum current used.* *(All. 1930)*

Each lamp consumes 50 watts. Therefore 360 lamps consume (360×50) watts.

Hence the total power consumed for 9 months (each month being of 30 days) at 6 hours a day $= 360 \times 50 \times 30 \times 9 \times 6$ watt-hour

$$= \frac{360 \times 50 \times 30 \times 9 \times 6}{1000} = (540 \times 54)\text{kilowatt-hours.}$$

The cost at 6 as. per kilowatt-hour $= \frac{540 \times 54 \times 6}{16} = 10{,}935$ rupees.

The voltage of the supply is 220 and the total power consumed $= (360 \times 50)$ watts. Now, watts $=$ amperes $\times$ volts. Since the lamps are in parallel and the P. D. is 220 volts, the maximum current

(*i.e.* the current in the main lead) $= \frac{360 \times 50}{220} = 81\cdot81$ amperes.

(5) *In a house there are 20 half-watt lamps each of 60 candle-power, the voltage of the supply being 220. Calculate (a) current used by each lamp, (b) energy consumed per hour, (c) cost of lighting the house for 20 hours at 4 annas per kilowatt-hour.*

(*a*) Electrical energy consumed by each lamp of 60 c. p. $= \frac{1}{2} \times 60 = 30$ watts. Watts $=$ volts $\times$ amps. $\therefore$ Current consumed by each lamp $= \frac{30}{220}$ $= 0\cdot136$ amp.

(*b*) 1 watt $=$ 1 joule per sec. $\therefore$ 30 watts $=$ 30 joules per sec.
So the energy consumed by 20 lamps per hour $= 20 \times 30 \times 60 \times 60$ joules $= 2{,}160{,}000$ joules.

(*c*) Total power consumed $= \frac{20 \times 30}{1000}$ K. W. $= 0\cdot6$ K. W.

$\therefore$ **Total cost for 20 hours $= 20 \times 0\cdot6 \times 4$ annas $= 3$ rupees.**

74. Electric Bell.—This is an apparatus (Fig. 82) where the magnetic effect of electric current is utilised for converting electrical energy into sound energy. It consists of a horse-shoe *electro-magnet* M and a *hammer* H connected to a soft iron piece, called the *armature*, carried by a spring S, one end N of which is rigidly fixed. When no current passes through the instrument, the other end of the spring makes contact with the point of the screw C, which being adjustable can be worked to establish contact if there is any looseness. The circuit includes the battery B and the key K. On pressing the

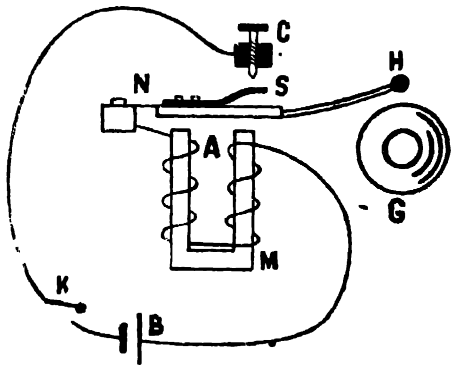


Fig. 82—Electric Bell

key, when a current passes through the electro-magnet, the armature is attracted by it and the hammer strikes the *gong* G. But when the armature is attracted, the circuit breaks at S. The electro-magnet being demagnetised does not act, the armature is brought back to its original position by the action of the spring, and makes the circuit complete again. So the armature is again attracted, and the hammer strikes the gong. By this process the bell keeps ringing intermittently as long as the current continues to flow.

This is commonly used as a **calling bell.** Leclanche' cells are suitable for working this instrument. A *Bell-push* K, which is simply a spring key, is generally included in the circuit.

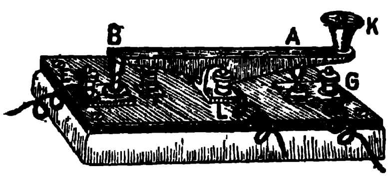


Fig. 83—Morse key

75. Telegraph.—The *electric telegraph* is another application of the electro-magnet. Telegraphy means the art of sending messages from one station to another at a long distance apart by a recognised code by the electrical method.

The essential parts of a line-telegraph system are (*i*) **Line wire** connecting two places between which messages are to be sent; (*ii*) **Battery**; (*iii*) **Transmitter** for sending the signal; (*iv*) **Receiver** for receiving the signal.

Transmitter.—Now-a-days **Morse key** (Fig. 83) is used as a **transmitter**. It consists of a brass lever AB having a fulcrum at the middle, which is permanently connected to the line. The lever has got a screw at the extreme end which touches a button below it in its natural position. At the other end there is a knob K which, when pressed, touches a button below, connected with the positive pole of a battery, and thus sends a current through the line-wire to the other station. How a Morse Key is actually connected is shown in Fig. 84. The duration of the current depends upon the time for which the knob is kept pressed. When the knob is released, the spring of the lever makes the screw at the other end touch the button below in its natural position which is connected with the receiving instrument.

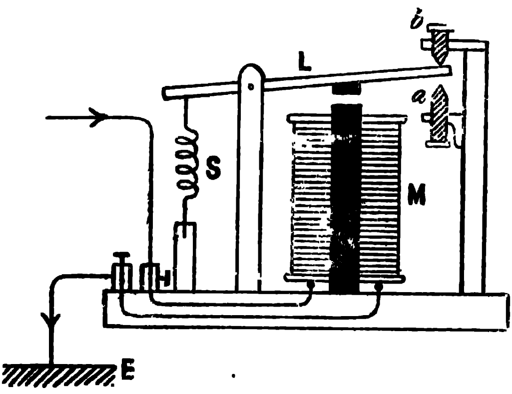


Fig. 83(a)—The Morse Sounder

Receiver.—The Morse Sounder Fig. 83(a) is used now-a-days as a **receiver**. It consists of an electro-magnet M with a piece of soft iron fixed on a pivoted lever L placed just over the core of the electro-magnet. As soon as a current from the sending station passes through the electro-magnet M, it attracts the lever which strikes against a screw 'a' below and makes a clicking sound When the current stops, the lever is released, and is brought back to its original position by the action of the spring S and so it strikes against the screw 'b' at the top. The signals are based on the duration of the

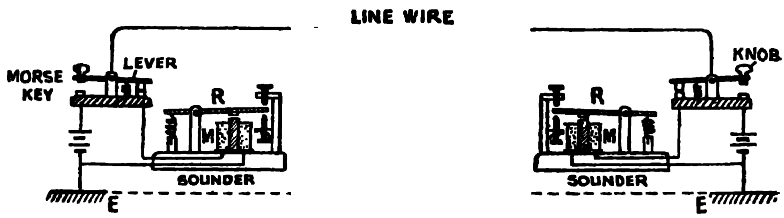


Fig. 84—Telegraph Circuit.

interval between the striking of the screw at *a* and that at *b*. Thus by manipulating the key, *long* and *short* sounds corresponding to *dashes* and *dots* can be produced and transmitted to the receiving station.

Principle of a Telegraph System.—Fig. 84 represents a simple telegraph system between two stations. On pressing the knob of the Morse-key in the sending station, the contact of the line wire is established with the positive pole of the battery, the negative pole being connected to the earth. A current goes along the line-wire through the Morse key to the sounder of the receiving station and operates it, as explained before. When the knob of the transmitting key is released, contact of the lever is now made with the sounder, which is then ready to receive signals from the other transmitting key. It should be noted that, in this case, only **one line wire** is necessary, *the return path for the current being completed through the earth.*

Relay.—For a very distant station the line-wire may be extremely long and so the current may be too feeble to operate the Morse sounder. In such a case an appliance, known as a **relay**, is used in the circuit near the sounder in order to magnify the effect. It consists of an electro-magnet M and a light soft iron armature attached to a lever L. The electro-magnet is placed in the line circuit (Fig 85). When the weak current passing along the line-wire is passed through the relay, the armature is attracted downwards and touches a contact screw P, which puts into circuit a local battery B, and thus the sounder S, which is in the same circuit as the battery, is operated efficiently.

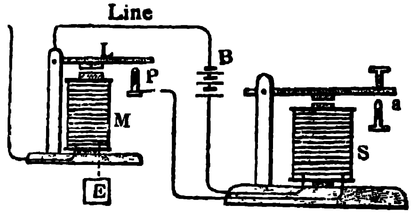


Fig. 85—Relay

N. B. It is to be noted that in the simple system described above, message can be transmitted over the line in one direction only at a time. In the more improved system, called the Dupleix system, which is mostly used, messages can be transmitted and received at the same time at the same station.

76 The Telephone.—This is an electrical arrangement by which speech can be transmitted from one station to another. The *telephone* was invented by Graham Bell in 1875, and the instrument, Bell Telephone, goes by his name. It consist of a long bar-magnet, M (Fig. 86), round one end of which there is a flat coil C of insulated copper wire. The two ends of the coil are connected to the two

binding screws T, T. In front of the coil C there is a thin iron diaphragm D held firmly in a wooden cap and almost touching the end of the magnet.

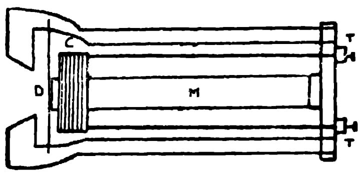


Fig. 86—Telephone

The conical opening in front of the diaphragm converges the sound waves directed to the diaphgram. The terminals T, T are connected to two line wires at the distant ends of which a similar apparatus is connected

Action.—The diaphragm, being just in front of the magnet, becomes magnetised by induction, and when the diaphragm is spoken into, it vibrates to-and-fro by the sound waves which fall on it, and so the number of lines of force passing through the coil is changed. Induced currents are thus set up in the coil. These currents, travelling through the line wires, change the strength of the field in which the iron diaphragm of the distant telephone is situated and throw it into the *same kind* of vibration as the first diaphragm. Thus, the second diaphragm reproduces the sound which caused the first one to vibrate.

The *Bell telephone* (Fig. 86) may be used both as a *transmitter* and a *receiver*. It is independent of any battery, but the induced current generated is so weak that it is not successful for a long distance. Secondly the quality of the sound received is also poor.

A battery connected in the line will improve matters a little no doubt, but for proper reception an improved receiver is necessary. Such a modern receiver consists of a U-shaped magnet having one coil round each pole, the two coils being connected in series and the terminals finally joined to the line-wires. Both the poles are placed close to each other behind the diaphragm, so that the line current will now traverse both the coils and the electro-magnetic action on the diaphragm will be almost doubled.

76. (a) Telephone Circuit.—The essential parts of a telephone circuit are (*a*) **a Transmitter,** (*b*) **a Receiver,** and (*c*) **a Line**

Bell's telephone is not used as a transmitter ; but a transmitter, working on a different principle, called the **Microphone transmitter** (Fig. 87) is used. Again, in modern practice an improved type of *Bell's Telephone* using a horse-shoe magnet provided with double coils over the two poles, as described above, is used as a *receiver*.

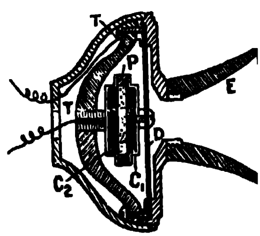


Fig. 87—Microphone Transmitter

The *microphone transmitter* consists of a shallow chamber, loosely filled with granules of carbon, having two thin carbonplates, one C_1 at the front and the other C_2 at the back The centre of a steel diaphragm D is rigidly fixed to the middle of the front carbon plate. In front of the diaphragm is the conical mouth-piece E which converges the sound-waves directed to it during a talk or speech. The diaphragm is connected to one of the terminals T, T, the other terminal being connected to the back carbon plate Usually, the plates C_1 and C_2 are separated at the edges by cotton wool pads P. The principle of this transmitter is that when the pressure at the surface of contact of two carbon particles varies, a large variation in the electric resistance of the contact takes places.

Action.— In Fig. 88 it has been shown how two stations can be connected by a telephone system using a microphone transmitter M and a suitable receiver T at each station. The two stations have been shown, for convenience of drawing the figure only, as upper station and lower station.

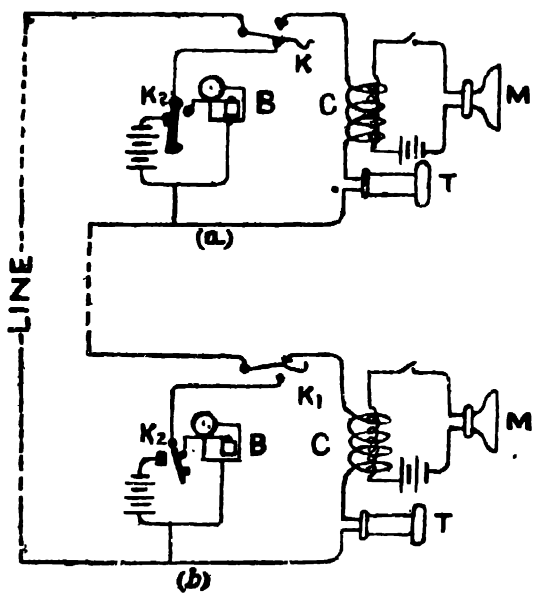


Fig. 88—Complete Telephone Circuit

The primary of a transformer or induction coil C is used in series with the transmitter and battery, in order that the resistance of the microphone circuit may be kept low so that any variation of resistance of the carbon granules may be as large a proportion of the total resistance as possible and would thus produce large variations in the current. The secondary, of many turns, is in series with the line and the receiver at the other station. In the position shown, the receiver T of the lower station is supposed to be in use When not in use, it is to be hung up on the key K_1, when the induction coil will be thrown out of circuit. In the figure, the receiver of the upper station has been shown hung up on the key (K), which thus joins the line to the

key K_2, and the distant station then can call up, since the line is in series with the electric bell B. To call up the distant station, the key K_2 must be pressed. This puts the battery in series with the line and the electric bell of the distant station. In the figure the key K_2 of the upper station has been shown pressed.

Owing to the disturbing effects of stray currents which often pass through the earth, the earth, in this case, cannot be used for the return circuit as in telegraphy. Hence **two line-wires** are necessary for a telephone circuit.

77. **Dynamo**.—*It is a machine for producing electric energy at the expense of mechanical energy.* If a closed coil of wire is made to rotate in a magnetic field so as to change the number of magnetic lines of force passing through it, an induced E. M. F. is generated in the coil. This induced E. M. F. depends upon the rate of change of the lines of force,—that is, it depends on the number of turns of wire, the strength of the magnet, and the rate of rotation of the coil. Suppose a coil of wire $ABCD$ is rotated between the poles N, S of a strong magnet or a powerful electro-magnet (Fig. 89). Let the coil be rotated from the horizontal position in the clockwise direction. When AB rises and CD falls, the direction of the induced current, according to Fleming's right-hand rule, will be as indicated in Fig. 89, *i.e.* from A to B and C to D. Now, consider the second half of the revolution, *i.e.* when the coil is again horizontal. This time the side DC will be in the position now occupied by AB, so DC begins to rise and AB begins to fall Hence the induced current flows in the direction $DCBA$, opposite the former direction. This shows that for a complete rotation of the coil the induced current (or E. M. F.) changes or alternates its direction at each half revolution. The strength of this current continuously increases from a zero value to a maximum one and then gradually diminishes to zero value again, during the first half revolution, after which the direction of the current is reversed, reaches a negative maximum value and then comes to the zero value again during the other half revolution. The current going through this series of changes is said to have completed one **cycle**, and the number of cycles completed per second is called the **frequency** of the current. The current produced in this way is called *alternating current* (*A. C.*), and the machine producing it is called an **alternating current** (A. C.) **dynamo** or simply an **Alternator**. This is the *principle* of the dynamo.

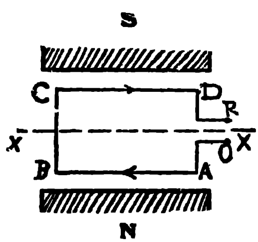


Fig. 89

It should be *remembered* that an alternating current may be used for lighting and heating purposes, but *it is unsuitable for electrolysis or for charging batteries.*

Parts—(*i*) The rotating coil is called the **armature**. and the magnetic field in a dynamo is produced by a strong electromagnet, called (*ii*) the **field magnet**. The armature consists of a very large number of turns of wire, and is rotated on a horizontal axle by any prime mover, such as a steam engine, oil Engine, Steam turbine, etc.

Besides the above two essential parts of a dynamo. *i.e.* the armature and the field-magnet, the following two parts are also necessary.

(*iii*) **Slip-Rings**.—These are two metal rings (Fig. 90) to which the ends of the armature coil are connected. These are rigidly mounted on the main shaft but are insulated from it, and they rotate with the armature, which is also properly insulated from the shaft.

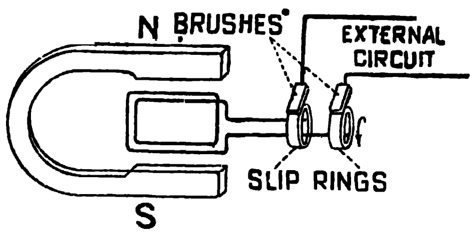


Fig. 90

(*iv*) **Brushes**.—These (Fig. 90) are made of carbon rods and are kept lightly pressed against the slip rings by means of springs, and are connected to the external circuit.

The current produced by the rotation of the coil (as explained above) is collected by means of the brushes bearing on the two slip rings and conveyed to the external circuit.

Direct Current (D. C.) Generator.—The alternating current produced in an A. C. generator can be made to flow in the same direction through the external circuit by a special device, called the **commutator**, and thus the machine can be converted into a **direct current (D. C.) generator**. The process is called **rectification**. The principle of construction of a D.C. generator is the same as an A.C. dynamo except that the two ends of the armature are fastened to two halves C, C' of a metal cylinder, insulated from each other (Fig. 91),

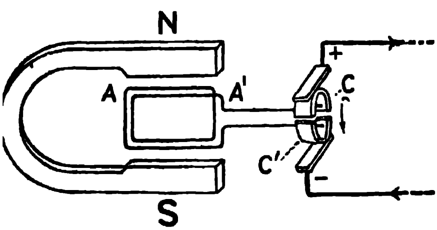


Fig. 91

called the **commutator**, instead of the 'slip rings' These two half cylinders are fixed to the main shaft but insulated from it, and it is so arranged that, during a half revolution of the coil, each half cylinder (say, C) makes contact with a particular brush (Fig. 91), and when the current is reversed during the other half revolution, the same half cylinder C is in contact with the other brush. So during the first half of the revolution (say, anti-clockwise as shown by arrow) the current flows from C' to C in the coil and from the positive to the negative brush through the external circuit ; and during the other half revolution, when the direction of the current is reversed, the current flows from C to C' in the coil, but during this time the positions of C and C' are also interchanged, and so now C' makes contact with the positive brush, and C with the negative brush. Hence in the external circuit the direction of the current is the same as before.

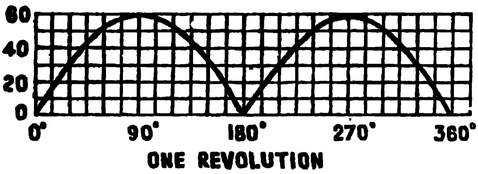


Fig. 92

The E.M.F. generated by a single coil connected to two split-rings (commutator), rotating between two poles, as described in the D. C. generator above, will be unidirectional no doubt, but will fluctuate between a maximum and zero values, as shown in Fig. 92. To make such a pulsating E.M.F. sensibly constant, a larger number of coils in series are spread over the armature, each coil being displaced from the next by a regular electrical interval, with double the number of segments on the commutator to which the coils are connected. The effect is increased by the use of a multi-polar magnet. The E.M.F. produced by each coil will be of same type as in Fig. 92, but at any instant each will differ in phase from the next by the same constant electrical degrees. The E.M.F. obtained at the brushes will be the resultant effect of super-position of these separate E.M.F.'s which are slightly but regularly varying in phase in succession. This resultant E.M.F. will be sensibly constant.

Maximum E. M. F. in a Dynamo.—The maximum E. M. F. produced by a dynamo is directly proportional to (*a*) the area of the armature coils, (*b*) the number of turns in the coils, (*c*) the intensity of the field produced by the field-magnet, and (*d*) the number of revolutions per second.

78. Current induced by Earth's Magnetism : Earth Inductor.—Faraday discovered that currents are induced in conductors moved in

the earth's field. Such conductors are called *earth inductors.* Their action is well shown by the following apparatus known as "Delezenne's circle" (Fig. 93).

It consists of a wooden ring RT having a groove on it which contains a coil of some turns of silk-covered copper wire. The two ends of the coil are connected to a *Spring Commutator* by which the current induced in the coil, although it will change in the coil at every half revolution, is made unidirectional through the galvanometer G. The ring can be rotated on the axis

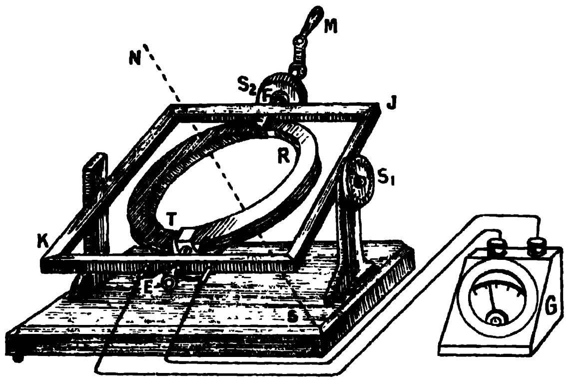


Fig. 93—Earth Inductor

EF by the handle M while the axis EF is itself fixed in a rectangular wooden frame JK movable about a horizontal axis. A pointer on the dial S_1 shows the inclination of the axis EF while another pointer on S_2 indicates the angular displacement of the ring as it is rotated.

In using the apparatus, the plane of the ring is placed at right angles to the line of dip and the axis EF normal to the magnetic meridian ; the plane of the coil is thus placed perpendicular to the earth's field shown by the dotted line N-S.

The coil is then rapidly rotated by the handle M. The current induced in the coil, reversing in direction twice in every revolution (vide Art. 77), being made continuous by the commutator E, produces a steady deflection in the galvanometer G. On reversal of the direction of rotation, the direction of the deflection is also reversed.

The current generated in the earth-inductor, as explained above, is essentially an alternating one. It is an illustration of a simple A. C. generator. The above coil may also be used to generate current by cutting the vertical component of the earth's field, instead of the horizontal field.

79. Electric Motor.—*It is a machine by which machanical energy is obtained at the expense of electric energy ;* so this may be called a **reversed dynamo.** If an electric current from an external source be sent through the armature of a dynamo, it will begin to rotate

between the pole pieces of the field-magnet like rotation of Barlow's Wheel (see Fig. 24). The field-magnet may be either a permanent magnet or an electro-magnet. Wherever a large power is required an electro-magnet is used. The electro-magnet may be excited by current from a separate battery or from the line from which the current for the armature is drawn, the latter method being the most common practice now-a-days. The supply of current to the coil of the field magnet may be made differently.

In **series-wound motor**, the coil of the electro-magnet is in series with the armature, whereas in the **parallel-wound motor**, it is in parallel to the armature.

In the **Compound-wound motor**, there are two coils over the pole-pieces of the field-magnet, one with a small number of turns is placed in series with the armature, while the other having a comparatively larger number of turns is connected in parallel to the armature. Thus the field here is a combination of a series and parallel fields. The operating characteristics of these different motors are different and each type of motor has its own field of applications. By joining the axle of the armature with any other machine, mechanical work can be obtained from an electric motor. Often times, the main shaft is provided with a pulley having a belting arrangement bearing on it, which transmits the motion to other shafts.

The blades of an **electric fan** are made to rotate by fitting them on the axle of the armature of a motor placed inside the outer case. Besides this, an electric motor is used in many other cases, such as printing machines, tram cars, cinema machines, water-pumps, etc.

Motors can be worked by alternating as well as by direct current, but direct current motors are more conveniently worked, and *a direct current machine can be worked either as a motor or as a generator.*

Tram Cars.—*Tram cars* are run by motors. Fig. 94 shows a

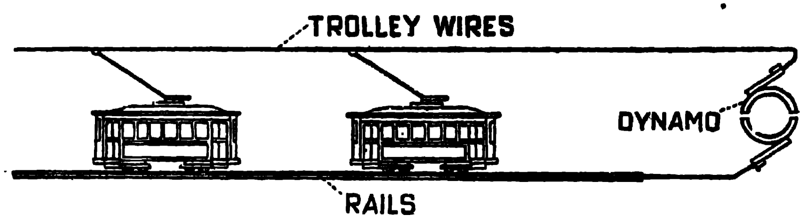


Fig. 94—Tram Car

typical street tram car. The current from the dynamo at the power station is conveyed along the overhead line, called the trolley

wire, to which is touched a small metal wheel fixed at the end of a long flexible pole on the roof of the car. The current from the overhead line is conveyed along a wire attached to the pole to the motors placed underneath the car. The current then goes to the rails through the wheels of the car and returns to the power station. The car stops whenever the wheel of the pole is separated from the overhead wire, or a break is made in the motor circuit.

80. Transmission of Electric Power.—The current which is used in electric lamps, fans, and for other purposes such as driving electric motors, etc., is supplied by an electric supply company from a power station generally several miles off from the places where the current is consumed. The current from the station reaches the consumers along wires, or **supply mains**, as they are called. But these mains have resistances due to which the electrical energy supplied at the station is wasted as heat.

If C amperes be the current flowing through the mains having resistance R ohms, the heat energy produced will be C^2R watts and this energy is wasted. So, for any given current, the loss of power in the mains will be small, if R is small, which can be secured in the following two ways :—

(*a*) *By using materials of low specific resistance, such as copper or aluminium*, and (*b*) *by using thick wires.*

But the cost of *thick* wires, especially when the distance is considerable, would be prohibitive. Again the loss can be reduced by reducing C, the current, for halving the current will quarter the loss(as *loss* $\propto C^2$). So, for a given *electric power* ($P = C \times V$), the current C can be halved by doubling the P. D. (V) at which the power is supplied ($P = C/2 \times 2V$) Therefore it is **economical** to transmit the current at a very **high voltage**. But for domestic purposes a voltage much greater than 220 is considered dangerous on account of the possibility of shocks, short circuits, etc., though for electric railways and other commercial work a voltage as high as 440 or 500 is more usual. Thus the arrangement should be to transmit the power at high voltages and then to convert it to a lower voltage before use. Unfortunately this transformation is not practicable with direct current where a voltage exceeding 500 or 600 volts is impossible, and for which dynamos and motors which should be used are far less efficient and there is much difficulty of ensuring proper insulations of wires, etc. For this reason an *alternating current*—a current which reverses its direction many times per second—is generally used for high voltage transmission.

By means of a "step up" transformer a voltage can be raised up, and it can be reduced by a "step down" transformer. At Niagara

Falls the current is generated by water power at a voltage of 2,000—12,000 volts, which is then raised to about 100,000 volts and transmitted to distant stations, where it is reduced to the voltage which is safe for domestic use. At Sierra Nevada in California, a high voltage power—even above 220,000 volts—has been possible to generate by alternating current which is transmitted to a distance of about 250 miles to Los Angeles

For supplying electricity for domestic and other similar purposes to a small locality, the energy can be generated by a Direct Current Generator or by an Alternator. The alternating current can be rectified into a direct current by a motor generating set. The long wires leading from the power station and running throught the locality are called the **mains,** one of which, in the case of direct current supply, is at a higher potential and is called the **positive wire**, while the other is kept at a lower potential and is called the **negative wire**. In the case of alternating current supply, the current frequently reverses its direction and so there is no fixed positive or negative wire, each wire being alternately at a higher and a lower potential with respect to the other. This variation of potential per second is called the **frequency**, or simply the **cycle**, of the alternator. The common frequency in most towns' supply (A. C.) is 50 cycles per second and it is described as "210 volts, 50 cycles," but frequencies of several million cycles per second are possible in wireless circuits. It should be noted here as a word of caution that an **alternating current is more dangerous than direct current at the same voltage**.

When an A. C. supply is described as "210 volts, 50 cycles", the actual voltage changes from 0 to about 300 volts one hundred times a second, so it is really more dangerous than the voltage described, *i.e.* 210 volts.

81. Advantage of A C. Supply.—An alternating current can be transformed by a suitable transformer (Art. 78) to a smaller current at a higher voltage, or *vice versa*, with very little loss of energy, which is not possible for a direct current (D. C). For example, an A. C. can be transformed down to run a 6 volt lamp, and again it can be raised up to 100,000 volts. Most power-stations use A. C. supply. because electric power can be transmitted at high voltages with far less loss of energy than at low ones, which can be transformed to convenient and safe voltages to be used anywhere.

The **disadvantage of A. C.** is that it cannot be used in electrolysis and charging batteries, though it is as good as D. C. for heating and lighting purposes.

82. Dangers from Electricity.—A large number of fires are very often due to defective insulation upon electric-light wires. When

the insulation of an electric fixture wire, as, for example, a lamp cord, has been damaged, the two bare wires may come together, which may set many materials on fire in an instant. Bodily harm also very frequently occurs from electricity, which may produce fatal results. A bare wire carrying currents at 110 volts or 220 volts pressure may sometimes be handled safely if the skin where the wire touches is perfectly dry, or if the person's shoes are dry (or have rubber soles). But, if the hand be wet, or if the person stands on a damp floor, the result may be fatal, as enough current may pass through the heart to paralyse it. The resistance of the human body is about 30,000 ohm, but most of it lies on the skin, which again, may be as low as only 200 and 300 ohms when the skin is wet, and under such circumstances fatal shocks have been known to be caused even by 100 volts.

Generally, currents at 220 volts (or still higher) pressure are always dangerous. Birds are very often seen to be on the street trolley wires and then fly away safely because they have made no connection to the earth. Similarly, you can safely hang from the same wire without establishing any path for the current to pass on to the earth through your body which would prove fatal.

Example.—*1500 kilowatts are transmitted at 60,000 volts through a cable of resistance 0·109 ohm per mile The diameter of the cable is 0·455 inch. Find the loss of power per mile of cable.*

What would be the diameter of the cable required if the same power were transmitted at 200 volts in order that the power lost per mile should note change.

Power lost $= E \times C = C^2 r$ watt. Here $C = \dfrac{1,500,000}{60000} = 25$ amp.

Hence power lost per mile $= (25)^2 \times 0·109 = 68·1$ watts.

For transmitting this power at 200 volts, the current required $= \dfrac{1,500,000}{200}$ $= 7,500$ amp. If r in. be the radius of the required cable, its area of cross-section is πr^2 sq. in. and that of the old one is $\pi \times \left(\dfrac{0·455}{2}\right)^2$ sq. in.

So the resistance of the new cable $= \left\{\dfrac{\pi \times (0·455)^2}{4 \times \pi r^2} \times 0·109\right\}$ ohms.

$\therefore$ Power lost $= (7500)^2 \left[\dfrac{\pi \times (0·455)^2}{4 \times \pi r^2} \times 0·109\right] = 68·1$ watts,

whence $r = 68$ in.; or the diameter of the required cable $= (68 \times 2)$ in. $= 136$ in. $= 11$ ft. 4 in. (Of course a cable of this diameter is impossible to use).

Questions

Arts. 72 & 73.

1. A railway carriage is lit up from a 15 volt battery by 12 lamps each taking 1·5 amps. and arranged in parallel. Find the resistance of a lamp and total power used in lighting the compartments. (Dac. 1934)

[*Ans* : 10 ohms ; 270 watts.]

2. Describe an electric glow-lamp. (*a*) Why does the filament of the lamp become hot while the wires leading to it remain comparatively cold? (*b*) An electric lamp is marked '40 watts, 200 volts'. Explain these terms. What will be the strength of the current passing through its filment? (*c*) 'The current consumption of such a lamp is one unit in 25 hours.' Comment on the statement. (C. U. 1932)

[**Hints.**—(*a*) This is because $H \propto r$ (see Art. 46). (*b*) The marking on the lamp means that when the lamp is run on a circuit which maintains a P. D. of 200 volts, the electrical energy is consumed at the rate of 40 joules every second or 40×10^7 ergs per second.

Ans : $C = 0·2$ amp.

(*c*) The lamp burning for 25 hours would consume (40×25) watt-hours $= \frac{40}{1000} \times 25 = 1$ kilowatt-hour. This is the Board of Trade Unit (B. T. U.) of current consumption (see Art. 50) and hence the above statement].

3. Why is an electric light bulb made air-tight and free from air? Account for the rise of temperature in the bulb when the current is turned on. (All. 1929)

4. What is a fuse wire? Why is it inserted in practical electrical circuits? (Pat. 1941)

5. Ten 220 volt half-watt lamps are installed in a house. Find out the resistance of the combination, the candle power of each lamp being 50. Find out the number of units (kilowatt-hours) consumed in a month of 30 days, if the lamps burn 5 hours a day. (All. 1929)

[*Ans :* 37·5 kilowatt-hours].

Art. 74.

6. Describe in detail with neat sketches the component parts of the electric bell?
(C. U. 1925, '28, '30, '43, '45 ; Pat. 1921, '31, '32, '40, '47 ; All. 1920, '24)

What kind of cell will you use for working such bells?

In a certain factory steel was once used by mistake instead of soft iron to make the cores of an electro-magnet for some bells. What should be the matter with the bells? (Pat. 1924)

[Hints.—As the steel core will not readily demagnetise itself after the current is stopped, the armature will be held up by magnetised steel core and will not easily move to-and-fro, and so the bell will not work properly.]

Art. 75.

7. Explain the principle of the electric telegraph. (C. U. 1915, '30 ; Pat. '32, '44, '47)

Draw a diagram of Morse's Sounder. (C. U. 1925)

Art. 76

8. Carefully explain the use of the microphone and telephone in the transmission of speech by electricity along wires. (Utkal 1948 ; Pat. 1920, '25, '27 ; Cf. '42 ; Cf. U. P. B. 1947)

9. Explain the action of a telephone with the help of a diagram. (C. U. 1920, '21, '24, '25, '26, '28, '30 ; Pat. 1931, '44 ; All, 1922, '24, '28)

10. Explain the construction and working of a Bell's telephone, and draw a plan of simple telephone connections between two stations. (Pat. 1930, '39)

11. Explain, giving neat diagrams, a telephone system, and state the function of each part. (Pat. 1937 ; Cf. C. U. '40)

12. Write short notes on the following :—
(*a*) telephone, (*b*) microphone. (C. U. 1938, Cf. '42)

13. Explain the principle and action of any two of the following :—(*a*) A calling bell ; (*b*) A relay ; (*c*) A carbon microphone transmitter. (Pat. 1944)

13(*a*). Describe and explain with the aid of neat sketches how sound can be transmitted from one place to another through a metallic wire electrically. (Utkal 1947)

Art. 77.

14. Describe the construction and action of a dynamo. (All. 1920, '21, '29 ; Pat. '44)

(*a*) Describe a simple method of obtaining an alternating current. How would you proceed to change the alternating current to a direct current ? (C. U. 1947)

15. Explain how the phenomenon of electro-magnetic induction has been utilised in transforming mechanical energy into electrical energy. (Pat. 1938)

Art. 78.

16. Describe how to move a wire forming part of a closed circuit in the earth's magnetic field so as to induce a current along the wire, and how to move it so that there may be no induced current along the wire.

If it be moved the same distance but twice as fast on one occasion as on another, what is the relation between the currents produced in each case ? (Pat. 1926)

[**Hints**—(*a*) (See Art. 78). No current will be induced along the wire, if it is moved along the direction of the earth's total intensity (See Fig. 43, Part V). (*b*) The current in the second case will be increased twice].

17. Write a brief essay on electricity in the service of man. (C. U. 1944)

Art. 79.

Describe an experiment to show that a mechanical force acts on a current conveying conductor, situated in a magnetic field. Show how this force is made use of in a direct current motor. (C. U. 1949)

(*See also* Art., Fig. 24)

18. Write a short note on "electric motor." (C.U. 1938, '42 ; U.P.B. 1943)

CHAPTER IX

Cathode Rays : X-Rays and Radio-activity

83. Electric Discharge through Gases.—With an induction coil the sparks pass between the terminals of the secondary, only when the two terminals are close enough. For example, a difference of potential of about 20,000 volts between the terminals is necessary for sparks to pass through a length, or *spark-gap* as it is called, of 1 cm. between the terminals. The potential difference (*sparking potential*) depends upon the length of the gap. The sparking potential should be increased with the length of the gap. This is because air and other gases are good insulators under ordinary conditions ; and so high

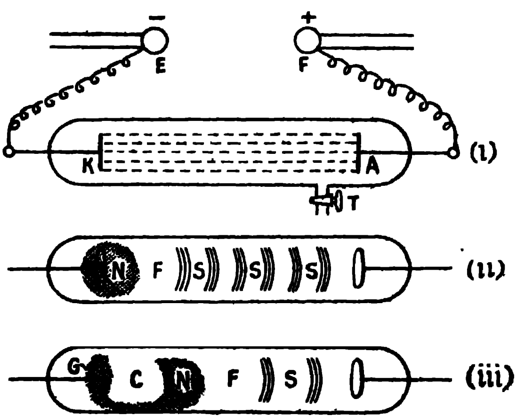


Fig. 95—Discharge Tube Phenomena

electrical pressures (which can be obtained by an induction coil or an induction machine, such as a Wimshurt machine) are required in order to set up a *discharge* through them.

84. Discharge tube phenomena—If air or any other gas is enclosed in a glass tube having two aluminium electrodes attached in platinum wires sealed in through its ends [Fig. 95 (*i*)] which are joined to the two terminals E and F of a powerful induction coil, no discharge will pass through the gas under ordinary conditions. If, however, the tube is gradually exhausted through the side tube T by means of an air pump, the insulating power of the enclosed gas will be gradually more and more reduced and the following phenomena will be observed successively.

(1) With reduction of pressure within the tube, when the pressure falls to about a millimetre of mercury and the voltage across the electrodes just maintains a current, there will be no visible discharge within the tube, but there will a luminosity confined to each electrode. This is known as **Dark discharge**.

(2) With further fall in pressure, a brilliant discharge will fill the whole of the tube stretching from the anode A (electrode connected to the positive terminal F of the induction coil) almost up to the cathode K (electrode connected to the negative terminal E). This is called the **positive column** [vide Fig. 95 (*i*)]. The colour of the discharge depends on the nature of the gas enclosed. With air the colour is red.

Ordinary **Geissler tubes** [Fig. 95 (a)] are tubes of this nature. These tubes are of various forms and show beautiful colour effects when an electric discharge is passed through them. Such unbroken bands of light in a positive column are commonly used now-a-days in evening advertisements.

Fig. 95(a)

(3) . When the pressure is reduced to about half a millimetre, the luminosity of the gas diminishes, and the positive column breaks down into a number of luminous discs, called *striations*, disconnected from each other by regular dark intervals. At this time a bluish patch of light appears around the cathode, which is called the **Negative glow** (N).

The negative glow appears distinctly separated from the striations by an ill-defined dark space F, which is called the **Faraday dark space** [see Fig. 95 (*ii*)]

(4) With further reduction in pressure, the whole series consisting of the striations, the Faraday dark space, and the negative glow pro-

ceeds towards the anode, and gradually a stage comes when the negative glow gets detached from the cathode leaving another dark space C in between ; this dark space is referred to as the **Crooke's dark space.** At this stage the cathode itself is left covered up with a pale velvety glow G, called the **cathode glow** [see Fig. 95 (*iii*)]. The number of striations at this stage is also small.

(5) With further reduction in pressure, when, say, the pressure falls to 10^{-3} to 10^{-4} mm. of Hg., the Crooke's dark space fills up the whole of the tube and the inside surface of the glass-tube becomes fluorescent, the colour of fluorescence depending on the composition of the glass. It appears bright green with soda-glass and blue with lead glass. At this stage of vacuum a beam of invisible rays is shot out normally from the cathode proceeding towards the anode. These rays have been carefully tested and they have been found to be nothing but a beam of electrons (negatively charged fundamental particles) which are constituents of all chemical elements. These have been called **Cathode rays.** The path of the rays may, however, appear slightly violet due to fluorescence of the residual gas molecules by collison with the rays.

When the tube is rendered almost a complete vacuum, no P. D., however great, can cause a discharge to pass from one electrode to the other through the inside of the tube. The discharge tends to pass across the outside of the tube.

The phenomena connected with the discharge through gases were first investigated by Sir W. Crookes, and afterwards by Sir J. J. Thomson, and other workers also have made various constributions to the subject. As a result of their investigations the properties of the cathode rays may be summed up as follows :—

Cathode rays.—When the pressure within a discharge tube is of the order of 10^{-3} to 10^{-4} mm. of Hg., and a current is maintained by the P.D. applied between the anode and the cathode, the whole of the tube appears dark except at the walls on which a fluorescence is visible whose colour varies with the composition of the glass-tube. In testing the origin of this fluorescence, Crookes, Thomson, and others have come to the conclusion that this fluorescence is caused by an invisible beam of negatively charged particles which are shot out normally from the cathode. Such beams are called cathode rays. The average mass of each particle is 9×10^{-28} gm. or roughly $\frac{1}{1840}$ of an H-atom, carrying a charge of $4{\cdot}77 \times 10^{-10}$ E.S. units. No smaller charge, or a smaller mass, has been insolated even to-day. Their nature has been found to be exactly the same, whatever is the gas used. So it is believed that

they are one of the fundamental particles with which all matter is composed, and as a matter fact they are nothing but electrons.

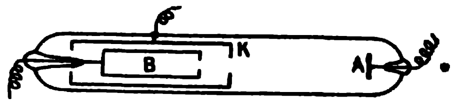


Fig. 96

Properties.—(*i*) The rays consist of an invisible stream of minute *particles each carrying a charge of negative electricity*, which can be proved by placing inside the discharge tube (having anode A and kathode K) a hollow metallic vessel B connected to an electroscope, placed outside, by means of a platinum wire fused into the tube, when the leaves of the electroscope will be found to diverge with negative electricity [Fig. 96]. These negatively charged particles are termed **electrons** (see Art. 7, Part VI), which have been shown to be the constituents of the atoms of all matter. The charge on an electron has been found to be $4{\cdot}77 \times 10^{-10}$ E.S. units.

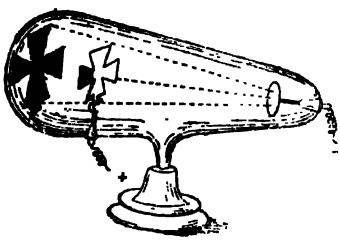


Fig. 96 (*a*)

(*ii*) These negatively charged particles travel in *straight lines* from the cathode with high speed varying from 10^9 to 10^{10} cms. per sec., *i.e.* from $\frac{1}{30}$th to $\frac{1}{3}$rd the velocity of light. This may be demonstrated by placing a metallic obstacle [aluminium cross, Fig. 96 (*a*)] in the path of the rays, when a well-defined shadow is thrown on the glass behind it.

(*iii*) The particles striking the inner surface of the glass tube cause it to glow with a greenish **fluorescent** light.

(*iv*) *The cathode rays always heat the material upon which they fall.* If the rays which are emitted normally from a cathode K of spherical shape are concentrated upon a thin sheet of platinum P placed at the centre of curvature, the platinum may be heated to redness.

(*v*) The cathode rays falling on the upper portion of a paddle wheel W of mica placed in a vacuous tube provided with glass rails G will cause it to roll along the rails, showing that *they exert mechanical pressure* by virtue of their kinetic energy [Fig. 96(c)].

(*vi*) ***They are deflected by a magnetic as well as by an electrostatic field.***

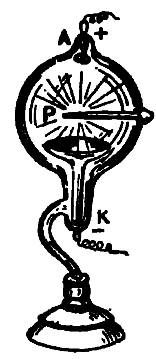


Fig. 96(b)

The rays from the cathode are allowed to pass through a brass plate in the same discharge tube. If a strong magnet is brought near the tube, the bright patch of light passing through the hole is seen to be deflected, the pencil of rays be having as if it were a flexible wire carrying an electric current flowing towards the cathode, and the direction of deflection being that in which a stream of negatively charged particles would be deflected.

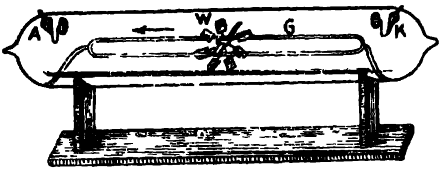


Fig. 96(c)

The rays can also be deflected by allowing them pass through a strong electro-static field.

(*vii*) The mass of an electron has been found by recent experiments to be $\frac{1}{1840}$ of the mass of a hydrogen atom, and the charge on an electron is 4.77×10^{-10} C.G.S. electro-static units.

(*viii*) *The rays can ionise a gas.*—When these rays pass through the gas, the gas molecules break up into positively and negatively charged ions. When an electric field is applied, the ions move to opposite electrodes and thus render the gas conducting.

(*ix*) Lenard has shown that cathode rays can pass through thin sheets such as of aluminium, gold, silver, etc.

85 **Positive rays.**—During discharge in a cathode ray tube, another type of rays has been discovered travelling *towards* the cathode, that is, *in a direction opposite to that of the cathode rays.* This was first observed by Sir J. J. Thomson. These rays are called **positive rays** These are also deflected, though to a smaller extent, by a magnetic and an electric field, and from the nature of the deflection it follows that they consist of positively charged particles. The mass of these particles is found to be almost equal to the mass of the atoms of the gas molecules enclosed in the tube.

86. X-rays.—Prof. W. Rontgen of Wurzbürg discovored in 1895 that when cathode rays proceeding with enormous speed strike the glass wall of an almost completely vacuous discharge tube, it emits another kind of rays having remarkable properties. These are called **X-rays**, because their nature was unknown in the beginning, or Röntgen rays, after the name of the discoverer.

Action.—In a modern X-ray tube (Fig. 97), the cathode rays in an evacuated bulb proceeding from the surface of a concave cathode are concentrated on a *target*, called the anti-cathode, of a very hard

metal such as tungsten which has a high melting point. The anti-cathode is usually placed at an angle of 45° to the axis of the incident cathode rays. The Cathode rays (*electrons*) strike the *anti-cathode*, which thereby becomes a source of X-rays. These X-rays consist of electro-magnetic waves, or pulses, each resembling a wave of light radiating outwards in straight lines and passing undeflected through the glass walls of the tubes.

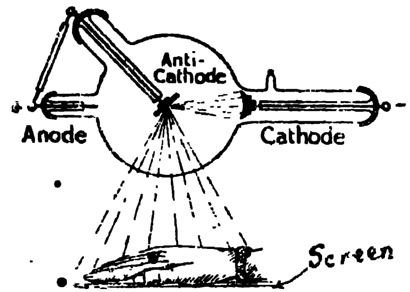


Fig. 97—X-ray tube

(a) X-ray Tubes—These may be divided into two classes; (*a*) *gas tubes*, (*b*) *electron tubes*, or *Coolidge tubes*.

(i) Gas-tube.—It is an evacuated spherical glass tube provided with three projecting side tubes, (as shown in Fig. 97) each of which has got a metallic electrode sealed into it. One electrode which serves as the cathode consists of an aluminium disc, just opposite to which there is the anode. Between the cathode and the anode there is another electrode, called the anti-cathode, which is made of some heavy metal, usually of platinum or tungsten, so that it may not be fused by the heat generated by the impact of the cathode particles. Hence in some tubes, which are used continuously, there is an arrangement for cooling the anti-cathode by circulating water round it.

(ii) Coolidge Tube.—In this tube the electrons are not generated by the cathode as such, but by a filament C of tungsten wire which forms the cathode, and which is maintained at red-heat by passing a current through it. When a potential is applied between the cathode C and the anti-cathode A, the electrons emitted by the red-hot tungsten loop impinge on the anti-cathode and thereby X-rays are emitted. The cathode stream thus produced can be focussed to a fine spot on the anti-cathode by surrounding the cathode with a molybdenum shield M. The tube is exhausted as completely as possible.

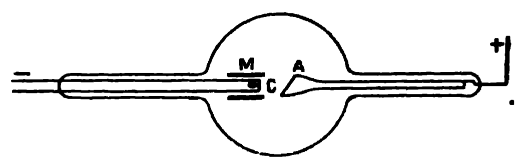


Fig. 97(a)

The **hardness** of the tube can be adjusted by adjusting the applied voltage. One **advantage** of this tube is that no cooling arrangement

is necessary in this tube as tungsten has got a very high melting point and the anti-cathode is made very massive. The P. D. applied between the electrodes being always greatly in excess of what is called the *Saturation Potential*, the current through the tube depends only on the electrons emitted by the filament, which again is governed by the temperature of the filament. This can be controlled by means of a rheostat placed in the circuit of the filament. Thus the current in the circuit can be regulated independently of the P.D. across it. That is, the intensity of the beam can be varied without altering the hardness of the rays. Besides this, due to its simplicity of construction, this tube is preferred to the other forms known as gas tubes.

(b) **Nature of X-rays.**—X-rays are of a nature similar to ordinary light, both consisting of electro-magnetic waves in ether, but in the case of X-rays, the waves are of exceedingly short wave-length, even far beyond the range of ultra-violet rays. The range of the wave-lengths of visible radiation is from 4000 to 8000 A. U., while that for ordinary X-rays varies from 1 to 3 A. U. The rays having a very high penetrating power are called **hard X-rays**, while those possessing less penetrating power are called **soft X-rays**.

(c) **Properties of X-rays.**—

(*i*) Like cathode-rays X-rays are invisible to the human eye, but unlike the cathode-rays they consist of waves, the wave-length of which is much shorter—about $\frac{1}{5000}$ of that of visible light.

(*ii*) X-rays penetrate easily through most of the solid substances which are opaque to ordinary light, such as aluminium, flesh, leather, wood, paper ; but substances like bone, and denser metals like lead, are opaque to X-rays. *So it is possible to obtain a picture of the bones through the flesh with the help of X-rays falling on an X-ray plate.*

The distance to which they can penetrate a substance is only approximately proportional to the density of the substance, but it depends actually on the atomic numbers of the elements in the substance. For this reason even fairly thin layers of such elements as lead, barium and bismuth, or their compounds, are opaque to X-rays. Soda glass is transparent but lead glass is opaque to X-rays.

(*iii*) They affect the photographic plates (silver-salts), and certain other chemical compounds—especially barium platino-cyanide—which becomes *fluorescent*. A photographic plate is also affected by ordinary light ; but in the case of X-rays the plates will be equally affected even if it is kept in an envelope of thick black paper.

A screen, prepared by painting *barium platino-cyanide* on a thick sheet of paper, held in the path of X-rays with the painted side towards the observer, glows with a bluish *phosphorescence*. If a hand be placed between the source of X-rays and the screen, a well-defined shadow showing the details of bones will be visible on the screen (Fig. 98). Similarly, if a purse with coins in it be held between the X-ray bulb and the screen, the coins will be visible through the purse If a covered photographic plate be substituted for the barium platino-cyanide screen, a photograph is obtained in the usual manner on development of the plate. Such a photograph is called a **radiograph**. Fig. 98 shows the X-ray plate of the hand of a person with a copper pice under it.

Fig. 98.

(*iv*) X-rays passing through air or any other gas have the property of rendering it a conductor of electricity, *i.e.* they can ionise a gas through which they pass.

• A small portion of the molecules of a gas gains or loses electrons by collision or otherwise under ordinary conditions. These charged molecules are called *ions* (see also Art. 57), the number of which can be greatly increased when the gas is exposed to X-rays. The gas then becomes a good conductor of electricity. This process is known as **ionisation**.

(*v*) They are not deflected by electrical and magnetic fields proving that they are not charged particles and so not of the same nature as cathode rays.

(*vi*) They cannot be reflected or refracted by ordinary means, but reflection and diffraction have been possible by passing the rays through crystals wherein the arrangement of atoms is far closer and the surface far smoother than can be produced artificially.

(d) Application of X-Rays.—There are various applications of X-rays of which only a few are given below :—

In Medical Science.—In medical science X-rays have been of great value in discovering broken bones, or fractures, locating foreign

bodies, patches in the lungs, etc. Occasional exposures to X-rays are sometimes used for cure of Cancer and other malignant growths. They are also used in watching the beating of the heart, in following food through the digestive tract, and also in detecting ulcers of the roots of the teeth.

In Industry.—They are used for discovering flaws in metal castings and other metal preparations, in defective weldings, hidden corrosion, etc. They are used in the examination of crystal structure, and they help jewellers in distinguishing true from false gems.

In Detective Department.—They are used by customs-officials in detecting smuggling. Any banned material kept in metal or wooden baggages are detected by passing X-rays through them.

87. **Radio-activity.**—In 1816 Henri Becquerel, a French Chemist, found that a photographic plate wrapped in black paper was affected by the mineral *Uranium* placed over the plate and left for some time in the dark. This property was seen to be common to all the uranium compounds. This is due to some kind of radiation emitted by the uranium. A substance which emits such radiations is said to be *radio-active*, and the phenomenon is called **Radio-activity**. Shortly after this, Madame Curie discovered a substance (which she named **radium**), which exhibits the radio-active properties to a remarkable degree.

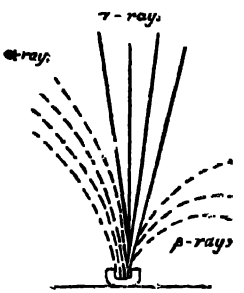


Fig. 99

It has been shown by Prof. Rutherford, in 1899, that the radiations emitted from radio-active substances are of three types (Fig. 99).

(1) **Alpha-rays** (*α-rays*)—These are *positively charged* particles and are deflected by a magnetic and an electric field. Such particles have been identified to be nothing but doubly ionised helium atoms. They have some penetrating power but their power of ionisation is great.

(2) **Beta-rays** (*β-rays*)—These are *negatively charged* particles (electrons) projected with very great velocity. They have lesser ionising power than α-particles but their power of penetrability is much greater.

(3) **Gamma-rays** (*γ-rays*)—They are neither deflected by an electric field nor by a magnetic field. They are not charged particles but they closely resemble X-rays and have much *greater penetrating power* than the **β-rays** or the X-rays.

Cosmic-rays :—It is now firmly established that the conductivity of the atmosphere increases with altitude and there is no difference in values between day and night. This is due to a radiation far more penetrating than even the hardest γ-rays, which reaches the earth from outer space uniformly from all directions. The name *cosmic rays* has been given to it. In the earth's atmosphere, these radiations have been found to be a mixture of electro-magnetic radiations, electrons and positrons. The penetrating component of these radiations is at present thought to be due to corpuscles of mass intermediate between the electron and the proton—abaut 150-250 times the electronic mass. These particles are called **mesotrons** or **mesons**. The charge associated with a *meson* may be either positive or negative. The particle is very unstable and readily breaks up into an *electron* or *positron* and a *proton*. Experiments go to confirm that the electronic and penetrating radiations are only secondary radiations which originate from a primary incident radiation of positive corpusles from outer space. Jeans and Eddington suppose that these radiations are the outcome of a continuous transformation of mass into energy somehow taking place in the glowing nebulæ while Blackett is of the opinion that these radiations have been present since the origin of this universe and is persisting by successive reflections from the boundary of the universe.

Questions

Arts. 83, 84 and 86.

1. Describe the general character of the discharge in a vacuum tube.

2. Show how Cathode rays and X-rays may be obtained, and mention the characteristics of these rays. (C. U. 1949)

3. Describe briefly what happens when an electric discharge is passed through a vacuum tube in which the pressure is being gradually reduced. How do you prove that the discharge from the cathode consists of charged particles ? What are Crooke's dark space and Faraday's dark space ? (C. U. 1933, '36, '39).

4. A glass tube having a side tube connected to an air-pump is fitted with two electrodes at the two ends. How would you arrange to pass an electric discharge through the tube ? Describe what happens when the discharges are passed with the exhaust pump running. (Pat. 1936)

5. Write a short essay on "X-rays". (U. P. B. 1947)

5(*a*). Epplain briefly how X-rays are produced. (All. 1926, '29, '32)

6. Describe an X-ray bulb and explain how it is worked. Mention some of the properties of the X-rays. (C. U. 1934, '38. ; Cf. 49)

CHAPTER X

Wireless Telegraphy and Telephony

88. Carrier Waves.—We have seen that in the case of the ordinary telephony, a current of electricity passes between the transmitting and receiving stations, and this current acts as a carrier of pulsations which make the diaphragm of the receiver to move in unison with that of the transmitter. But how is sound carried in wireless telephony? We know that sound is produced by the mechanical vibrations of material things and is carried through the air by waves, which, falling on suitable bodies at some distance, generate exactly similar vibrations and thus reproduce the same sound made at the source. Owing to rapidly increasing area over which sound-wave spreads, as it radiates from the source, the intensity of the wave becomes rapidly diminished, and so sound cannot be transmitted to a very long distance. By the use of electricity, however, ordinary restrictions of distance have been overcome in the transmission of sound; and though in wireless telephony no electric current flows between the transmitting and receiving instruments, as in wire telephony, its place as a **carrier agent** of sound fluctuations is taken by **electro-magnetic waves** like heat and light waves, but of longer wave-length.

These electro-magnetic waves, set up in the ether, radiate in all directions with the enormous velocity of 186,000 miles per second. The frequency of these electric (ether) waves is about 1,000,000 per second, while that of the sound-waves is anything between 50 to 15,000 per second. The transmitter of an wireless set is an apparatus for producing a continuous succession of electro-magnetic waves, known as the **carrier waves,** by creating **oscillating electric currents** in an earth-connected long wire, called the **Aerial.**

89. Oscillatory Circuit.—Before considering how oscillating electric current can be produced, let us take a mechanical illustration. Every one must have seen that when a spring door is deflected from its normal position of rest and then released, it returns to its position of rest, but having acquired some speed it 'overshoots the mark', and cannot stop at its position of rest due to inertia. The door then continues to move to-and-fro several times, the amplitude of vibrations gradually becoming less until it finally comes to rest. Instead of releasing the door all on a sudden from some distance, it can be gradually made to stop at its position of rest by continually offering

resistance to its movement. So it is seen that there are **three** *necessary* **conditions** for this oscillatory movement :—

(1) There must be a return force ; that is one which tends to restore the door to its original position ; or, in other words, it must have **elasticity**.

(2) It must have **inertia**.

(3) The **resistance** must be **low**.

These three conditions must be fulfilled by any type of oscillation, and so the above conditions must be met by an electric circuit to be oscillatory. The first of the above conditions, *i.e. elasticity*, is supplied by a *condenser*. A condenser consists of two or more tinfoils separated by a dielectric, say glass or mica.

The tinfoil merely acts as a means of distributing the applied electro-motive force uniformly over the surface of the dielectric. If the two ends of a charged condenser is joined by a conductor of low resistance, say a thick copper wire, a quantity of electrons, much greater than the required number to discharge the condenser, rush in from one of the plates to the other, and the charges on the plates are momentarily reversed, *i.e.* the positive plate receives more than the required number of electrons (negative charges) to be discharged and so it becomes negatively charged, and the negative plate by losing more than the required number of electrons becomes possitively charged for the moment. Now a quantity of electrons again rush into the negative plate, *i.e.* in the opposite direction, and the charges are again reversed. This process is repeated several times like the oscillation of the spring door until the condenser is finally discharged, but each time the quantity of the electrons transferred is much less than the preceding discharge, and in this way electric *oscillation* is rapidly *damped* down as the oscillatory spring door gradually stops at its normal position.

Besides the above quality of a condenser equivalent to 'elasticity', an oscillatory circuit must also possess the property equivalent to 'inertia' in ordinary matter in virtue of which every body tends to oppose any change of state. If a coil of wire is included in the electric circuit, *the self-induction of this coil gives 'inertia' to the current* in the circuit ; for when the current begins to flow, there will be an induced electro-motive force of self-induction (see Art. 68), which opposes the flow ; and, after the current continues to flow, the electro-motive force of self-induction opposes the stopping of the current. For the same reason that the spring door swings past its position of equilibrium, the inertia (self-induction) of the current flowing in any circuit

consisting of a condenser and a coil of wire prevents the current from stopping at the exact moment the condenser is fully discharged *i.e.* the self-induction of the coil causes it to 'overshoot' the mark and thus *the discharge becomes oscillatory.*

Fig. 100 shows the essentials of an oscillatory electric circuit, which consists of a condenser, a coil of wire, and a short spark gap, the two ends of which are joined to the terminals of an induction coil. On passing a current through the induction coil, a succession of sparks will pass through the gap. It has already been explained that the discharge of a condenser may be compared to the swinging of a spring door, when taken to one side from its equilibrium position and released all on a sudden. Due to elasticity the door will continue to move to-and-fro several times, the amplitude gradually becoming less due to friction and also resistance offered by air, until it will finally come to rest. The movement in this case is oscillatory. Similarly, when the condenser is discharged in this manner, the electricity does not

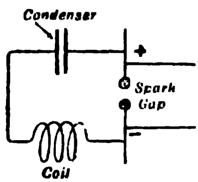


Fig. 100—Oscillatory Circuit

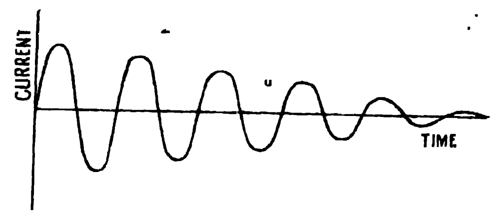


Fig. 101

rush across in one direction but surges backwards and forwards, first charging up one plate and then the other, *i.e.* the charges are momentarily reversed, until its energy is damped down and finally exhausted by the resistance of the circuit. The current passing between the terminals may be represented graphically by Fig. 101. An oscillation of this type is called a **damped oscillation**.

Of the three necessary conditions of an oscillatory movement, for which a simple arrangement is shown in Fig. 100, low resistance in this case of spark discharge is obtained by having the spark gap as short as possible, the *capacity* of the condenser is equivalent to the quality of "elasticity", and as a coil of wire has got a better inductive property, *i.e.* 'inertia', than a straight wire, a coil of wire has been introduced in the circuit (Fig. 100).

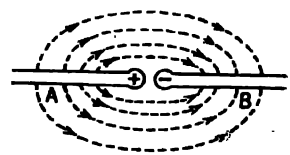
Ordinarily electric lines of force connect the plates of a charged condenser, but during discharge, these electric lines of force collapse, and they are replaced by circular lines of magnetic force lying in planes perpendicular to the conducting wire, which are set up by the

current surging through the circuit. So when an electric discharge passes through the spark gap (Fig. 100), the to-and-fro motion of the current causes the radiation into space of electric and magnetic lines of force. These groups of electric and magnetic lines of force following one another in rapid succession give rise to what are termed **electro-magnetic waves** capable of travelling through space in every direction with an enormous velocity, *i.e.* the velocity of light (186,000 miles per second), and hence can reach any part of the world in only a part of a second.

It should be noted that the only difference between the waves set up in the above manner and light itself lies in their frequency, *i.e.* the number of vibrations per second. So they differ in wave-length and in the manner of their detection. These electro-magnetic waves are detected by special apparatus, whereas **light waves** are perceived by the eye and possess much greater frequency. Their differences are thus merely of degree.

90 Magnetic and Electric strain in the Ether. (Electro-magnetic Waves).—In order to understand more fully the radiation of electro-magnetic waves by means of an oscillatory electric circuit we should consider the condition of the medium where electric oscillations take place.

We know that a suspended magnet is attracted by another magnet and begins to move towards it even when the other magnet is some distance away. The suspended magnet must be moving due to some force and the force must be acting through some **medium**. If we wish to move a body from one place to another, it can be done by pushing it with a rod or by pulling it by means of a rope, that is, in either case there must be a medium through which the force is transmitted ; and the medium, which in this case is the rope, or the rod, must be under **strain** at the time of transmitting the force.



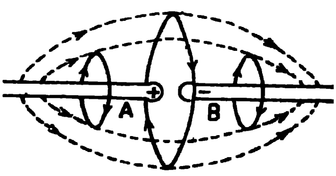


Fig. 102

Similarly the force, in the case of the magnets referred to, is transmitted through a medium which is *ether* (and not air, as the above magnetic phenomenon takes place also in a vacuum), and the ether is strained at the time of transmitting the force of attraction from one magnet to the

other. The lines of magnetic force (vide Figs. 24, 25, etc., Part V) are all **lines of magnetic strain** in the ether by which magnets and magnetic substances are affected. In the same manner we have **lines of electric strain** in the ether between two charged bodies, one positively and the other negatively. We have seen in Art. 11 that when a current flows in a wire, lines of magnetic force appear round the wire in concentric circles. These are lines of magnetic strain in the ether, and it should be noted that a magnetic field is created by an electric current *in motion*, and there is *no magnetic strain line* in ether when electricity is *at rest*, though there are electric strain lines present. In Fig. 102(*a*) ether between two charged bodies *A* and *B*, one positively and the other negatively, is electrically strained and there are electrical strain lines (indicated by dotted lines) between them. Now, if *A* and *B* are discharged by a spark passing from *A* to *B* there will be a *flow of electricity* from *A* to *B* (but according to electron theory it is from *B* to *A*) and so magnetic strain lines will at once appear round about the path of the spark, and it should be carefully noted that these magnetic strain lines (indicated by continuous lines) are *at right angles* to the electric lines [Fig. 102 (b)].

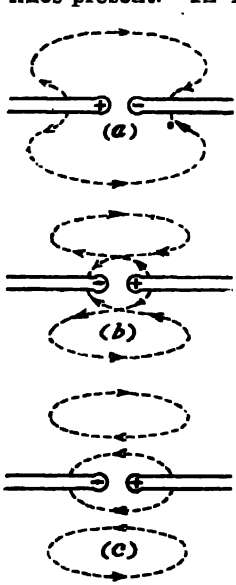


Fig. 103

In an oscillatory circuit, as soon as a discharge takes place, the electric lines of force will rapidly contact and so will be in motion, thus constituting the electric current, and they will give rise to circular lines of magnetic force [Fig. 102 (b)]. The discharge being oscillatory the charges on the knobs will be reversed and the lines of electric force will re-appear with their directions reversed. Now at the time of contraction, these lines do not approach the rods as rapidly as their opposite ends move along them and so they will assume a pear shape, as shown in [Fig. 103 (a)]. At a later instant the ends of the lines will overlap, as shown in [Fig. 103 (b)], and the main length of the line will be separated and thrown off as an independent closed loop of electric strain [Fig. 103(c)]. The detachment of loops constitutes **electric radiation.**

In this way a series of detached electric loops will travel outwards with a velocity of 186,000 miles per second, the direction of the electric strain being opposite in successive loops. These loops in travel-

ling outwards with the velocity of light will give rise to loops of magnetic strain which are also in opposite directions in consecutive loops. The magnetic strain is always at right angles to the direction of the electric strain (Fig. 104). These two sets of strain, electric and magnetic, constitute **the electro-magnetic wave**. This wave is transmitted with an energy, which is partly electric and partly magnetic, in a direction perpendicular to the plane containing both the directions of the electric and magnetic strain.

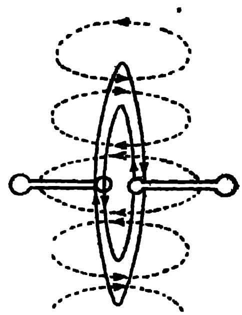


Fig. 104

91. Historical.—Sir Oliver Lodge first showed that when a spark passed across an air-gap in one circuit, another spark jumped across another gap in a separate circuit placed at a little distance, provided the second circuit was "**in tune**" with the first, *i.e.* the natural frequency of electric oscillation of the second circuit was the same as that of the arriving electric waves. In 1888 the famous German scientist Hertz first showed that the disturbance in the intervening medium was of the nature of wave motion traversing the ether. He generated electro-magnetic waves, detected them at a distance, measured their wave-lengths, and also proved that they could be reflected and refracted like waves of light. Signor Marconi discovered means of transmitting and receiving these electro-magnetic waves over long distances.

92. Transmission of Electric Waves.—Fig. 105 represents a simple wireless transmitting circuit. At each discharge along the spark gap S, obtained by the induction coil I. C., oscillations occur, due to which electro-magnetic waves are generated and emitted by the aerial A, which travel through space with the velocity of light. By altering the capacity of the condenser C and also by altering the number of turns of wire in the coil, *i.e.* the length of the coil, included in the oscillating circuit, the frequency, and hence the wave-length of waves generated, can be changed and thus the circuit can be '**tuned**' (see Art. 93), as it is called. If a key is included in the battery circuit of the induction coil, sparks can be generated by means of it; the number of these sparks depends on the duration of the depression of the key and signals based on these

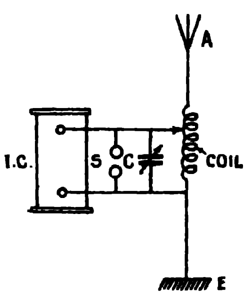


Fig. 105—Transmitting circuit

sparks can be transmitted, as done with the Morse code in ordinary telegraphy (Art. 75). Marconi first showed that in order to increase the radiating property of the circuit by setting up much stronger waves by a comparatively smaller strength of current, *i.e.* to radiate out more energy in the form of waves so that the waves are capable of detection at greater distances, one plate of the condenser should be connected to a long wire, called the **aerial**, and the other to earth, as shown in Fig. 105. It is found that the long wire itself can serve as one of the plates of the condenser, so the aerial and the earth may be considered to form two opposite plates of a huge condenser.

93. Wireless Telephony.—Wireless telephony which is the art of wireless transmission of speech or music has been possible owing to the discovery of methods for generating electro-magnetic waves which are uniform in amplitude and not broken into isolated groups as those generated by *sparks*. Such waves are termed **Continuous undamped waves**. Because of thir uses in the transmission of speech or music, they are also called **Carrier waves**. The difficulty with the spark-method of transmission is that the note due to the sparks themselves will be always audible in the telephone of the receiving circuit ; for, the sparks to be inaudible have to be produced at a frequency greater than 16000 which is impossible to be attained in practice. Moreover, sparks being intermittent in character are incapable of transmitting the continuous waves which are characteristic of sound.

If the wave-form characteristic of any note can be superimposed upon a system of radiating electro-magnetic waves of the continuous type, then wireless telephony becomes a possibility. The process is called **Modulation**. The continuous electro-magnetic waves may be generated from an aerial by using a *Poulson arc system*, or by means of triode valves, the latter method being almost universal (Art. 97). The characteristics of the sound may be superposed on the carrier waves by the following simple method.

Suppose a microphone is directly inserted in the aerial of an wireless transmitter ; any sound-waves falling on the diaphragm of the microphone will cause fluctuations in the current oscillating in the aerial but it will not affect the frequency and wave-length of the radiated waves. These modulated waves will cause similar fluctuating currents to be produced in the receiving aerial of a distant station and the receiving telephone will reproduce the original sound. The above method of inserting the microphone directly in the aerial is

applicable with small currents only in the aerial. Other methods are used in modern broad-casting stations.

93 (a). Tuning.—We have seen in Art. 35, Part III, that if two tuning forks mounted on resonance boxes have the same frequency of vibration, that by striking one of them the other responds in sympathy. This phenomenon of resonance is due to the fact that sound-waves starting from the first fork impinges upon the second one, which gives out waves of the same frequency ; but the second fork will remain unaffected, if the frequency of the impinging waves is different.

The same thing happens also in the case of electro-magnetic waves. When these waves cut across and pass beyond the aerial of a receiving station, an E. M. F. is set up in the long wire, as we know that when a conductor cuts across a number of magnetic lines of force, or magnetic lines of force cut across a fixed conductor, an induced E. M. F. is set up in the conductor. The direction of the induced E. M. F. will be rapidly changing owing to the rapid change of direction of magnetic lines of force. This oscillating E. M. F. sets up a feeble oscillating current in the aerial. In order that the receiving circuit may respond efficiently to the incoming waves, it is necessary to adjust it so that the natural frequency of the circuit as a whole corresponds with the frequency of the waves, or, in other words, the aerial and other parts of the circuit should be properly 'tuned.'

94. Reception of Electric Waves : Crystal Receiving Set.—Fig. 106 represents a simple receiving set which contains three essential parts, namely, the **aerial** *A*, the **rectifier** *D*, and the **telephone** *T*. In the aerial, oscillatory currents are set up by the incoming waves. These induce oscillations in the coil of the second oscillatory circuit, which is to be 'tunned' with the oscillatory circuit at the transmitting station, that is, the capacity and inductance of the circuit are adjusted so that the natural frequency of the circuit as a whole corresponds with the frequency of the incoming waves. As the telephone included in the circuit will not respond to all such rapid alterations of current of the oscillating circuit, certain crystal, such as **Galena** or **Carborundum**, is included in the circuit, which conducts electricity in *one direction only*. Thus the crystal acts as a **rectifier**

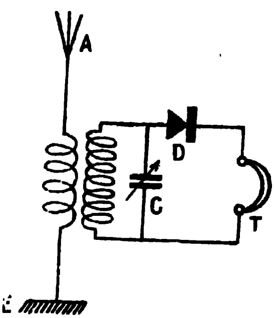


Fig. 106—Crystal Receiving Circuit

by converting the oscillatory current into a unidirectional current, which acts on the diaphragm of the telephone and produces sound. Thus the telephone converts the variations of the unidirectional current into sound. The process of converting a high frequency oscillatory current into a unidirectional current is known as **rectification.**

95. Thermionic Valve.—A thermionic valve is an essential part of all modern wireless sets. The **Fleming Valve,** a discovery of Dr. J. A. Fleming in 1904, consists of a vacuum electric lamp with its tungsten filament (F) surrounded by a hollow cylinder of copper or nickel (P) supported from a platinum wire sealed through the glass (Fig. 107). Conventionally, this metal cylinder is represented by a *flat plate* placed above the filament as in Fig. 108. The filament is such that it can be made incandescent by two or three accumulators.

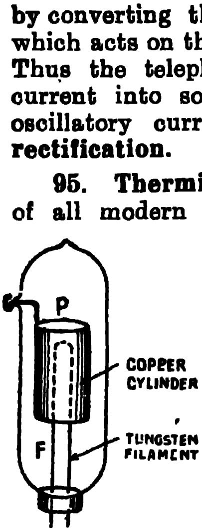


Fig. 107

Action of the Fleming Valve.—To understand the action of the valve it is only necessary to remember that a current through the filament means the flow of millions of electrons along it at an enormous speed : the speed being very great, some of the electrons escape from the filament just as particles of steam are evaporated from the surface of heated water. The space around the filament thus becomes more or less occupied by the emitted electrons. This constitutes what is called the *Space-charge.* Suppose, then, that a *positive* charge be given to the plate. Some or all of the electrons in the space-charge will be attracted to the plate. This flow of electrons towards the plate is eqivalent to an ordinary electric current in the opposite direction. If the plate should be charged up negatively, the electrons would be repelled and no current will pass. If the potential given to the plate due to positive charges be gradually increased, the electronic current will also increase more and more, electrons being attracted to the plate ; but ultimately, a potential may be attained when all the electrons emitted by the filament will be attracted to the plate at the rate at which they are emitted and the current will assume a limitting stage when the current will not increase any further on further increasing the potential. This value of the potential of the plate is ordinarily referred to as the *Saturation potential.* The valve is called a thermionic valve because its action depends on the electrons, *i.e.* particles charged with electricity, emitted from a hot filament.

In Fig. 108, it has been shown how the positive charge on the

plate P may be maintained by connecting it to the positive terminal (through the galvanometer G) of a battery of cells B, the negative of which is connected to the negative of a battery A used for heating the filament F. As explained above a current will pass through the plate circuit, the circuit between the filament and the plate being completed by the electrons emitted by the filament. This current will be readily detected by the galvanometer G. Suppose now, the battery B is reversed having its negative terminal joined to the plate. There will be no current flow now, because the electrons will be repelled by the negative charge of the plate.

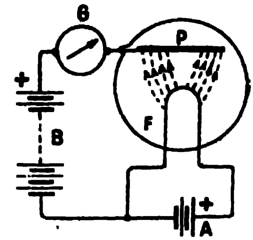


Fig. 108

Thus the lamp allows the current to pass in one direction only and may therefore be used to act in the same way as a crystal valve.

In Fig. 109 it has been shown how a **Fleming Valve may be used as a rectifier** in a receiving circuit. The circuit is similar to that in Fig. 106, the Fleming valve taking the position of the crystal rectifier D. The oscillations picked up from the aerial charge up the plate alternately, positively and negatively. When the charge on the plate is positive, a current traverses the telephone but no current passes when the charge is negative.

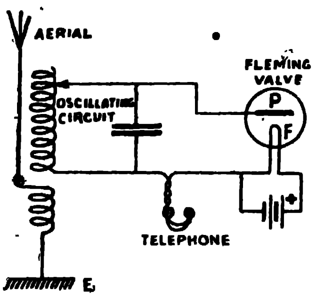


Fig. 109

96. The Triode (or Three-Electrode) Valve.—The Fleming valve was greatly improved upon by Dr. Lee de Forest in 1907, who inserted a third electrode known as "**grid**" between the plate and the filament. Fig. 110 represents a Triode valve. The filament is horizontal and is of tungsten and is surrounded by a spiral of molydenum wire which acts as the *grid* and around these is the *plate* which is a thin cylinder of nickel. A slight positive charge on the grid increases the upward flow of electrons from the filament to the plate through the grid and thus the current strength is increased ; but when the grid has a negative charge, electrons may be driven back and the current flowing through the plate may thus be regulated. Thus a very small change of potential on the grid causes a great difference of the current strength in the telephone. So **the grid helps to magnify or amplify smaller oscillations**. In practice, the grid G is connected to the ærial A (Fig. 111) and any feeble oscillations which reach the

valve create small but rapid alterations of potential in the grid. The grid is thus automatically charged positively and negatively each time the current in the aerial reverses, and hence rectifies the

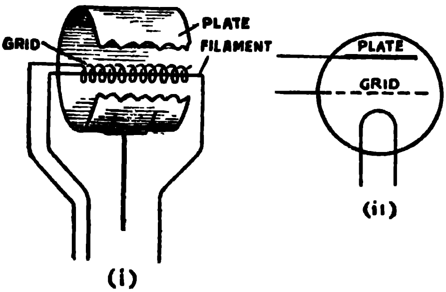


Fig. 110—Triode Valve

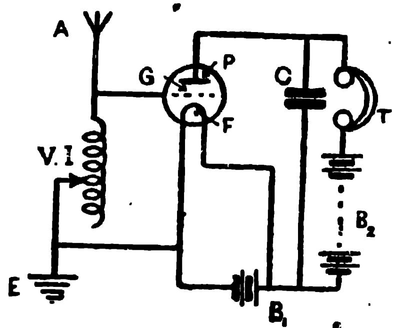


Fig. 111 - Triode Valve used in a Receiving Circuit

current by allowing current in one direction only. So a greatly amplified direct current is produced in the plate filament circuit by which the telephone is worked.

97. The Triode Valve used as Oscillator in a Transmitting Circuit. —In Fig. 112 it has been shown how by means of a *reaction coil* in the plate circuit of a triode valve, it is possible to generate undamped high frequency electromagnetic oscillations. The windings of the coil in the plate circuit and the coil in the grid circuit can be so arranged that an increase in the plate current will cause a fall in the potential of the grid which will therefore diminish the plate current; similarly, a fall in the plate current will cause an increase in the grid potential to be followed by an increase in the plate current. It is thus clear that once the oscillations are started, they will be maintained constant as long as the energy supplied by the battery is maintained.

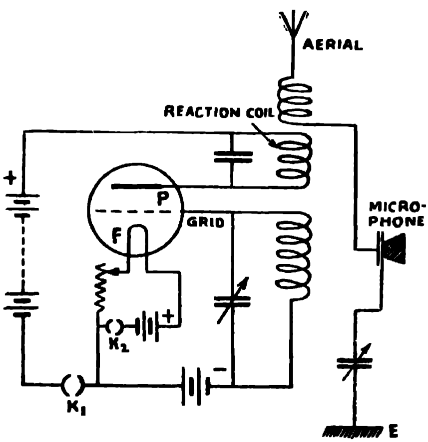


Fig. 112—Triode Valve used as Oscillator in a Transmitting Circuit

These undamped oscillations may be modulated, in wireless telephony, by modulating the current of the coupled aerial by inserting in it a microphone. The method described is only suitable for transmitters of small output. In actual transmitters used in broad-casting ordinarily modulation of the plate current is used.

98. The Photo-electric Cell.—This consists of a vacuous bulb on half the wall of which is a clean deposit of some alkali metal such as *sodium, potassium, or caesium*. Two wires are sealed into the bulb, one end, almost at the centre of the bulb, forming the anode and the other connecting the metal deposit serving as the cathode. When in darkness, no current flows in the bulb, but, when exposed to light, the metal, sodium or potassium, as the case may be, emits electrons which travel to the anode, and this stream of electrons constitutes an electric current. The intensity of this current being directly proportional to the intensity of light incident on the metal, the current can be modulated. This current, though very feeble, can be amplified by a series of valves arranged as in a wireless circuit. This modulated amplified current passing through a loud-speaker of a talkie-machine produces the sound. The photo-electric cell (which is not really a generator of current like other cells) is an extremely sensitive instrument, which has become very important commercially in not only **Talkie** pictures but also in **Television**.

99. Television.—Television is made possible by photo-electric cells. By television not only the speech is transmitted by wireless waves but the picture of the speaker is also transmitted along with his speech.

100. Sound Films.—In "talkies", a sound record is made on a film, and it is generally the edge of the film that carries the sound record. Sound-waves falling on the diaphragm of a microphone produce varations in electric current, which operates a light-bulb. So the current of varying intensities allows light in varying amounts to pass and excite narrow bands of the films. Thus, on developing the film, density of the successive portions of the film will vary according to the amount of light falling there, and so a dark band of varying densities will appear on the edge of the film.

When a film is being shown, a beam of light is directed through the film into a photo-electric cell, where variations of electric current will be produced corresponding to the varying intensities of light coming through the film. The varying currents are amplified and passed through the coil of a loud-speaker, whereby the resulting vibrations in the diaphragm reproduce the original sounds.

Questions

1. Explain the principle of wireless telegraphy.
2. Describe a simple wireless receiving set. (All. 1945 ; U. P. B. 1947)
3. Describe a triode valve. (All. 1945)
4. Write an essay on Wireless Telegraphy and Telephony. (All. 1944, '46)
5. Write a short note on "Wireless". (U. P. B. 1948)

APPENDIX (A)

AERONAUTICS

CHAPTER I

The Atmosphere

Before proceeding to study the principles on which the flight of an aircraft depends, the following facts about the atmosphere should be remembered.

1. Air Resistance.—Due to the fact that air has weight and that it is always subject to convection currents, air offers resistance to bodies in contact with it, and this resistance depends upon its properties of (*a*) *viscosity*, (*b*) *elasticity*, and (*c*) *inertia*.

(*a*) **Viscosity.**—It is the property in all fluids (Air is a fluid) corresponding to friction in solids, which causes one layer of air to move with the layer next to it (see p 111, Vol. I). Due to viscosity eddies are formed in air when it is disturbed by a body passing through it, and these eddies cause many of the phenomena of flight.

(*b*) **Elasticity.**—The tendency of the air-particles to reoccupy former space from which they have been disturbed is due to the property of elasticity of air.

(*c*) **Inertia.**—It is a property common to all matter due to which air will tend to persist to be at rest, or in steady motion, and will resist any attempt to change such rest or motion.

2. Temperature.—There is a gradual drop in temperature with altitude in the atmosphere, the average fall in temperature being 1°F. for every 300 ft. (see Art. 134, Vol. I).

3. Pressure.—The average pressure of the atmosphere at sea-level is about 14·7 lbs. wt. per sq. in., which changes from place to place and from day to day with changes of weather and temperature. This pressure decreases with increase of altitude (see p. 165, Vol. I) and it is greater near the earth's surface than at greater altitudes. It has been estimated that about one half of the total weight of the atmosphere is concentrated in the first 18,000 ft. It should be remembered that the pressure exerted by the *air in motion* may be greater or less than the pressure exerted by *air*, when *stationary*, according to the nature

of its motion, and from these pressures the forces of *lift* and *drag* (discussed later) on an aircraft are obtained.

4. **Density.**—The pressure of the atmosphere produces density, and so the density of air is greatest at sea-level and decreases with altitude. At sea-level the density of air is about 0·08 lb. per cu. ft. and at 20,000 ft. it is only 0·041 lb. per cu. ft., which is about one-half of the first value. It is the density of air which makes all flight possible, as an aircraft is supported in the air by forces entirely dependent on its density ; the less the density, the less the weight lifted and more difficult does flight become, and in a vacuum the flight becomes impossible.

5. **Humidity.**—The presence of water vapour in the atmosphere diminishes its pressure and so the density is also diminished.

CHAPTER II

Air Resistance

6. **Streamlines.**—Whenever a body is moved through air (or any other fluid), or the fluid flows past a body, there is always produced a definite resistance to its motion. This resistance is usually termed "**Drag**" in aeronautical work. The effect of this resistance in the viscous fluid is to set up displacement in the shape of eddies in the fluid.

In such cases two modes of flow take place : (*a*) *turbulent flow*, and (*b*) *streamlined flow*. Turbulent flow may arise due to friction and excessive velocity, and thus the eddy resistance becomes much greater, but if a body is so shaped as to produce the least possible eddy motion, then it is said to have a *streamline shape*, and the lines round the body interposed in the fluid showing the directions, and shapes of these disturbances are called **Streamlines**. These streamlines enable us to understand the nature of the flow of the fluid past the body.

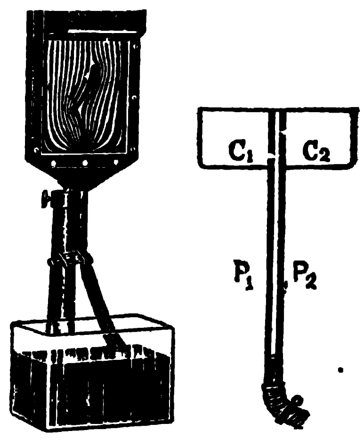
As it is difficult to investigate the disturbances on an aircraft, while in actual flight, most of the aeronautical experiments for studying the phenomena of flight are carried out by scientists in the laboratory by using some form of **Wind Tunnel**, in which air is made to flow past a model of aeroplane which remains at rest relative to

the tunnel. The effect is the same as if the body were made to move through *still* air, because it is the *relative motion* of air to the aircraft, or the aircraft to air, which is investigated.

Air speed and Ground speed.—True *air speed* of an aircraft is the speed relative to the air, that is, the speed with which it would travel in the absence of wind ; while *ground speed* means its speed relative to the earth, or the actual speed over the ground. For instance, if the normal speed (air speed) of an aircraft flying from A towards B be 100 m.p.h. while wind is blowing at 40 m.p.h. from B towards A, the aircraft will reach B with an actual speed (ground speed) of 60 m.p.h.

It is possible to study and photograph streamlines and eddy motions by introducing smoke into the air-flow in wind tunnels, or coloured jets into the Water tank experiment described below.

Experiment.—The apparatus for demonstrating streamline flow of liquids consists of a rectangular reservoir at the top divided into two compartments C_1 and C_2 [Fig. 1(b)] by two glass plates P_1 and P_2 separated by a distance of about 1 mm. These plates have equidistant perforations inside the reservoir (as C_1), the perforations in P_1 being alternate to those in P_2. One of the compartments C_1 is filled up with clear water and the other C_2 with a coloured water, say, water coloured with potassium permanganate. Now, the liquid flowing down between the plates from both the compartments collects at the bottom and finally flows out through a rubber tube provided with a pinch-cock. On opening the pinch-cock clear water from C_1 and coloured water from C_2 will flow down between the plates through alternate perforations. The violet coloured tracks will show the parallel streamlines along which the water flows, and they finally curve inwards towards the end. Due to the colouring material the streamlines are made visible to an audience. The actual apparatus is shown in Fig. 1(a), where a thin body made of gutta perch has been introduced in the stream between the plates to show the distortion of streamlines.



(a) (b)
Fig. 1—Water tank Experiment

due to its shape. Similarly, small bodies of different shapes can be introduced to show how the streamliness are distorted in each case.

7. **Effect of shape.**—One great object of the designer of aeroplanes is to reduce the eddy resistance to an absolute minimum, and much experimental work has been carried out with this in view. Results show that the shape of a body has a striking effect on the amount of drag produced, and that enormous advantage is gained by adopting a "streamline" shape, the example of which in nature is the outline of a fish. When air flows past a perfectly streamlined body, no eddies are created in its neighbourhood.

Fig. 2 shows some of the streamlines flowing past bodies of different shapes. It will be noticed in Fig. 2(*a*) that, in the case of a flat plate, the air-flow breaks up after passing the edge of the plate into a series of eddies and vortices, the size and nature of which will also be influenced by both the velocity of the air-flow and the linear dimension of the plate. It will also depend on the inclination of the plate to the direction of air-flow. Fig. 2(*b*) shows that owing to its position both sides are affected by the air-current. Streamlines at the bottom are deflected downwards and eddies are formed at the lower edge, whilst on the top there are similir eddies and also regions of lower pressure due to the distortion of the straight-line motion of the air-current. When, however, the obstacle has got a suitably curved shape as in Fig. 2(*c*), the air or fluid passes over and behind the body in unbroken smooth lines, termed stream lines, and the obstacle giving rise to a definite streamline pattern is usually called a *Streamline body*.

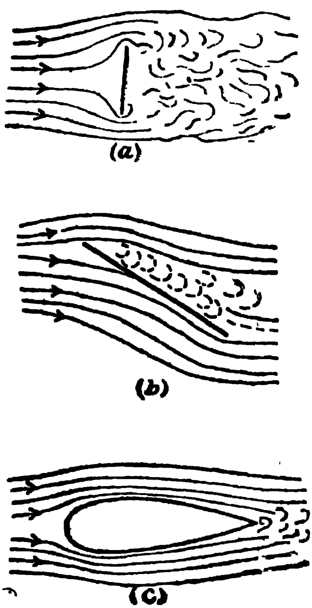


Fig. 2—Effect of shape

On comparing the flow past a rough obstacle with that past a streamline body, we notice that in the former case large portions of the fluid spin around as if they were detached portions of the fluid. These isolated portions of the fluid are called *eddies*. A ball thrown in air and moving with spin will require more energy than when it is moving without spin. An eddy differs from a fluid moving in a streamline manner in the same way

as a ball moving with or without spin in air. For an aeroplane having rough shapes, the energy of the spinning fluid of the eddies must ultimately be derived from the engine, and so, such bodies will tend to slow down the machine and produce inefficient flight. *Stream-line shapes are*, therefore, *necessary for the efficiency of the aircraft.*

8. The above experiments show that the resistance offered to a body in *relative motion* with a fluid is made up of two parts : (*a*) *Eddy-resistance* ; (*b*) *Skin friction.*

(a) **Eddy Resistance** —This is the portion of the resistance due to eddies formed when a viscous fluid flows past a solid body and we no longer get a smooth streamline flow. The extreme example of this type of resistance is the case of a flat plate held at right angles to the fluid or wind [Fig. 2 (*a*)]. The resistance in this case is very large and the pressure in front of the plate is greater, and that behind the plate is less, than that of the atmosphere which causes a kind of "sucking effect" on the plate. The space immediately behind the plate is not traversed by streams of air and is called a *dead space.*

(b) **Skin Friction.**—This is the resistance due to friction between the surface of a body and the layer of air next to it. Also, due to the viscosity of the air, layers near the surface will tend to retard those further away.

9. **Resistance Formula.**—It can be proved and verified by experiments that when a body is passing through air, the resistance R on it depends, within certain limitations, on the following factors.—

(*i*) *The shape of the body* ; (*ii*) *the surface* ; (*ii*) *frontal area of the body exposed (in sq. ft.)* : (*iii*) *the square of the velocity* V *of air-flow (in ft. per sec.)* ; (*iv*) *the density* ρ *of the air* (*in lb. per cu. ft.*) ; (*v*) *acceleration of gravity* g (*in ft. per sec*^{2}.).

Thus, we have the resistance formula $\mathbf{R} = \dfrac{K\rho AV^2}{g}$ lb.-wt., where K is a constant depending on the shape of the body, the value of which for a flat *plate* is 0·6 and that for a streamline by 0·03.

10. **Bernouilli's Theorem** —Before proceeding further we should consider here a theorem, known as *Bernouilli's Theorem*, which states that in the flow of an ideal (*i.e.* not viscous) fluid *the sum of the Potential energy, Kinetic energy, and the pressure energy is a constant.* This theory can be roughly verified by an experiment with the Venturi tube illustrated in Fig. 3.

11. **Venturi Tube Expt.**—A metal tube AB tapering at the two ends and having a narrow neck in the middle is connected to a hori-

zontal glass tube $T\ T$, containing a coloured liquid by means of a number of narrow manometers (h). When the reservoir R is half-filled with the coloured liquid, the liquid stands at the same level in all the tubes. This happens at normal pressure over all the tubes.

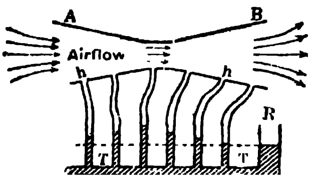


Fig. 3—Venturi Tube

When, however, an air-stream from a wind tunnel passes through the Venturi tube, the speed of the air in AB will change from place to place due to which the liquid level in several of the inner manometers will rise. The greatest increase in height will be observed in the narrowest part of the tube, where the speed of the air is also greatest. But, as the liquid level rises in the manometers due to reduction of pressure, we have this somewhat unexpected fact that *the pressure of the air falls when its speed increases.*

As the change of potential energy is negligible, the increase of speed (and hence of kinetic energy) is obtained by losing some of its pressure energy Hence it illustrates the Bernouilli's theorem stated above. This Venturi effect, as it is called, is employed in many scientific devices in order to produce a reduced pressure.

CHAPTER III

Aerofoils (or Wings) : Flat and Cambered Surfaces : Lift and Drag

12. Principles of Flight.—Let us proceed now to consider the question of why it is that an aeroplane is capable of flying through air. In order that an heavier-than-air machine can fly, there must be some means of forcing the air downwards so as to provide the equal and opposite reaction which will lift the weight of the machine, and in the conventional aeroplane this is provided for by wings, which are inclined at a small angle to the direction of motion. The necessary force driving the machine forward is obtained by the thrust of a airscrew. The wings (or aerofoils) are always slightly curved, but let us consider the case of a flat plate first, as in the original attempts of flight, flat surfaces were used.

13. Flat Plate inclined to Air Current.—For simplicity we suppose that a flat plate AB is at rest and that the air current flows past the plate AB which is inclined at an angle α to the direction of the air-flow (Fig. 4). In Fig. 2 (*b*), it has been found that in this position both sides of the plate are affected by the air-current, due to which pressure of air on the top surface is decreased while that underneath the plate is increased. Each of these pressure-changes produces forces R_1 and R_2 acting upwards on the plate giving rise to a resultant force R, which is practically normal to the surface when the angle α is small. The force R_1 arising from the decrease of pressure pulls the plate up, and the force R_2 arising from the increase of pressure pushes the plate up (*vide* Venturi tube expt.). The force R_1, called the **Total Reaction**, can be resolved into two components at right angles—one horizontal, D, and the other vertical, L, acting upwards. The component L, called the *lift*, balances the weight of the plate, and the component D, called the *drag*, resists the motion through the air. Obviously, the L component which supplies the lifting force to the plane is of profound importance. For equilibrium the L component must equalise the weight W of plate. If W is greater than L, the plate will fall, and if less, it will rise.

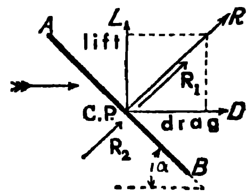


Fig. 4

Actually in practice the flat surface is inefficient as a means of lifting because of the total resistance offered, and therefore the total engine power which has to be employed, is very high in comparison with the lift obtained from it.

14. Aeroplane and Kite.—The flight of an aeroplane is much like that of a kite floating in air (see p. 31, Part I). In the case of the aeroplane the rush of air past the wings is due to the motion of the aeroplane itself through the air rather than to a wind, as is the case with the kite. The tension of the kite string here corresponds to the forward thrust of the propeller. The L component balances the weight of the machine, while, for equilibrium, the D component must be counterbalanced by an equal force which is obtained by the action of the screw-propeller. On increasing the propeller speed, the forward thrust and R increase. Consequently, the L component becomes greater than the weight and so the aeroplane rises. It should be noted that the air-pressure depends only on the rate and direction with which the air and the body meet, and the result is the same whether the body moves to meet the air, or the body remains still and the air flows

against it. Obviously the greater the velocity with which the aeroplane and air meet the greater will be the air pressure.

15. Cambered Surface—The advantage of using a suitably curved (or *cambered* as it is termed) surface, instead of a flat one, was soon discovered by which a much greater lift, especially when compared with the drag, could be produced. In this, the eddy disturbances due to the distortion of the streamlines can be minimished and so the efficiency of the system can be increased. Thus the modern aerofoil has both the top and bottom surfaces cambered. The top camber is greater than that of the bottom surface as due to this the lift component has an appreciably higher value over a wider range of the angle of incidence. The additional advantage of the curved surface is that it automatically provides a certain amount of thickness which is necessary for structural strength. The thickness is expressed as a percentage of the chord and for general use the best top surface camber is about 11 per cent. of the chord, while for high speed it should be only 7 or 8 per cent.

Airflow and Pressure over Aerofoil

16. Some Definitions.—A transverse section of a wing (or aerofoil, as it is called) of an aircraft is shown in Fig. 5, where along the front of the aerofoil at A is the **Leading edge**, and at the rear at B is the **Trailing edge**.

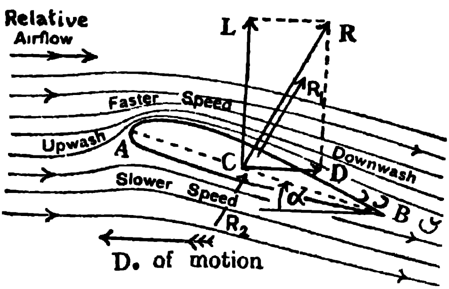


Fig. 5—Airflow over aerofoil

The line AB joining the centres of curvature of the leading and trailing edges is called the **chord**.

Camber is the curvature of the aerofoil of both the top and bottom surfaces. The greatest height of the top or the bottom surface, when divided by the chord length, is called the *camber* of the respective surface. Camber decides the thickness of the aerofoil.

Angle of Attack is the angle between the chord line and the relative airflow, which is the direction of the airflow with reference to the aerofoil.

[N. B.—The angle of attack is often referred to as the **angle of incidence**, but it is better not to use this term in order to avoid

confusion with the Rigger's angle of incidence, which is the angle between the chord line and some fixed horizontal datum line in the aeroplane. For a given aeroplane this angle is fixed, whereas the angle of attack may alter during flight.]

The total length of the aerofoil perpendicular to the section is called the *Span* ; and the ratio of the span to the chord is called the **Aspect ratio.**

17. Airflow past an Aerofoil.—Experiments show the following results when a typical aerofoil moves through air at a small angle of attack (see Fig. 5) :—(*a*) A slight upward deflection, called "*upwash*" occurs in front ; and (*b*) a considerable downward deflection, called "*downwash*", occurs behind the wing. The downwash is important as it affects the direction of the air striking the tail plane or other parts of the aeroplane in the rear of the main planes ; (*c*) A smooth streamline airflow takes place over the top and bottom surfaces ; (*d*) The streamlines are closer above the top surface than over the bottom ; (*e*) Above the top surface the speed of airflow is increased and below the bottom surface it is decreased ; (*f*) The pressure of the air above the wing is reduced below the normal atmospheric pressure due to the increased speed of the airflow ; and (*g*) the pressure below the wing is increased due to the decreased speed.

Though the facts stated in (*f*) and (*g*) appear to be puzzling at first, it can be explained by the Venturi tube experiment. Here the upper surface is somewhat similar in shape to the lower half of the Venturi tube and the closer streamlines above the highest part of the chamber resemble those passsing through the neck of the Venturi tube.

As stated in the case of a flat plate the decrease of pressure above the wing surface produces a force R_1—which is the important part of the total force—pulling the wing up and the increase of pressure below the wing gives rise to a force R_2 pushing the wing up. These two upward forces give us the resultant force R acting approximately at right angles to the chord line. But the decrease of pressure above the wing surface is more important, for to this is due the greater part of the lift force. Roughly about two-thirds of the total load on the wing may be attributed to this decrease of pressure while about one-third may account for the increase of pressure on the lower surface.

[**Note.**—It should be noted here that the common idea is that the airflow moving away from the upper surface of the wing causes a partial vaccum and thus provides a lift force, but this is wrong. In fact, the greater will be the increase of speed as the air is drawn closer on to the upper surface of the wing, and by the consequent reduction of pressure the upward force produced will be greater.]

18. (a) Centre of Pressure.—The point in the chord line through which the total force R may be considered to act is known as the **centre of pressure**. It has no fixed position but varies according to the speed and the angle of attack.

(b) Distribution of Pressure over an Aerofoil.—The distribution of pressure over the surface of an aerofoil has been experimentally determined and its study is of great importance. The method consists in distributing a number of glass tubes, which are placed parallel to the direction of motion, over the upper and lower surfaces of the aerofoil. These are connected to a manometer, and different pressures are ascertained. Fig. 6 shows the pressure distribution over an aerofoil at an angle of attack of 5°, from which following observations are made :—(a) The pressure is not evenly distributed, both the decreased pressures on the top surface and the increased pressures on the lower surface are most marked over the front portion of the aerofoil ; (b) the greatest pressure decrease (and hence the largest forces) occur on the top surface, and it is near the leading edge and over the highest part of the camber ; (c) the decrease in pressure over the top surface is greater than the increase on the lower surface.

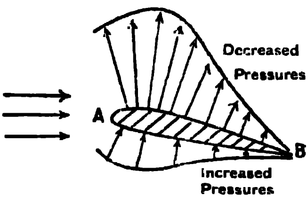


Fig. 6—Pressure Distribution

From this it is seen that the shape of the top surface is of great importance. It is the top surface, which by means of its decreased pressures, provides the greater part of the lift, and, at some angles of attack, this decrease of pressure on the top surface gives us as much as four-fifths of the lift.

(c) Movement of Centre of Pressure.—Experiments show that the distribution of pressure over the aerofoil changes considerably with the change of the angle of attack and consequently the centre of pressure (C. P.) moves. The position of C. P. is usually defined as a certain proportion of chord behind the leading edge. The movement of C. P. is an inconvenient property of the aerofoil, for unless its centre of gravity (C. G.) and C. P. coincide there will be a turning effect about C. G. To understand this suppose that for a certain angle of attack the C. G. and the C. P. coincide. Now, when the angle of attack increases there will be a forward movement of C. P. and so there will be a turning moment about C. G. equal to Rx, where R is the total wind thrust and x the distance between C. P. and C. G. This moment will rotate the aerofoil and still further increase the angle of attack and thus the equilibrium will be disturbed.

In any case, large moments of C. P. will make the aeroplane difficult to control and so in a good aeroplane the movement of C. P. should be limited which is obtained by a suitable bi-convex cross-section or by increasing the aspect ratio, for example, by tapering the wing.

19. Lift and Drag.—In practice the direction of motion of an aeroplane is not always horizontal, and so the L component is not always vertical. It is usual to split up the *Total Reaction R* into two components, L and D, *relative to the airflow* :—the component L which is always perpendicular to the direction of the airflow (or motion) is called **Lift**, and that parallel to the direction of airflow is called **Drag**, which is always opposite to the direction of motion. Lift is used to balance the weight of the aeroplane and so keep it in the air in level flight. Other parts of the aeroplane as tailplane, elevator, etc., may provide further lift forces when desired. *Drag is the enemy of flight* and every effort must be made to reduce it to a minimum. Only in normal level flight the lift is vertical and the drag horizontal, but if, in turning the wings of an aeroplane assume a nearly vertical position, then the lift (L) is nearly horizontal. Lift is always perpendicular to the direction of motion and drag is always opposite to it.

20. Lift and Drag Formulæ.—In Fig. 5, R is the resultant force on a transverse section of the wing of an aircraft whose angle of attack is $\sphericalangle$ and whose velocity is V. We have already seen in Art. 9 that the total reaction (or resistance) $R = \dfrac{K\rho AV^2}{g}$ lb.-wt.

We have in Fig. 5, the lift component $L = R \cos \sphericalangle$, and the drag $D = R \sin \sphericalangle$, whence

$$L = K \cos \sphericalangle \frac{\rho A V^2}{g} \text{ lb.-wt.} \ldots (1) ; \text{ and } D = K \sin \sphericalangle \frac{\rho A V^2}{g} \text{ lb.-wt.,} \ldots\ldots\ldots (2)$$

where ρ represents the air density (in lb. per cu. ft.), A the surface or plane area of the wing projected on the plane of the chord (in sq. ft.), V the velocity of air-speed (in ft. per sec.), and g the acceleration of gravity (= 32·2 ft. per sec.)

Since K is a constant for some given conditions in a machine we may write the symbol K_L for $K \cos \sphericalangle$ and K_D for $K \sin \sphericalangle$, which are spoken of as **Lift and Drag Co-efficients** respectively.

[**Note.**—(*a*) These symbols should not be confused with L and D which give the actual lift and drag in pounds weight, and K_L and K_D are constants only. (*b*) The above relations are strictly true when $\sphericalangle$ is small, for we are not justified in assuming that R is at right angles to AB for large angles of attack].

Then, we have, $L = K_L \dfrac{\rho A V^2}{g}$; and $D = K_D \dfrac{\rho A V^2}{g}$; and hence, dividing one by the other, we get the important relation, $\dfrac{L}{D} = \dfrac{K_L}{K_D}$ and L/D is known as the **Lift-Drag Ratio.**

Note that when L is exactly equal to the weight W of the aerofoil we get, for a normal horizontal flight, $W = \dfrac{K_L \rho A V^2}{g}$.

21. Factors affecting the L/D Ratio.—The factors affecting the L/D ratio are :—(*i*) *The angle of attack.* We get a maximum L/D at an angle of attack of about 4° (see Fig. 7) :

(*ii*) *The airspeed.*—Both lift and drag are directly proportional to the square of airspeed. Hence increase in airspeed will increase the lift and drag, other factors remaining the same.

(*iii*) *Increase in wing surface or plan area* (*i.e.* the area projected on to the plane of the chord). This will increase the lift and drag when the plane is flying at the same speed and the same angle of attack in air of the same density. (In practice, however, the angle of attack rarely remains constant even for a very short time).

(*iv*) *Increase in density of the air.* V and α remaining the same, the increase in density will increase and the decrease of the density will decrease, the lift and drag.

22. Lift and Drag Curves.—In order to get some idea of what happens when the angle of attack of a typical aeroplane wing is gradually altered, we shall consider the lift and drag curves shown in Fig. 7. Considering the curve drawn with the lift co-efficient L and the angle of attack, it will be seen that there is a definite lift at 0°, and that the lift increases steadily between 0° and 12°, where the graph is practically a straight line. The maximum value reaches at about 15° after which the lift begins to decrease rapidly. This rapid falling off is called *stalling*, and the angle of attack at which the lift reaches a maximum value is known as the **Stalling angle.**

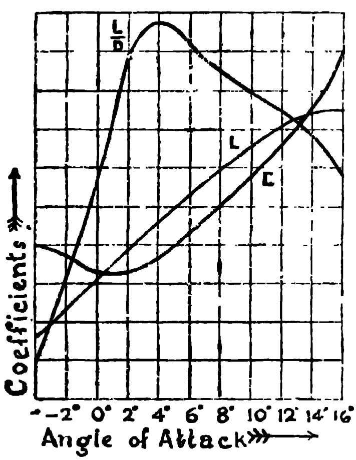


Fig. 7—Lift and Drag Curves

Now for the *Drag Curve* we find that its value is always positive. It is least at about 0°. The

increase of drag up to the stalling angle is not very rapid, but after it the increase becomes more and more rapid.

23. **L/D Ratio Curve.**—We have considered lift and drag separately, but it should be realised that the ratio L/D under varying conditions is of great importance. We know that an aeroplane travelling through the air must employ power to create a propeller-thrust in order to overcome the drag of the aerofoils, and so it is desirable to require as little power as possible for a given lift, or in other words, for the sake of efficiency we want as much lift, but as little drag, as possible for our aerofoil. In fact, we want the highest possible values for the L/D ratio for any given working range. From Fig. 7 we find that the lift is highest at about 15° and the least drag we get at about 0°, so at neither of these angles we really get the best conditions for flight, that is, the best lift in comparison to the drag, or the **best lift-drag ratio**. This shows the importance of the curve showing L/D ratio of the aerofoil against the angle of attack. Here we find that the greatest value of L/D occurs at about 3° or 4° at which angles the lift is about 20 times the drag. Thus it is seen that an ideal aerofoil must be moving at an angle of attack of about 3° or 4°, when it will give its best all-round result. This angle at which the best result is obtained is sometimes called the **Optimum angle**.

Note.—In Fig. 7, the values of lift and drag co-efficients are taken instead of the total lift and drag as the former will be practically independent of the air-density, the scale of the aerofoil, and the velocity employed, whereas the total lift and drag will depend on the actual conditions at the time of the experiment.

24. **Stalling.**—At values greater than that corresponding to the maximum lift, the lift falls off rapidly and this rapid falling off is called **stalling**, when the aeroplane is said to be stalled. Stalling is accompanied by a loss of lift as well as much increase in drag. The airflow no longer shows a smooth streamline flow and it finally changes into a turbulent flow. It is extremely dangerous if stalling happens at the time to be near the ground.

One of the devices in reducing the risk of stalling is the **Handley Page slot**, which is shaped rather like a wing and fitted on to the leading edge of the main wing. On moving forward the slot at a time when the angle of attack of the aerofoil is increased, a smaller angle of attack is presented to the on-coming air causing an increased airflow over the wing surface, and the lift is restored.

25. **Aerofoil Characteristics.**—The lift and drag co-efficients of an aerofoil depends on the shape of the aerofoil, and they will change with changes in the angle of attack. The result of experiments on

aerofoils can be easily demonstrated by drawing graphs to show how L co-efficient, D co-efficient, and L/D ratio alter with changes in the angle of attack. These three may be called the *characteristics of the aerofoil.*

26. (a) The Ideal Aerofoil.—The characteristics of the ideal aerofoil are given by the curves in Fig. 7. Thus the ideal aerofoil should have—

1. *A high maximum lift co-efficient* in order to lower the landing speed for the safety of the plane. The higher the lift co-efficient of the aeroplane, the lower will be its landing speed and greater will be the safety of the plane.

2. *A low minimum drag co-efficient*—not only at a certain angle of attack, but it should remain low over a large range of angles. Thus the aeroplane will have a low resistance and will be able to attain high speed.

3. *A high Lift-Drag ratio* for the sake of efficiency, good weight carrying capacity for a small expenditure of engine-power and so less expense

4. *A small movement of centre of pressure* to improve stability.

(b) Compromises.—In actual practice, however, we find that no aerofoil will meet all the requirements. Therefore some sort of compromise is made just as in the case of a good hydro-static balance. We cannot get an aeroplane which will serve all our different purposes, and the shape of the aeroplane is the first, and perhaps the greatest, compromise to be made. So different degrees of cambering is made according to the different purposes the aeroplane is desired to serve. For instance, for high speed the *top surface* camber should be about 7 or 8 per cent of the chord, while for general use it should be about 10 or 11 per cent of the chord

Both lift and drag are increased by increasing the chamber of the upper surface. The alterations in the chamber of the *buttom surface* of the aerofoil have a much smaller effect. Modern aerofoils have their lower surface flat or slightly convex.

27. (a) Normal Horizontal Flight.—Without taking into account the forces on the tail unit, an aeroplane, when flying straight and level —which we refer as normal horizontal flight—may be said to be under the influence of the four main forces,—

(1) The lift L of the main planes acting vertically upwards through the centre of pressure.

(2) The weight W of the aeroplane acting vertically downwards through the centre of gravity.

(3) The thrust T of the propeller airscrew pulling horizontally forward along the propeller shaft.

(4) The drag D acting horizontally backwards. This is the total drag on the aircraft consisting of the drag of the aerofoils and also of the remaining parts of the aeroplane.

(b) Conditions of Equilibrium.—In the ideal case when the aeroplane is flying level at a steady speed in a fixed direction, that is to say, the main condition of equilibrium of those four forces, which must obey the simple laws of mechanics, is that all the forces would act through the same point. Then we have—

(*i*) $L = W$. This condition will keep the aeroplane at a constant height. If $L > W$ (This is secured by increasing airspeed by increasing engine power), the aeroplane will ascend, and if $L < W$ the aeroplane will descend.

(*ii*) $T = D$. This condition will keep the aeroplane moving at a constant speed. If $T > D$ the aeroplane will move with an acceleration and if $T < D$ there will be retardation. In practice, however, these forces are never constant owing to varying conditions, such as the weight of the aircraft does not remain constant in value, L is not constant as the angle of attack may change due to wind-thrust, the position of C.G. is not constant Due to these difficulties the ideal arrangement of the forces is not possible.

Now when the size and position of forces change, the turning effect of the aircraft is controlled by the pilot by (*i*) control column movement (discussed later on) and (*ii*) mainly by adjustable tail plane.

CHAPTER IV

Aeroplanes and their Controls : Manœuvres

28. Component Parts of the Aeroplane.—We have already mentioned about some parts of an aeroplane and especially we have dealt with one of its main parts, *i.e.* the wings or aerofoils. Let us state here that an aeroplane mainly consists of the following parts :—

(*a*) *Fuselage* ; (*b*) *Wings* or *Aerofoils* ; (*c*) *Propeller* or *Airscrew* ; (*d*) *Tail plane* ; (*e*) *Aileron* ; (*f*) *Elevator* ; (*g*) *Rudder & Fin.*

The Fuselage.—The main body of the machine is referred to as the fuselage, which must be large enough to contain engine, tanks, pilot, bombs, goods, passengers, etc., that the machine has to carry.

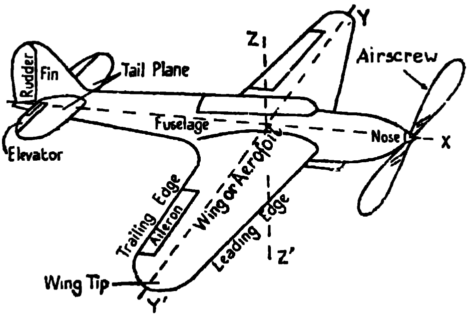


Fig. 8—Aeroplane

Tail plane.—It is a small plane fitted at a considerable distance behind the main planes in order to provide the upward or downward forces necessary to contact the unruly action of the four main forces mentioned in Art. 27.

29. The Propeller or Airscrew.—The theory of airscrew is too advanced to be considered here, but a general idea of the work of an airscrew will be given here.

A propeller, also called an airscrew, is much like an ordinary electric fan in appearance, but while a fan sucks air from behind and throws it forward, an airscrew sucks air from the front and throws it backwards. The result is that due to reaction the fan tends to move backwards, while the airscrew is thrust forward, and thus pulls the aeroplane along with it. The thrust of a propeller is the force with which it drives the air backwards or urges aeroplane forwards. The propeller is the means by which the power of the engine, which rotates it, is transformed into a forward thrust and thus gives the aeroplane a translational velocity. Thus, the aeroplane forces its way through the air by means of propellers rotating in a vertical plane, and we may say in effect that an airscrew screws itself through the air pushing or pulling the aeroplane to which it is attached. The propellers are situated either in front of the body of the machine, when it will cause tension in the airscrew shaft and will thus pull the aeroplane forward (in which case the aeroplane is called a **tractor**) ; or in the rear of the body when it will push the plane forward (in which case it is called a **pusher**). Airscrews vary in the number of blades from two to four, but the two-bladed variety is the easiest to manufacture and slightly more efficient. The shape of each part of an airscrew blade, taken in a direction at right angles to its length, is found to be similar to that of an aerofoil.

The diagram (A, B, C, in Fig. 9) shows several cross-sections

taken at various distances from the centre. The airscrew also derives similar forces from the airflow to those giving lift and drag in the case of wings, but owing to variations in camber, chord, and speed, the lift and drag components increase and decrease from section to section. The airscrew may be considered to be exactly like an aeroplane wing, but that, instead of moving in a straight line and supporting the aeroplane, the airscrew moves in a spiral path and produces the thrust which overcomes the drag of the aeroplane. Due to their different functions the plane form of an airscrew blade differs from that of a wing ; and the airscrew blade is twisted so that the angle to the shaft of the propeller is greater at the base than at the tip, while the angle of the wing is almost the same throughout. Thus the forward thrust of the airscrew corresponds to the upward lift of the aerofoil, and drag in this case is represented by the resistance of the air to the rotary motion of the airscrew.

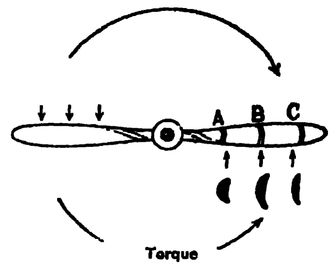


Fig. 9—Airscrew Torque

The total airscrew thurst is the sum of the thrusts on each blade section, and it is the force which pulls the aeroplane through the air. The total drag on all the blade sections constitutes a couple—known as **airscrew torque**—which resists the rotary motion of the airscrew (Fig. 9) and opposes the **engine torque** (or *the turning moment*) applied to the airscrew shaft by the engine. The airscrew torque has to be overcome by the engine torque. This is analogous to the thrust and drag in the case of aeroplane.

30. (a) Pitch.—The airscrew is a screw which screws its way through the air in the same way as an ordinary screw does through wood but some important differences are to be noted. In the case of an ordinary screw the distance moved forward in one revolution is a fixed quantity and is called the *pitch* of the screw, the value of which depends on its geometric dimensions, and is usually called the *geometric pitch.* But, in the case of the airscrew, the distance moved forward in one revolution (called the *advance per revolution*) is not a fixed quantity as it depends entirely on the forward speed of the aeroplane. Another important difference between the airscrew and the ordinary screw is that the airscrew has no actual grip on the air comparable to an ordinary screw in wood, and there is a certain amount of '**slip**' so that the distance moved forward is less than the geometric pitch. This distance is not also constant as it varies with the speed of the

aeroplane. Thus the '*slip*' of a screw is the difference between the distances it should travel theoretically and its actual progress.

(b) **Pitch Angle.**—We should all know that the twisted appearance of the airscrew blades is not without any meaning—rather it is the product of highly skilful design. The sections of the blade near the tip are moving with a much greater velocity than those near the root, and so most of the thrust is produced by the portions near the tip. For this reason the *pitch (or blade) angle* is not the same throughout the air-screw blade in order that every part of the airscrew may move the same distance forward during one revolution of It Other things being equal, a large propeller moving comparatively slowly gives more thrust than a small one driven at high speed. The **pitch** (or **blade**) **angle** is the angle which the chord of any given blade section makes with the horizontal plane when the airscrew is laid flat on this plane, its axis being vertical.

The *Experimental Mean Pitch* is the distance the airscrew moves forward in one revolution when the thrust is zero, and when the thrust and efficiency of the airscrew is a maximum, the pitch is called the *Effective pitch.*

(c) **Efficiency.**—The efficiency of an airscrew is the ratio of the useful work done by it to the work put into it by the engine. In actual flight for the same rotational speed of the airscrew, a forward motion —which means some useful work done—may be attained at which each blade section meets the airflow at an angle of attack of about 3°, which is the most efficient angle of attack for an aerofoil having its maximum lift-drag ratio. So, here the ratio of airscrew thrust to torque is a maximum ; and so at this speed the screw has maximum efficiency.

31. Fixed Pitch and Variable Pitch Air-screws.—It has been been seen that only at a particular speed of the aircraft a fixed-pitch airscrew has got its maximum efficiency at a given rotational speed, but, in practice, the actual speed of an aircraft varies over a more or less wide range. An airscrew whose pitch can be varied by the pilot, when in flight, is called a **variable pitch** airscrew, the mechanism of which is rather complicated, though, this is very effective for all conditions of flight. But whether a variable pitch airscrew is advisable or not depends on the speed-range of the aeroplane. For a high speed-range, a variable-pitch airscrew is essential, and when the maximum speed is relatively low, a fixed-pitch airscrew will work quite well. With this type of airscrew an aeroplane might be brought home safely when in danger, which would have been impossible with the

fixed-pitch type. So for a modern machine of high-speed range a V. P. airscrew is essential.

Stability and Balance

32. Stability and Balance.—If an aeroplane, when disturbed, tends to return to its original position, it is said to be **stable** and the **stability** of the machine means its capacity to return to some particular condition of flight after it is slightly disturbed from that condition.

Note.—**Stability** should not be confused with **Balance**. Suppose an aeroplane flies with one wing dipping than the other and it may, when disturbed from this state, return to its former position. Such an aeroplane is not unstable but only out of its proper balance.

33(a). Stability.—An aeroplane may rotate about three axes all mutually at right angles to each other and all passing through the centre of gravity of the aircraft. These axes are as follows :—*The Longitudinal* (or *rolling) axis* XOX' running from nose to tails ; *the Lateral* (or *pitching*) *axis* YOY' in the same horizontal plane, and *the Normal* (or *yawing) axis* ZOZ'.

(1) The rotatory motion of the aeroplane about the lateral axis is called **Pitching** caused mainly by a wind-gust resulting in the nose rising or depressing. During pitching the longitudinal axis moves in a vertical plane.

The capacity to correct pitching is defined as *Longitudinal Stability*.

(2) Any rotatory motion of the aeroplane about the longitudinal axis is called **Rolling**, resulting in one wing rising and the other dropping. The lateral axis moves in a vertical plane during rolling. The ability of the aeroplane to correct rolling is called *Lateral stability*.

(3) The rotatory motion about the normal axis is called **Yawing.** It results in the nose and tail being deflected to one side, and in this both axes move. The capacity to correct yawing is called *Directional stability*.

(**b**) **Longitudinal Stability.**—This is achieved by the tail plane by setting it at an angle less than that of the main planes. Suppose that due to wind-gust the nose of the machine is thrown up. The tail plane is then turned so that it presents an angle of attack less than that of the main planes and thus a force is obtained on the tail plane in such a direction as is necessary to counteract the movement of C. P. of the main planes, which is detrimental to stability, and thus to bring the machine to equilibrium position. Another condition for

longitudinal stability is that the position of the centre of gravity of the aeroplane must not be too far back.

(c) **Lateral Stability.**—During normal flight the lift on the wings is vertical, and equal and opposite to the weight, but when a roll takes place one wing drops and the other goes up. In this position the lift is inclined and is no longer in the same straight line as the weight. As a result of these two non-parallel forces, the machine cannot be in equilibrium and moves bodily sideways, called **Side slip**, in the direction of the lower wing. To overcome this lateral instability a small positive *dihedral angle* is introduced between the two wings by setting the wings to be inclined upwards by a small angle to the lateral axis. Now the vertical component of the lift on the lower wing is increased, the angle of attack being greater, and that on the other side is decreased and thus a couple is introduced which brings the aeroplane to the normal position. Lateral stability depends also on the positior of the centre of gravity of the aeroplane.

[The **dihedral angle** is the angle between each plane and the horizontal for the normal position. It is positive when the plane is sloping upwards and negative when sloping downwards].

(d) **Directional Stability.**—This is secured by fitting a small aerofoil vertically at the centre of the tailplane. This acts in a way similar to that of the tailplane and produces a force which opposes any tendency to spin round the normal axis. This small aerofoil is known as the '**Fin**', which is the most important factor, for, both by its surface area and position, a correcting turning moment is obtained from it.

Lateral and directional stability are inter-relative. A roll is followed by a yaw and *vice versa*, and the study of the two cannot be separated.

34(a). Control.—It is no doubt necessary that an aeroplane should be stable but that is not enough. It is also necessary to control the machine to force it to take any desired position, or to correct any tendency of the machine to wander from any desired path. When the pilot desires to bring about such changes he has at his disposal three movable control surfaces which are operated from the cockpit by means of cables or rods : (*i*) *the elevator* ; (*ii*) *aileron*, and (*iii*) *rudder*,

(b) **Longitudinal Control and the Elevators.**—Longitudinal control is the control of pitching and is obtained by the elavators which are flaps hinged behind the tailplane by which the angle at which the machine is flying can be altered and thus the nose of the machine can be raised or lowered as desired. Elavators are operated by means of the control column (also called *Joystick*) situated in front

of the pilot's seat. By pulling the joystick backwards, the elevators are raised by which action the aeroplane begins to ascend, and the opposite action takes place by moving the joystick forward.

(c) **Lateral Control and the Ailerons.**—Lateral control is the control of rolling of the lateral axis and is obtained by the ailerons, which are flaps hinged at the rear of the main wings near each wing tip They are connected together so that when one flap is depressed. the other on the opposite wing-tip is raised. When a machine has been titled through an angle laterally by a wind-gust, the pilot rights the aeroplane by depressing the ailerons. Thus by the aid of the ailerons the aeroplane may be *banked*, that is, the machine may fly with one wing lower than the other. The ailerons are operated by moving the control column by the hand or sometimes by a control wheel like the steering wheel of a motor car.

(d) **Directional control and the Rudder.**—It is the control of yawing, or rotation about the normal axis, and is obtained by the rubber, which is a vertical flap hinged on to the rear of the fin. This is operated by a *rudder bar* in the cockpit and worked by the pilot's feet. On pressing the right foot forward the rear of the rudder will be moved to the right and the aeroplane will turn to the right and so on. The function of the rudder is to keep the machine in its correct course ; and it is also used in conjunction with the ailerons for *turning* the machine.

In general, the movement by the rudder will give rise to a side force on the fin, movement of the elevator will produce a force on the tailplane, while the movement of aileron increases or reduces the lift on the wing, as the aileron is pulled down or pulled up.

It should be noted that in each of the above cases the control surfaces are placed as far as possible from the centre of gravity of the machine so as to provide sufficient leverage to alter its position.

(e) **Engine.**—Besides the above control units, the engine is also considered as another unit, the primary function of which, from the point view of control, is to vary the height at which the machine is flying for a given angle of attack, speed, etc.

(f) **Stability and Control.**—The difference between stability and control should be clearly noted. Stabilising devices, such as the tailplane and fin, restore the aeroplane to its original path of flight after a disturbance has occurred, while, on the other hand, the pilot uses the control surfaces, such as the elevators, etc., to manœuvre the machine into any desired position ; but this change of attitude will be resisted by the inherent stabilising devices. The control surfaces

should therefore be effective enough to overcome the action of the stabilising devices.

35. Manœuvres.—The various manœuvres which an aeroplane may be required to perform are given below :—

(1) **Take-off and Landing.**—In *take-off*, the throttle of the engine is opened, and the machine moves over the ground gaining speed, while the pilot depresess the elevators, thus raising the tail. The machine then rises up attaining the minimum speed to be sustained in air.

Landing is done by bringing down the speed of the aircraft until it is brought into contact with the ground. Landing may be '**slow**' or '**fast**'.

(2) **Gliding.**—In this the engine is throttled down until the speed of the engine is just sufficient to keep the engine going. Now the thrust T disappears and the aircraft must be kept in equilibrium by the forces of lift, drag, and weight only, that is, the total reaction, or the resultant of the lift and drag, must be exactly equal and opposite to the weight. The angle between the path of the glide and the horizontal is called the *gliding angle*, which is the same as the angle between the lift and the total reaction.

(3) **Climbing.**—In order to make a climb, the pilot holds the control column backward to have the angle of attack between the normal and stalling values.

(4) **Banking.**—Banking is accomplished by moving the ailerons over, so that one wing drops and the other rises. In this the lift force, in addition to lifting the machine, supplies a component towards the centre of the turn, so that a large force is obtained for pulling the machine into a circular path and settling it down to the steady condition.

Besides these, other different manœuvres done by expert pilot are as follows :—(5) *Side slip* (see p. 526) ; (6) *Ceiling* (see Art. 35) ; (7) *Loop* ; (8) *Spin* ; (9) *Roll* ; (10) *Zoom* ; (11) *Nose-dive.*

36. High Altitude Flying.—It has already been pointed out that with the increase of altitude the density, pressure, and temperature of the atmosphere all decrease, and these cause important modifications in the forces acting on an aircraft. Here we shall consider only the effects of the decrease in the density of air ; which may be summarised as follows :—(a) Decrease in lift and drag ; (b) Falling off in the power of the engine ; (c) Decrease in airscrew thrust.

(*a*) Lift and drag depend on the density of air. At higher altitudes the density of air is considerably less than that at ground level and

so the lift can no longer balance the weight of the aircraft. It is necessary, therefore, either to increase the speed of the aircraft, leaving the angle of attack the same, to obtain sufficient lift to balance the weight, or to increase the angle of attack (see Art. 21), which in turn will increase the drag.

There is, however, a limit to the possible increase of speed as it depends on the power of the engine which is also limited, and further there is also a limit to the increase of the angle of attack, as, we know, when this is made too great, the lift will decrease instead of continuing to increase, or, in other words, there will be stalling of the machine.

(*b*) As the pressure of air decreases with height, the weight of petrol-air mixture taken into the cylinder of the engine for combustion is reduced and so there is a considerable falling-off in the power of the engine. This may be remedied to a certain extent by *super charging, i e.* by forcing the mixture into the cylinder with a pump. But ultimately atmospheric pressure becomes so small that, with all existing engines, there is a height at which the power begins to fall off inspite of the supercharger, and we find that sooner or later a height is reached which cannot be exceeded. This height is called the **Ceiling** of the aeroplane.

(*c*) In rarefied air the airscrew-thrust is sufficiently reduced even when the engine and propeller revolutions per minute are sufficiently increased. In such cases variable pitch airscrews are usually employed to compensate for the loss to some extent.

The low temperature and low pressure at high altitudes clearly affect the comfort of the pilot and other passengers. For the low temperature, heavy warm clothing, often electrically heated, may be used, and for oxygen deficiency in the lungs, oxygen is supplied from cylinders. But at sufficiently high altitudes, the pressure in the lungs becomes so low that oxygen cannot be forced in and the oxygen deficiency finally endangers life.

At heights more than 10,000 ft. symptoms such as drowsiness, breathlessness, muscular weakness, etc., become pronounced, and above about 25,000 ft., it becomes dangerous. Apparatus for the artificial administration of oxygen is always necessary for high altitude flights, and with its aid flying upto heights of about 35,000 ft. may be safely undertaken.

Besides this, discomfort or pain in the ear is often felt by the pilots due to changing atmospheric pressure.

From what has been said above, it is clear that there is a limit to the height at which a particular aeroplane can fly, depending, of course, on the general design of the aeroplane and on the engine-power available.

Questions

1. What is meant by 'stream-line' flow and 'stream-line' body? What is the importance of stream-lining all parts of an aircraft which are exposed to airflow.

2. Write short notes on any three of the following :—(a) Airships; (b) Aerofoil; (c) Parachutes; (d) Stream-line and Turbulent flow; (e) Stalling. (Pat. 1944)

3. What is meant by "Aerofoil"? How does the aerofoil of an aeroplane determine the efficiency of it?

Explain fully how the wings of an aeroplane support it high up in the air. Indicate the forces that act on the machine. (Pat. 1939)

4. Write a note on the flight of an aeroplane, indicating the part played by the more important portions of it. (Pat. 1942)

5. Describe what happens when a flat plate moves through air, and explain why aeroplane parts are stream-line shaped. (Pat. 1939 ; Cf. '44)

6. Explain what is meant by 'stream-lines' flow. Describe an experiment to demonstrate the deformation of stream-lines by an obstacle.

Discuss the flow of air past a flat plate moving through air with a high velocity, with its plane inclined at a small angle to the direction of motion. Show how a lifting force is produced on the plate and explain how it varies with the angle of incidence of the plate. (Pat. 1945)

7. What are cambered wings in an aeroplane? Explain their action. Also explain with neat diagrams the actions of the tail, elevator, fin, and rudder. (Pat. 1938 ; '49)

8. What is a 'cambered wing'? Draw a neat diagram of a section of it perpendicular to its span, and indicate in it the lengths known as the 'chord', the 'upper camber', and the 'lower camber'. What are the essential points that are kept in view in its construction? (Pat. 1940)

9. What are the functions of :—(a) the propeller; (b) the wings; and (c) the ailerons in an aeroplane? Is there any limit to height to which an aeroplane can ascend? Give reasons. (Pat. 1942)

10. How do you get the 'lift' that supports an airplane in the air, and what is the corresponding 'wing-drag'?

Define co-efficients of 'lift and drag', and prove that in horizontal flight (when lift must be equal to the weight of the machine)

$$V=\sqrt{\frac{W}{dsk_L}},$$ where V is the velocity of the machine, s the area of the wing, k_L the co-efficient of 'lift', and d the density of air supposed uniform. (Pat. 1940)

11. A stream-line body having a frontal area of 1 sq. ft. moves through air with a speed of 180 m.p.h. Calculate the 'drag' on the body assuming the value of 'drag' co-efficient to be 0·04 and that density of air 0·056 lb. per cu. ft.

[*Ans :* 2·44 lb.-wt.]

12. Define 'Centre of Pressure'. How does the C. P. of an aerofoil move with the increase of the angle of attack from 0° to 20°.

13. What are the factors on which the 'lift and drag' of an aircraft depend.

14. Criticise the following statements :—(a) 'Lift' increases as the angle of attack of the wing increases ; (b) 'lift' is always vertical ; (c) 'lift and drag' are affected only by air-speed and angle of attack.

15. Draw a neat sketch of an aeroplane showing its essential parts, and explain fully the control system in it. (Pat. 1948)

16. Draw a sketch showing the four principal forces acting on an aeroplane in normal horizontal flight.

17. What is an 'air-screw' ? Explain how it gives the forward motion to an aeroplane. (Pat. 1939)

18. Describe the parts of an aeroplane which ensure its stability in all possible modes. Illustrate, by neat sketches, the mechanisms to control its motion in various directions and indicate how the pilot manipulates them in taking a turn. (Pat. 1941 ; Cf. '44)

19. How can you distinguish the difference between stability and control. Name the axes about which pitching, rolling and yawing of an aircraft takes place. Which control is used to produce each motion ?

20. At a certain speed of normal horizontal flight of an aeroplane the ratio of its lift to drag is 7·5 to 1. What are the values of lift, thrust, and 'drag' when there is no force on the tailplane ? The weight of the aeroplane is 3500 lbs.

[*Ans :* lift = 3500-lb.-wt. ; thrust = 467 lb.-wt. ; drag = 467 lb.-wt.]

21. What is the true air-speed of an aeroplane at a certain height weighing 60000 lbs. and having a wing area of 1300 sq. ft. The 'lift' co-efficient is 0·5 and the density of air at that height is 0·056 lb. per cu. ft.

Calculate also 'thurst' and 'drag' when the value of L/D ratio is 8.

[*Ans :* Speed = 248 m.p.h. ; thrust = 7500 lb.-wt. = drag = 7500 lb.-wt.]

APPENDIX (B)

1. Trigonometrical Ratios—Let ABC be an acute angle represented by θ (Fig. 1). From any point D in AB drop a perpendicular DE on BC. It can be shown geometrically that where'ver the point D be taken on AB, the ratio, DE/BD, *i.e.* $\frac{\text{perpendicular}}{\text{hypotenuse}}$, is constant and bears a fixed relation to the magnitude of the angle θ. This ratio is called the *sine* of θ. Similarly, the ratio BE/BD is also constant and is called the *cosine* of θ. So, we have the trigonometrical ratios as given below :—

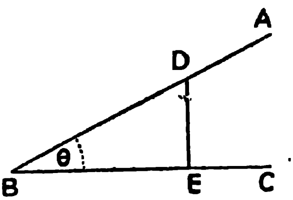


Fig. 1

$$\frac{DE}{BD} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \textbf{sine } \theta \text{ and is written, } \textbf{sin } \theta\,;$$

$$\frac{BE}{BD} = \frac{\text{base}}{\text{hypotenuse}} = \textbf{cosine } \theta \text{ and is written, } \textbf{cos } \theta\,;$$

$$\frac{DE}{BE} = \frac{\text{perpendicular}}{\text{base}} = \textbf{tangent } \theta \text{ and is written, } \textbf{tan } \theta,$$

$$= \frac{DE/BD}{BE/BD} = \frac{\sin\theta}{\cos\theta}.$$

2. Values of Trigonometrical Ratios.—The values of these ratios can be geometrically deduced for angles of 0°, 30°, 45°, 60°, and 90° which are given below [see Fig. 2 (*a* and *b*)].

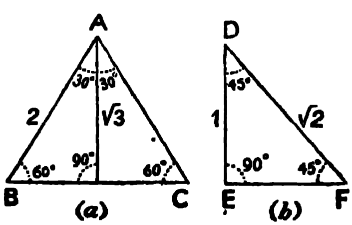


Fig. 2

The important values are tabulated below :—

Angle	Sine	Cosine	Tangent
0°	0	1	0
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	1	0	∞

3. From the above table it should be noted that,

Sin $0^\circ = \cos 90^\circ = 0$; $\sin 90^\circ = \cos 0^\circ = 1$;
Sin $30^\circ = \cos 60^\circ = 1/2$; $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$;
Sin $45^\circ = \cos 45^\circ = 1/\sqrt{2}$.

4. The inverses of sine, cosine and tangent are cosecant, secant, and cotangent respectively. That is, $\operatorname{cosec} \theta = \dfrac{1}{\sin \theta}$; $\sec \theta = \dfrac{1}{\cos \theta}$; $\cot \theta = \dfrac{1}{\tan \theta}$.

5. It follows geometrically from Art. 1, that,

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta.$$

6. **Signs of Trigonometrical Ratios.**—According to the conventions followed,

(*i*) for all angles in the first quadrant, the signs of all ratios are positive ;

(*ii*) for all angles in the second quadrant, only the sign of *sine* is positive and the signs of other ratios negative ;

(*iii*) for all angles in the third quadrant, only the sign of *tan* is positive and the signs of other ratios negative ;

(*iv*) for all angles in the fourth quadrant, only the sign of *cos* is positive and the signs of other ratios negative.

APPENDIX (C)

GRAPHS

Graph.—A graph is a representation, by means of a curve, of the relation between two *variable quantities.*

Axes of Co-ordinates.—Every point in the graph must be plotted with reference to two fixed straight lines XOX' and YOY' in the plane of the paper. These two straight lines are at right angles to each other, which divide the plane into four spaces XOY, YOX', $X'OY'$, $Y'OX$. These spaces are denoted by the first, second, third, and fourth *quadrants* respectively (Fig. 1).

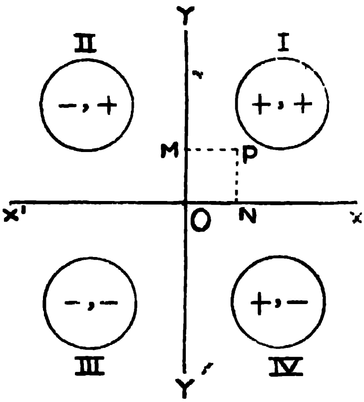


Fig. 1

The position of any point P in the plane can be located by knowing its perpendicular distances PN and PM from the two axes XOX', YOY'. These distances (PN and PM) are called the **co-ordinates** of the point P, PN being known as the **ordinate,** and PM the **abscissa** of the point P. The lines of reference XOX', YOY' are called the *axes of co-ordinates*, or simply *the axes*, the line XOX' being known as the **X-axis** and YOY' as the **Y-axis**. The point O is called the **origin** for which the co-ordinates for both the axes are zero, and the point is denoted as (O, O). Thus the ordinate of a point lying on the X-axis is O, and the abscissa of a point on the Y-axis is also O.

It should be noted that in the *first quadrant,* both the X and Y-co-ordinates are *positive*, in the *second quadrant*, X-co-ordinate is positive but the Y-co-ordinate is *negative* ; in the *third*, both the co-ordinates are *negative* ; and in the *fourth* quadrant, X-co-ordinate is *positive*, but the Y-co-ordinate is *negative.* As a general rule it may be expressed thus :—Ordinates *above* the X-axis are taken as *positive*, and ordinates below the X-axis are taken as *negative.* Similarly, abscissæ to the *right* of Y-axis are taken as *positive*, and abscissæ to the *left* of Y-axis are taken as *negative.*

Thus, the position of any point $A(-4, 3)$ will be in the second, and that of the point $B(-3, -4)$ will be in the third quadrant.

Choice of Axes.—In all physical problems there are two variables, of which one is the **independent** and the other the **dependent** variable. For instance, in the case of a simple pendulum, we know that time t for one complete oscillation depends upon l, the length of the pendulum. Thus, here t is the dependent and l the independent variable. As a rule **plot the independent variable along the X-axis, and the dependent variable along the Y-axis.**

Choice of Units.—To choose the unit for the ordinate or the abscissa, find the difference between the highest and the lowest values of it (given in the problem), and divide this by the number of available divisions of the graph paper along the same side. Thus get the approximate value of each division and then choose the next best possible value. Since in the graph paper the tenth or the fifth lines are generally drawn thicker, attempt should always be made to choose the units in such a manner that the larger divisions are multiples or submultiples of 5. If each division represents values, which are divisible by 10, such as 10, 100, 1000, or ·1, ·01, ·001, the ploting of points will be easier. Beginning from the origin write down the values along the X-axis and the Y-axis every 5 or 10 divisions apart.

Rules.—In drawing a graph for any physical experimental data the following rules may generally be observed—

(1) Obtain data for at least 6 points in the graph and tabulate the values for the X-axis (*independent variable*) and Y-axis (*dependent variable*).

(2) If there are both positive and negative signs in the given data, then the origin, *i.e.* the point of intersection of the two axes, should be in the middle of the graph paper, but, if the signs are all positive, the origin can be shifted to the extreme lowest position on the left of the paper in order to have a graph of *larger size.*

(3) Choose the units explained before, and plot the points marking their positions in the diagram by × or O sign.

Different suitable scales may generally be chosen for the two axes, but in some cases, as when area is to be calculated from the graph, equal scales will be convenient.

(4) The point of intersection of the two axes need not always be the zero of each axis.

(5) From the positions of the points, judge the nature of the graph and draw a smooth curve by joining the plotted points.

(6) The curve should necessarily pass through all the points, but keeping the nature of the graph intact, it should pass through *as many points as possible.* The point (or points) which does not lie on the curve is probably an *error* in the corresponding observation.

(7) The units should be so chosen that *the curve may cover as much of the graph paper as possible.*

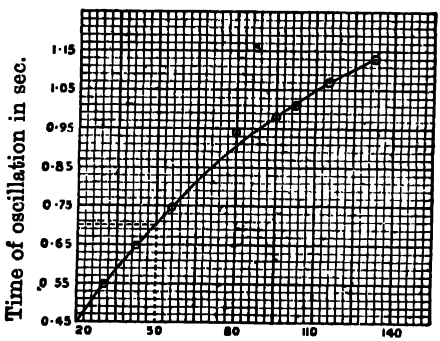
Examples

1. The following readings were obtained with a simple pendulum :

Length in cm.	20	30	42	55	70	80	95	102	115	130
Time of oscillation in seconds	·45	·55	·65	·74	·835	·94	·98	1·01	1·07	1·14

Represent by a graph the relation between the length and time and find from your graph the time of oscillation of a simple pendulum of length 50 cms. (*C.U.* 1915)

Here we find that the time of oscillation depends upon the length of the pendulum, so *time* is the *dependent variable* and should be plotted along the *Y* axis, and the *length*, which is the *independent variable*, should be plotted along the *X*-axis.



Length in cm.
Fig. 2

The difference between the highest value (1·14) and the lowest value (0·45) of time is 0·69, and the number of available divisions on the graph paper is 40. Therefore the approximate value of each division on the *X*-axis should be at least $\frac{\cdot 69}{40} = 0{\cdot}0017$.

Take each small division on the X-axis to represent 0·0020, which is the next best possible value.

Take one small division to represent 4 cms. on the *X*-axis.

Since the length begins from 20, and the time from 0·45, it is necessary to start from the origin as zero.

Write down the values of the ordinates every 5 divisions apart and begin 0·45 as the zero value of the ordinates, and similarly take 20 cms. as the zero value of the abscissæ.

Now plot the points and draw the graph (see Fig. 2).

To get the time of oscillation of the pendulum of length 50 cms., draw a straight (dotted) line through the point marked 50 cms. on the X-axis parallel to the Y-axis cutting the curve at a point, the ordinate of which has the value 0·71, which is the required time.

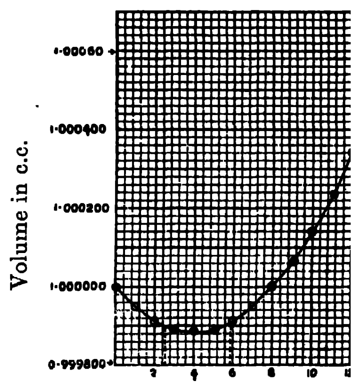
2. *From the following data plot a curve showing the variation in the volume of a mass of water with the temperature. Find graphically the two temperatures, at which the volume of 1 c.c. of water at 0°C. becomes 0·99990 c.c.* (C. U. 1909)

Temp.	Volume	Temp.	Volume
0	1·000000	7	0·999952
1	0·999948	8	1·000008
2	0·999911	9	1·000068
3	0·999889	10	1·000147
4	0·999883	11	1·000287
5	0·990891	12	1·000344
6	0·990914	13	1·000462

Here we find that on changing the temperature the volume is changed, so *temperature* is the *independent variable* and should be plotted along the the X-axis, and *volume*, which is the *dependent variable*, should be plotted along the Y-axis.

The difference between the highest value (1·000462) and the lowest value (0·999883) of volume is 0·000579. The number of available divisions on the Y-axis is 40. Therefore, the approximate value of each division of the Y-axis should be at least $\frac{0·000579}{40} = 0·000144$.

Take each small division to represent 0·000020, which is the next best possible value. Take 2·5 small divisions to represent 1°C. on the X-axis.



Temperature in Centigrade
Fig. 3

Write down the values of the ordinate every 5 divisions apart taking 0·999800 as the zero reading, and also write the values of temperatures on the X-axis.

Plot the points and draw the graph (Fig. 3). To get the value of the temperature corresponding to 0·99990 c.c., draw a straight line through the point (0·99990) parallel to the X-axis cutting the curve at two points, the abscissa of the first point being 2·41 and that of the second point being 5·8

nearly. Therefore the required temperatures are $2^\circ\!\cdot\!41$ and $5^\circ\!\cdot\!8$. Here the unknown result is determined by what is known as **"Interpolation."**

3. The battery resistance 'b' ohms for a current 'c' ampere was found in a certain test as follows :—

b	4·2	4·8	5·0	5·8	7·6	8·5	11·0
c	0·21	0·16	0·14	0·14	0·066	0·06	0·04

Illustrate the results graphically. Are they consistent with Ohm's Law? *(Pat. 1920)*

Plot '*b*' along the *X*-axis and '*c*' along the *Y*-axis (Fig. 4).

Units—1 small division on the *X*-axis represents 2 ohms.

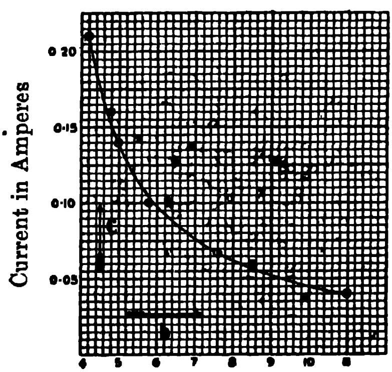
1 small division on the *Y*-axis represents 0·005 amp.

According to Ohm's Law the product of current strength and the corresponding resistance should be constant, which is not the case here. Hence the results are not consistent with Ohm's Law.

4 A copper rod is found to be 5·009, 5·0018, 5·0027 metres long at temperatures 10°C., 20°C., 30°C. respectively. Find by means of a graph its length at 0°C.

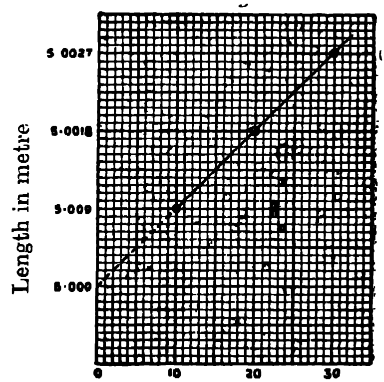
Units—each small division on the *X*-axis represents 1°C. *(C.U.1913)*

" " *Y*-axis " 0·00009 metre



Resistance in Ohms

Fig. 4



Temperature in Centigrade.

Fig. 5

Here some space is left below the point 5·0009 on the Y-axis in order to allow the curve (Fig. 5) to cut the Y-axis below this point, if necessary.

The graph obtained is a straight line which being produced meets the Y-axis at a point, the value of which is 5 metres from the graph. Thus the required length at $0^\circ C$. is 5 metres.

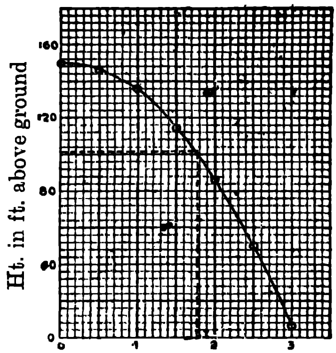
This method of determining the unknown result by producing the curve is known as "**extrapolation**".

5. *Draw a curve on the squared paper supplied to indicate the height above ground, at intervals of half a second, of a body falling freely from rest at a height of 150 ft.*

Find from your graph the position of a particle after 1·67 seconds. (*C. U. 1912*)

The space traversed by a body falling from rest $= \frac{1}{2}gt^2$, and hence the height above the ground at any time $= (150 - \frac{1}{2}gt^2)$ ft.

Taking $g = 32$ ft. per sec., the distance fallen through, and so height above the ground at intervals of half a second, is calculated and the following table is prepared.



Time in sec.

Fig. 6.

Time in seconds	0	0·5	1	1·5	2	2·5	3
Height fallen, in feet	0	4	16	36	64	100	144
Height above ground in feet	150	146	134	114	86	50	6

Units.—1 small division on the X-axis represents 0·1 sec.

1 small division on the Y-axis represents 4 ft.

The position of the body at the end of 1·67 sec., obtained from the graph (Fig. 6), is nearly 103 ft. above the ground.

PHYSICAL TABLES

(1) UNITS

Quantity	F.P.S Unit	C. G. S. Unit
Length	foot	centimetre
Mass	pound	gram
Force	poundal	dyne
Work	foot-poundal	erg
Power	horse-power	ergs per second

A force equal to the weight of 1 pound = 32·2 poundals. A force equal to the weight of 1 gram = 981 dynes.

(2) METRIC EQUIVALENTS

LENGTH

1 cm.	0·3937 inch = 0·032 ft	...	1 inch = 2·54 cms.
1 metre	39 37 inches		
	= 2·28 feet	...	1 foot = 0·305 metre
	= 1·09 yards	...	1 yard = 0·914 metre

1 kilometre = 39370·790 in. = 3280·899 ft. = 1093·633 yd. = 0·621 mile.

AREA

1 sq. inch = 6·45 sq. cm	...	1 sq. cm.	= 0·155 sq. in.
1 sq. foot = 0·093 sq. metre	...	1 sq. metre	= 10·764 sq. ft.
1 sq. yard = 0·836 sq. metre	...	1 sq. metre	= 1·196 sq. yds.
1 sq. mile = 2·590 sq. kilos	...	1 sq. kilo.	= 0·386 sq. mile.

VOLUME

1 cub. in. = 16·387 c.c.	...	1 c.c.	= 0·061 cub. in.
1 cub. in. = 0·061 litre.	...	1 litre	= 61·02 cub. ins
1 gallon = 4·546 litres	...	1 litre	= 1·76 pints.
		1 litre	= 0·22 gallon.

1 gallon = 0·1604 cub. ft. = 10 pounds of water at 62° F.

MASS

1 grain (gr.) = 0·065 gram (gm.) ... 1 gram = 15·432 grains.
1 ounce (oz.) = 28·35 grams. ... 1 m. gram. = 0·015 grain.
1 pound (lb.) = (16 oz. 7000 grains) = 453·6 grams. (1 gram. = 0·0022 lb.)
1 pound = 0·453 kilogram ... 1 kilogram = 2·205 pounds = 0·0009 ton.

FORCE

1 gram weight = 981 dynes.
1 pound weight = $4·45 \times 10^5$ dynes = 32 2 poundals.
1 poundal = 1 lb. wt. $\div g$ = 13,825 dynes.

(3) MENSURATION

$\pi = 3·14159$; $\pi^2 = 9·87$; $\log \pi = 0·4972$; $\log \pi^2 = 0·9943$
Radius of a circle = r ...... Circumference of circle = $2\pi r$.
$\sqrt{2} = 1·4142$; $\sqrt{3} = 1·7321$.

AREA

Square (side l) = l^2
Rectangle (breadth b) = $l \times b$
Parallelogram = base × perpendicular height.
Triangle = $\frac{1}{2}$ base × perpendicular height.
Circle = πr^2
Surface of cube (side l) = $6l^2$
Surface of sphere (radius r) = $4\pi r^2$
Curved surface of cylinder (radius r, height h) = $2\pi r \times h$.

VOLUME

Cube = l^3
Cylinder = area of base × perpendicular height.
Cone = $\frac{1}{3}$ (area of base × perpendicular height).
Prism = area of base × perpendicular height
Sphere = $\frac{4}{3}\pi r^3 = \frac{2}{3}$ circumscribing cylinder.

(4) USEFUL DATA

The weight of 1 cu. ft. of water = 62·5 lbs. approximately.

,, ,, ,, air at 0°C. and 1 atmosphere = 0·0807 pound.

The weight of 1 cu. ft. of hydrogen at 0°C. and 1 atmosphere = 0·0056 pound.

1 foot pound = $1·356 \times 10^7$ ergs.

1 horse-power hour = 33,000 × 60 foot-pounds.

Volts × amperes = watts.

$$\left\{\begin{array}{l}\text{1 standard atmosphere} = \text{760 millimetres or 30 inches of mercury ;}\\ = \text{1033 grms.-wt. per sq. cm.} = (1033 \times 981) = 1\ 013 \times 10^6 \text{ dynes per. sq. cm.}\\ = \text{14·7 pounds per sq. inch.} = \text{2116 pounds per sq. foot.}\end{array}\right.$$

Height of standard water barometer = 760 × 13·596 mm. = 29·92 × 13·596 inches.

A column of water of height 2·3 feet corresponds to a pressure of 1 lb. per sq. inch.

Velocity of light = 186,326 miles per sec. ; 3×10^{10} cms. per sec.

(5) DENSITY OR MASS PER UNIT VOLUME (IN GRAMS PER C. C.)

METALS

Aluminium	2·7	Bismuth	9·8
Antimony	6·7	Copper	8·9
Gold	19·3	Nickel	8·9
Iron (cast)	7·2	Platinum	21·5
,, (wrought)	7·8	Silver	10·5
,, (steel)	7·7—7·9	Tin	7·3
Lead	11·37	Zinc	7·1
		Quartz	2·65

ALLOYS

Brass	8·4—8·7	Bronze	8·7

LIQUIDS

Alcohol	0·79	Paraffin	0·70—0·82
Aniline	1·02	Olive oil	0·91—0·93
Benzene	0·89	Turpentine	0·87
Ether	0·72	Petrol	0·68—0·78
Glycerine	1·26	Petroleum	0·878
Kerosine	0·8	Water (4°C.)	1·00
Milk	1·03	,, (25°C.)	0·99708
Mercury (0°C.)	13·596	Sea water	1·026
Oil Linseed	0·94	Methylated spirit	0·83

COMMON SUBSTANCES

Cork	0·22—0·25	Paraffin	0·9
Ice	0·92	Porcelain	2·3
India-rubber	0·9—1·3	Quartz	2·6
Chalk	1·9—2·8	Salt (common)	2·2
Glass (crown)	2·4—2·6	Sand	2·6
Glass (flint)	2·9—4·6	Slate	2·3
Marble	2·7	Sugar	1·6
Ivory	1·8	Wood (teak)	0·7—0·8
Guttapercha	0·97	Wax (bees)	0·96

(6) MELTING POINTS

Bees wax	63°C.	Tin	232°C.
White wax	68°C.	Tungsten	3400°C.
Butter	28°—33°C.	Paraffin	45°—56°C.
Ice	0°C.	Sugar	160°C.
Copper	1083°C.	Sulphur	115°C.
Iron	1527°C.	Mercury	—39°C.
Platinum	1773°C.	Glass	1100°C.
Lead	327°C.	Naphthalene	80°C.

(7) BOILING POINTS

Alcohol	...	78°C.	Ether	...	35°C.
Aniline	...	182°C.	Glycerine	...	290°C.
Chloroform	...	61°C.	Mercury	...	357°C.
Water	...	100°C.	Turpentine	...	158°C.

(8) CO-EFFICIENT OF EXPANSION

Co-efficients of Linear Expansion of Solids

Aluminium	...	0·000022	Iron	...	0·0000114
Brass	...	0·000019	Lead	...	0·000029
Copper	...	0·000017	Platinum	...	0·000009
German-Silver	...	0·000018	Silver	...	0·000019
Glass	...	0·0000083	Tin	...	0·0000214

Co-efficients of Cubical Expansion of Liquids

Alcohol (ethyl)	...	0·00122	Olive oil	...	0·0007
Aniline	...	0·00085	Sulphuric Acid	...	0·0095
Glycerine	...	0·00053	Turpentine	...	0·00094
Mercury	...	0·00018	Water (10°—30°)	...	0·000203

Co-efficients of Cubical Expansion of Gases

The co-efficient of increase of volume of all gases at constant pressure and the co-efficient of increase of pressure of all gases at constant volume may be taken to be $= \frac{1}{273} = 0·003665$.

(9) SPECIFIC HEATS OF SOLIDS

Aluminium	...	0·21	Iron	...	0·11
Brass	...	0·09	Lead	...	0·03
Charcoal	...	0·19	Marble	...	0·22
Copper	...	0 095	Sand	...	0·19
Glass	...	0·16—0·19	Silver	...	0·056
Gold	...	0·03	Tin	...	0·055
Bismuth	...	0·03	Zinc	...	0·093
India-rubber	...	0·48	Paraffin	...	0·64
Ice (0°C.)	...	0·50	Sulphur	...	0·163
Common salt	...	0·20	Nickel	...	0·11

Specific Heats of Liquids

Alcohol	...	0·62	Mustard oil	...	0·50
Aniline	...	0·50	Mercury	...	0·033
Glycerine	...	0·58	Turpentine	...	0·43
Paraffin oil	...	0·53	Water	...	1·00

Specific Heats of Gases *(at Const. press.)*

Air	...	0·237	Oxygen	...	0·217
Steam	...	0·465	Hydrogen	...	0·41

(10) LATENT HEATS OF FUSION (*Calories*)

Water	...	80·0	Mercury	...	2·8
Sulphur	...	9·4	Lead	...	5·4
Silver	...	21·0	Bismuth	...	12·6

(11) MAXIMUM PRESSURE OF WATER VAPOUR (In Millimetres of Mercury)

Temperature (Centigrade)	Pressure (mm.)	Temperature (Centigrade)	Pressure (mm).
—10°	2·1	40	55·13
0	4·57	50	92·30
2	5·29	60	149·2
5	6·54	70	233·5
8	8·04	80	355·1
10	9·20	90	525·8
12	10·51	95	634·35
15	12·78	100	760·0 = 1 atoms.
18	15·46		
20	17·51	150	3569·0 = 4·7 atoms.
25	23·69	200	11647 = 15·4 atoms.
30	31·71		

(12) THERMAL CONDUCTIVITIES (in C. G. S. units)

Air	...	0·00005	India-rubber	...	0·0004
Aluminium	...	0·48	Lead	...	0·080
Brass	...	0·26	Mercury	...	0·0148
Copper	...	0·22	Silver	...	0·98
Glass	...	0·0005	Water (0°C.)	...	0·0012
Iron	...	0·16 to 0·18	,, (30°C.)	...	0·001

(13) VELOCITIES OF SOUND AT 0°C.

Substances	Feet per sec.	Metres per sec.
Air	1090	332
Oxygen	1041	317
Hydrogen	4163	1270
Carbon dioxide	856	262
Coal gas	1609	493
Iron	16820	5130
Glass	16410	5000
Brass	11480	3500
Water	4714	1437
Marble	12500	3810

(14) REFRACTIVE INDICES

Alcohol (Ethyl) ...	1·37	Glycerine ...	1·47
Diamond ...	2·47	Ice ...	1·31
Crown glass ...	1·53	Water ...	1·33
Flint glass ...	1·62	Turpentine ...	1·47
Ether ...	1·36	Rock-salt ...	1·55
Carbon disulphide ...	1·63	Canada balsam ...	1·53
Paraffin ...	1·44	Benzene ...	1·50
Quartz ...	1·55	Aniline ...	1·58

(15) CRITICAL ANGLES

Crown glass ...	41°·45 C.	Turpentine ...	43°·15 C.
Diamond ...	24°·25 C.	Water ...	48°·5 C.
Rock-salt ...	40°·30 C.		

(16) ELECTRO-MAGNETIC WAVES

Kind of waves	Wave-length in cm.	Detector
Electro-magnetic waves (used in Radio)	$2·5 \times 10^{6}$ to 5×10^{2}	Radio set
Do (in Television)	10×10^{2} to 5×10^{2}	
Short E. M. waves	40 to 0·04	,,

Kind of waves	Wave-length in cm.	Detector
Infra-red waves	0·04 to 8×10^{-5}	Heating effect
Visible light ,,	8×10^{-5} to 4×10^{-5}	Eye
Ultra-violet ,,	4×10^{-5} to $1·4\times10^{-5}$	Chemical action
X-rays	1×10^{-5} to 6×10^{-10}	Phosphorescence
Gamma Rays	$1·4\times10^{-8}$ to 1×10^{-10}	Ionisation

(17) E. M. F. OF CELLS

Bichromate ...	2	volts.	Leclanche′ ...	1·46	volts
Bunsen ...	1·94	,,	Clark at 0°C. ...	1·4491	
Daniell ...	1·09	,,	,, at 15°C. ...	1·433	
Grove ...	1·92	,,	Cadmium at 15°C....	1·0184	

(18) SPECIFIC RESISTANCES

Substances	Specific resistance in Ohms-cm. at 0°C.
Copper	$1·77\times10^{-6}$
Iron	9·8 ,,
Mercury	9·41 ,,
Silver	1·6 ,,
Tunsgten	5·5 ,,
Platinum	9—14 ,,
German Silver	20—23 ,,

(19) Electrical Units

Quantity	Practical Unit	Value in C.G.S. Units
Resistance	ohm	10^9
Current	ampere	10^{-1}
E. M. F.	volt	10^8

Quantity	Practical Unit	Value in C.G.S. Units
Quantity	coulomb	10^{-1}
Inductance	henry	10^{9}
Capacity	farad	10^{-9}
	microfarad	10^{-15}

(20) Hydrogen Atom and Electron

Mass of a Hydrogen atom ... $1{\cdot}662 \times 10^{-24}$ gm.
No. of molecules in 1 c.c. of Hydrogen at N.T.P. ... $2{\cdot}705 \times 10^{19}$
Mass of electron ... 9×10^{-28} gm.

$$= \frac{\text{mass of hydrogen atom}}{1840}$$

Charge on electron $= 4{\cdot}77 \times 10^{-10}$ *E.S.* unit.

(21) CONVERSION TABLE

To reduce	Multiply by	To reduce	Multiply by
Inch to centimetre	2·55	Cu. ft. of water to lbs.	62·4
Sq. in. to sq. cm.	6·45	Miles per hr. to ft. per min.	88
Cu. in. to cu. cm.	16·39	lbs. per sq. in. to atmospheres	0·07
Grams to grains	15·4	Grams per sq. cm. to lbs. per sq. in.	0·014
Pounds to grams	453·6	Atmospheres to lbs. per sq. in.	14·7
Ounces to grams	28·35	lbs. deg. F to ft. lbs.	772
Grains to grams	0·065	H. P. to watts.	746
Gallons of water to lbs.	10	H.P. to ft. lbs. per min.	33000
Cu. ft. to gallons	6·24		
Cu. ft. to litres	28·3		
lbs. of water to litres	0·454		

Definitions of Units

AMPERE—The practical unit of electric current. It is a current which when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0·001118 gm. per second.

The ampere = one coulomb per sec. = 10^{-1} E.M.U. = 3×10^{9} E.S.U.

Amperes = Volts/Ohms = Watts/Volts = $(\text{watts/ohms})^{\frac{1}{2}}$

Amperes × volts = amperes^{2} × ohms = watts.

ANGSTROM—Unit of wave-length = 10^{-1} metre or 10^{-8} cm.

ATMOSPHERE—Unit of pressure.

Normal pressure = 14·7 pounds per sq. inch = 760 mm. (or 30 in.) of Hg. at 0°C.

BRITISH THERMAL UNIT (B. Th. U.)—The amount of heat required to raise the temperature of one pound of water through 1°*F*.

CALORIE (or, THERM)—The amount of heat required to raise the temperature of one gram of water at 4°C. by 1°C.

COULOMB—Unit of quantity. It is the quantity of electricity transmitted per second past any point in a conductor traversed by a current of one ampere. 1 Coulomb = 10^{-1} E.M.U. = 3×10^{9} E. S. U.

Coulombs = amperes × seconds.

DAY—Mean solar day = 1440 minutes = 86400 seconds = 1·0027379 sidereal day.

DIOPTRE—Unit of 'power' of a lens. The number of dioptres = the reciprocal of the focal length in metres.

DYNE—C.G.S. (or, absolute) unit of force. It is the force which acting on a mass of 1 gram gives to it an accleration of 1 cm. per. sec. per sec.

The wt. of 1 gram = (1 × 981) dynes.

ELECTRO-CHEMICAL EQUIVALENT (E. C. E.)—It is the weight in grams deposited in an electrode by one coulomb of electricity.

ERG—C. G. S. (or, absolute) unit of work or energy = 1 dyne acting through 1 cm.

FARAD—Practical unit of electric capacity. It is the capacity of a condenser when one coulomb of electricity will raise its potential by one volt = 10^{-9} E. M. U. = 9×10^{11} E. S. U.

The MICRO-FARAD (or, one millionth part of a Farad) is more commonly used as a unit of capacity = 10^{-15} E. M. U.

FOOT-POUND—The work done in raising a mass of one pound through a vertical distance of one foot.

FOOT-POUNDAL—The British (F. P. S.) unit of work = foot-pound/g. (g = the accleration due to gravity).

GRAM-CENTIMETRE—The gravitational unit of work = g ergs.

HEAT—Absolute zero temperature = −273°C. = 491°·4F = −218°·4 R.

HEFNER-UNIT—Photometric standard used in Germany.

HORSE-POWER—The British Unit of power = 33,000 ft.-lbs. per minute = 550 ft.-lbs. per second = 746 watts.

JOULE—C. G. S. practical unit of work $= 10^7$ ergs.

JOULE'S (or MECHANICAL) EQUIVALENT—The mechanical equivalent of heat $= 4{\cdot}2 \times 10^7$ ergs per calorie $= 4{\cdot}2$ Joules per calorie.

KILODYNE—1000 dynes (About one gram).

MEGADYNE—One million dynes (About one kilogram).

MICRO—A prefix indicating the millionth (10^{-6}) part.

MILLI—A prefix denoting the thousandth (10^{-3}) part.

OHM—Unit of electric resistance. The international Ohm is the resistance offered by a column of mercury 106·3 cm. long of uniform cross-section, at 0°C., and weighing 14·452 gm. $= 10^9$ E. M. U.

PENTANE CANDLE—Photometric standard used in England.

POUNDAL—The British (F.P.S.) unit of force. It is the force which acting on a mass of one pound imparts to it an acceleration of 1 ft. per sec. per sec.

The wt. of 1 pound $= g$ poundals.

RADIAN $= 180°/\pi = 57{\cdot}29578°$.

VOLT—The practical unit of potential difference. The International Volt is defined as the electro-motive force which, when applied to a conductor having a resistance of one international ohm, will produce a current of one international ampere $= 10^8$ E.M.U. $= 1/300$ E. S. U.

Watt—Unit of electric power.

Watts = volts × amperes = $(\text{ampere})^2 \times$ ohms.

1 watt $= 10^7$ ergs per sec. = 1 joule per sec.

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